Sovereign risk and bank fragility*

Kartik Anand Jochen Mankart

Abstract

We develop a mode of bank risk-taking with strategic sovereign default. Domestic banks invest in real projects and purchase government bonds. While increased bond purchases crowd out profitable investments, they improve the government's incentives to repay, which lowers its borrowing costs. But banks' holdings of government bonds are inefficient since they do not account for how their portfolio choices influence the government's incentive to default. In particular, when the government debt level is high, banks hold too few government bonds. In such situations, introducing regulations to limit banks' holdings of domestic government bonds would be detrimental to welfare.

Keywords: sovereign debt, financial intermediation, financial repression, bank fragility.

JEL classifications: G01, G21, G28.

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1 Introduction

Overview. Over the past two decades, public debt as a share of economic output has steadily grown in both advanced and emerging economics (Adrian et al., 2024). This, coupled with challenging economic and geopolitical conditions, recently led the European Central Bank (ECB) to warn that the high debt levels can have negative financial stability effects (European Central Bank, 2024). The warning encapsulates concerns over the sovereign-bank "doom-loop" that is said to have been at the root of the 2009-2012 euro area sovereign debt crisis (Schnabel, 2021; Brunnermeier et al., 2016). According to its proponents, deterioration in a sovereign's creditworthiness reduced the market value of banks' holding domestic sovereign debt. The resulting banking distress increased their chances of being bailed out, which further impinged on sovereign's creditworthiness. This loop can be curtailed by limiting banks' holding of domestic sovereign debt (European Systemic Risk Board, 2015; Weidmann, 2013).

These arguments, however, fail to account for how banks' holdings of domestic sovereign debt shape the government's incentive to default. Insofar that the government cannot selectively default on foreign investors (Broner et al., 2010), domestic banks' holdings of sovereign bonds are of material concern to a government's decision to default or not. Accounting for this, we present a model of bank risk-taking with strategic sovereign default risk. We show that the government's incentives to repay are increasing in the banks' holdings of domestic sovereign debt. The reasoning is two-fold. First, the larger is the share of government debt held domestically, the lower is the domestic tax revenue paid out to foreign investors. So the costs to the domestic economy from the government repaying is smaller. And second, the larger is the share of bonds held by domestic banks, the larger are the disruptions and therefore the costs to the economy if the government defaults. This, in turn, disciplines the government against defaulting. Our analysis, thus, suggests that measures aimed at limiting banks' holdings of domestic sovereign debt may be detrimental to welfare. In such situations, the disciplining effects of banks' holdings of sovereign debt, which lower the interest rate the government needs to pay, are of greater social benefit than the costs of crowding-out of real investments.

Approach and main results. We consider a model of competitive domestic banks funded by insured deposits and equity and subject to limited liability that decide between purchasing domestic government bonds and investing in the real economy. Government bonds are also purchased by foreign investors. The return on investing in the real economy is subject to an aggregate shock that impacts all banks. Following the realization of the shock, the government, which is only concerned about domestic welfare, chooses to either default or repay bondholders. Importantly, we suppose that the government cannot selectively default on a particular segment of bondholders. So if the government defaults, both domestic banks and foreign investors suffer losses. Government default also engenders a dead-weight loss on the domestic economy, which further impacts the balance sheets of domestic banks. If, however, the government repays, this involves transferring resources to foreign investors, which negatively impacts domestic welfare.

The relationship between sovereign risk and bank-risk taking depends on the level of government debt. For low levels of government debt, an *asymmetric nexus* arises in which sovereign risk is lower than bank default risk. This implies that banks default in states of the world where the government defaults, but not necessarily visa-versa. In this case, banks internalise how changes to their portfolios influence their default risk, taking the price of government bonds as given. In particular, the optimal portfolio choice for a bank trades off reducing the likelihood of default by holding safe government bonds versus achieving higher but riskier returns by investing in the real economy.

For high levels of government debt, the relationship is characterised by a *symmetric nexus* wherein sovereign default and bank default are perfectly synchronised. In this case, the government's failure condition becomes the de-facto failure condition for banks. This, however, implies that banks are unable to directly influence their own default risk by changing their portfolios. So if the government prefers repaying bondholders over defaulting, then all banks have strictly positive equity values. But, if the government prefers to default over repaying, then banks' equity values are zero. Thus, bank equity value is discontinuous at the point where the government is indifferent between repaying and defaulting.

To analyse the socially optimal level of banks' holdings of government bonds, we derive the constrained efficient allocation of the social planner who maximise aggregate domestic welfare while accounting for how banks' portfolios impact on the government's default incentives. We show that the social optimum depends on the nature of the nexus. Under the symmetric nexus and when the dead-weight loss from sovereign default is high, banks hold too few government bonds. This is because banks fail to account for the social benefit that holding more government bonds increases the government's incentives to repay. This, in turn, lowers the interest rate that the government must pay to bondholders and thereby the domestic tax burden. In such situations, policies aimed at limiting banks' holdings of government bonds are welfare reducing. In contrast, policies that encourage banks to increase their holdings of government bonds improve welfare. In what follows, we refer to such policies and bank regulation constituting a form of '*financial repression*'.[1]

By contrast, under the asymmetric nexus, when the dead-weight loss of a default is low, banks over-invest in government bonds. Since the level of government debt is low, the government's incentives to default remain low. Therefore following a marginal increase in banks' holdings of government bonds, the improvement in the government's incentives to repay is smaller than the loss to domestic welfare from crowding out investment to the real economy. It is precisely in this case that policies to limit banks' holdings of domestic government bonds are welfare improving

Our results inform the current debate on regulating banks' holdings of domestic government bonds. First, we argue that "history matters", i.e., the outstanding stock of government debt is crucial for determining the nexus between banks and the sovereign. This, in turn, determines the desirability of limiting banks' exposure to sovereign debt. Second, our results also show that the size of default costs are crucial. Limiting banks' exposure is welfare improving when sovereign default costs are low. But, the contrary holds too: limiting banks' exposure is welfare reducing when sovereign default costs are high. Thus, regulations that limit banks' holdings of domestic government bonds should be accompanied by introducing measures that reduce the costs of sovereign defaults.

[1]The term dates back to the work of McKinnon (1973) and Shaw (1973) and is used to capture a range of policies that redirect private capital to governments. This is also related to the notion of *moral suasion*, i.e., informal government pressure on domestic banks to buy more domestic government bonds, which Ongena et al. (2019) argue may have contributed during the European sovereign debt crisis.

Related literature. Our paper relates to the growing theoretical literature on sovereign risk and bank risk-taking (see e.g., Ari, 2018 and Crosignani, 2021).[2] These papers find that riskier banks tend to buy more risky domestic government bonds because of their limited liability status. These papers, however, assume that sovereign risk is exogenous and non-strategic. We depart by considering how strategic sovereign default interacts with bank risk-taking.[3]

Uhlig (2013) and Farhi and Tirole (2017) consider how banking supervision can influence banks' risk-taking in the presence of sovereign default risk. Banks load up on risky domestic government bonds because of lax domestic financial supervision. We show that banks may load up on domestic government bonds when it is in their (private) interest to do so. Limited liability implies that they typically do not care about states of the world in which the government defaults since in these states they default as well. However, we also show that in some states of the world, banks hold too few government bonds. In such situations, laxer supervision than usual might be one way to get closer to the social optimum.

Our normative result on the appropriateness of financial repression stems from a pecuniary externality: banks do not internalise the effect of their portfolios on the price of sovereign bonds. In related work, Chari et al. (2020) develop a model of optimal financial repression in a closed economy. Like us, they also show that financial repression is optimal when the government faces large refinancing needs.

[2]Other theoretical contributions on on the sovereign-bank nexus include König et al. (2014), Cooper and Nikolov (2018) and Leonello (2018). While these papers focus on the role of government guarantees in propagating risks, we focus on how banks' holdings of sovereign bonds influence strategic sovereign default.

[3]Our result on the synchronicity between the bank and government default thresholds in the symmetric nexus shares a family resemblance with results in Allen et al. (2015) and Gale and Gottardi (2020) on how banks and firms align their bankruptcies. An important driver behind the similarity in the results is the segmentation of funding markets.

Since they focus on a closed economy, the benefit that lowering the interest rate on sovereign debt leads to a lower outflow of tax revenue if the government chooses to repay, is absent in their model. An important difference is that they abstract from bank risk-taking which is crucial in our model. Banks enjoy limited liability which sometimes induces them to hold too much sovereign debt. This happens when the crowding-out of real investments, and therefore future tax revenue, is relatively larger than the benefit that a lower interest rate on government bonds provides.

Our paper also contributes to the literature on the costs of sovereign default. Gennaioli et al. (2014) present a model where banks hold government bonds to store liquidity for future investments. As such, a government default dries up liquidity in the banking sector, thereby reducing credit and output. In our model, banks hold government bonds for investment purposes. We, thus, explore how bank risk-taking influences sovereign default risk. Broner et al. (2010) argue that even if a sovereign could perfectly discriminate between defaulting on foreign bondholders but not on domestic ones, the full costs of a sovereign default will be borne by domestic bondholders who buy bonds from foreign bondholders in a secondary market. In our model, default is non-discriminatory and impacts both domestic and foreign bondholders.

The remainder of the paper is organised as follows. Section 2 describes the model and Section 3 derives the equilibrium and shows the workings of the model. In Section 4 we determine the social optimum and discuss our normative results within the recent policy debates on regulating banks' holdings of sovereign debt. Section 5 concludes. All proofs are relegated to the appendix.

2 Model environment

We now present our model to explore how endogenous sovereign default risk shapes bank risk-taking. There are two dates, $t = 0$ and $t = 1$ and a single good that is used for both consumption and investment. The economy consists of 'domestic' and 'foreign' agents, all of whom care about consuming at $t = 1$. Domestic agents include distinct unit masses of risk-neutral bankers and infinitely risk-averse households. In addition, a domestic government is responsible for insuring households' deposits, repaying bond holders and providing a public good. It chooses its policies to maximise aggregate domestic welfare. Foreign agents consist of a large pool of risk-neutral investors. The only source of uncertainty is an aggregate shock, $A \geq 0$, that is realized at $t = 1$.

An important assumption in our setup is that domestic and foreign capital markets are segregated. As such, foreign investors cannot hold deposits in domestic banks and domestic bankers and households cannot invest abroad. A link between the two markets is, nevertheless, provided by the government who issues bonds to all domestic and foreign agents.

Domestic bankers. The representative domestic banker owns and operates a domestic bank. All domestic banks are identical, operate under perfect competition and enjoy limited liability. The banker is endowed with $k > 0$ at $t = 0$, which is invested as bank equity. The representative banker's utility function is $U^B = G/2 + c_1$, where $c_1 \geq 0$ is the bank equity value and $G \geq 0$ is the level of the public good provided by the government, which is shared by all domestic agents.

The bank borrows $h > 0$ from households at $t = 0$ by issuing one-period debt contracts (deposits) that carry an interest rate $r_d > 0$. The bank can invest $\ell \leq k+h$ in a project (real economy) at $t = 0$ that yields $A\ell^{\alpha}$ at $t = 1$, where $\alpha < 1$. The aggregate shock, $A \geq 0$, is a random variable drawn at the start of $t = 1$, that is common for all banks. It is distributed according to the known cumulative distribution function $F(A)$. We denote the corresponding probability distribution function by $f(A)$. The bank can also purchase $b \equiv k + h - \ell \leq 0$ worth of government bonds at $t = 0$ with a gross return of $(1 + r_g)$ at $t = 1$ if the government repays and 0 if the government defaults.[4]

The bank repays depositors in full at $t = 1$ if the returns from investing in the real economy and purchasing government bonds are sufficiently high. But, if the returns are low, the bank defaults. In this event, all of the bank's resources are transferred to the depositors and the bank's equity value is zero.

Domestic households. The representative domestic household is endowed with $d > 0$ of the consumption good. At $t = 0$, the household invests in bank deposits, domestic government bounds and a safe storage technology that yields one unit of the consumption good at $t = 1$ per unit invested. In what follows, we denote the amounts invested in deposits and government bonds by h_d and h_b, respectively, while the remainder, $d - h_d - h_b$, is placed in storage. The $t = 1$ utility function for the representative domestic household is $U^H = G/2 + \min_{\{A\}} c_1$, where $c_1 \geq 0$ are the accrued returns, which depends on the aggregate shock. Thus, households' risk-aversion only directly influences their private consumption, while the level of public

[4]We abstract from the role of sovereign debt restructuring, which would generate a positive repayment even if the government defaults. However, this would not qualitatively alter our results.

good provision by the government is taken as a given.

Domestic government. At $t = 0$, the government has a stock, $S > 0$, of legacy debt that needs to be refinanced. To this end, the government issues an infinitely divisible one-period bond with face value $S(1 + r_g)$, where r_g is the endogenous net interest rate.

At $t = 1$, the government is endowed with $T > 1$, has powers to tax households' and chooses to either default or repay bond holders. Default is non-discriminatory and so, both, foreign and domestic agents suffer losses on their bond holdings. In particular, the losses suffered by the domestic bank impair its ability to adequately manage projects, thereby reducing project returns by a fraction $\delta \leq 1$. We offer two possible explanations for this assumption. First, banks use government bonds and other liquid assets to manage credit lines for firms. Following the government default, banks are unable to service the credit lines, which hamper the real economy (Bofondi et al., 2017). And second, insofar that the losses borne by the bank following the government default reduce its charter value, this increase the scope for shirking or absconding by the banker (Keeley, 1990; Calomiris and Kahn, 1991). These actions, in turn, further reduce the value of the bank's investments.

While the cumulative losses suffered by domestic banks impinge on their abilities to repay their debts, the government insulates depositors from any losses by credibly insuring their deposits. This is achieved by encumbering a portion of the endowment, T, for deposit insurance.[5] The remainder – after covering the depositors'

[5]We, thus, argue that domestic depositors are senior claimants on the government's resources. This line of reasoning can be motivated by appealing to political economy considerations where domestic depositors might vote out an incumbent government during an election if they suffer large losses (Rosenbluth and Schaap, 2003).

losses – along with additional tax revenue raised from households, can be used to repay bond holders. Anything that is left over constitutes the public good provided by the government.[6]

Foreign investors. Foreign investors are deep-pocketed. At $t = 0$, the representative investor can either purchase government bonds or invest in the world capital market at rate $\bar{r} > 0$.

Timing. At $t = 0$, the government issues bonds; domestic banks issue deposits to households and invest in projects and government bonds; domestic households invest in bank deposits, government bonds and their storage technology; foreign investors invest in government bonds and world capital markets. At $t = 1$, the aggregate shock, A, is realised; the government chooses whether to repay or default on its debts; banks either repay households in full or default and are protected by limited liability; the government provides the public good; domestic bankers, domestic households, and foreign investors consume.

3 Equilibrium

We solve the model by backward induction.

Definition 1. *The symmetric pure-strategy sub-game perfect equilibrium comprises of: (i) the representative bank's allocation between government bonds and real project,*

[6] This model environment allows us to side-step the issue of pricing of deposits as we show in Appendix A. While such an exercise could be done, for example, along the lines of Carletti et al. (2020), this would greatly complicate the model and is not central to our analysis.

$\{b^*, \ell^*\}$, the interest rate on deposits, r_d^*, and a critical bank default threshold, $\widehat{A}_B^*$; (ii) the representative household's allocation between bank deposits, government bonds, and storage, $\{h_d^*, h_b^*, d - h_d^* - h_b^*\}$, and (iii) the interest rate that the government must pay to roll over its debt, r_g^*, and a critical sovereign default threshold, $\widehat{A}_S^*$, such that

1. *At $t = 1$, the government repays whenever $A \geq \widehat{A}_S^*$, given the bank's and households' allocations and interest rates on its bonds and bank deposits.*

2. *At $t = 1$, the bank repays whenever $A \geq \widehat{A}_B^*$, given the government decision, the bank's and households' allocations and the interest rates.*

3. *At $t = 0$, the bank invests b^* and ℓ^* in government bonds and the real economy, respectively, to maximize its expected equity value, given its own and the government's default thresholds and interest rates.*

4. *At $t = 0$, households allocate h_d^* in deposits, h_b^* in government bonds and $d - h_d^* - h_b^*$ in storage, given the bank's and government's default thresholds and interest rates.*

5. *At $t = 0$, the return on government bonds, r_g^*, renders the marginal foreign investor indifferent between purchasing government bonds and investing in world capital markets, and the bank sets the interest rate, r_d^* to maximize its expected equity value.*

Without loss of generality, we suppose that in the presence of deposit insurance, households invest entirely in bank deposits, $h_d^* = d$ and that the interest of deposits is $r_d^* = 0$. This is tantamount to assuming that the bank chooses the interest rate on deposits to render households indifferent between lending to the bank and investing in their storage technology. Moreover, due to the ever-present risk of the government defaulting, it is never optimal for households to hold government bonds. A detailed micro-foundation for this result can be found in Appendix A.

3.1 Some simplified benchmarks with exogenous sovereign risk

Before solving the full model, it is instructive to consider some simple benchmarks where key model features are absent and with exogenous sovereign risk. This, in turn, allows us to better appreciate the roles played by the different frictions.

Banks' allocations without deposit insurance. Suppose that the government does not guarantee bank deposits. In that case, infinitely risk-averse households will not hold deposits or government bonds. Instead, they invest entirely in the storage technology. Thus, the bank can invest only up to its initial endowment of capital. Denoting the government's (exogenous) failure threshold by $\widehat{A}_S$, the bank's optimal investment is given by

$$\ell^{AUT} = \arg\max_{\ell} \int_0^{\widehat{A}_S} (1-\delta) A\, \ell^{\alpha} dF(A) + \int_{\widehat{A}_S}^{\infty} \big[A\, \ell^{\alpha} + b(1+r_g)\big] dF(A)\,,$$

subject to the balance sheet constraint $b = k - \ell$. Foreign investors are the marginal buyers of government bonds. Consequently, the interest rate, r_g, is obtained from their binding participation constraint, i.e.,

$$\big(1 - F(\widehat{A}_S)\big)(1 + r_g) = 1 + \bar{r}\,. \tag{1}$$

The first-order condition for the bank's investment is given by

$$\alpha\ell^{\alpha-1}\left((1-\delta)\int_0^{\widehat{A}_S} AdF(A)+\int_{\widehat{A}_S}^{\infty} AdF(A)\right)=1+r_g\,.$$

The bank invests up to the point that the marginal project return is equal to the expected return from holding government bonds. Inserting the participation constraint, we obtain that the bank's optimal investment is given by

$$\ell^{AUT}=\left(\frac{\alpha\left[(1-\delta)\int_0^{\widehat{A}_S} AdF(A)+\int_{\widehat{A}_S}^{\infty} AdF(A)\right]}{1+\bar{r}}\right)^{\frac{1}{1-\alpha}}\,. \tag{2}$$

Banks' allocations with deposit insurance. Next, suppose the government introduces deposit insurance. In this case, households deposit their entire endowments with the bank and the net return on deposits is zero. We obtain that, conditional on the government repaying all bond holders, the bank defaults due to limited liability whenever $A<\widehat{A}_B\equiv\frac{d-(1+r_g)b}{\ell^{\alpha}}$. Insofar that $\widehat{A}_B>\widehat{A}_S$, i.e., the bank is able to repay depositors whenever the government repays bondholders, but not visa-versa, the bank's optimal investment is given by

$$\ell^{*}=\arg\max_{\ell}\int_{\widehat{A}_B}^{\infty}\left[A\ell^{\alpha}+b(1+r_g)-d\right]dF(A)\,,$$

subject to the balance sheet condition, $b=d+k-\ell$ and the pricing of government bonds. We obtain

$$\ell^{*}=\left(\frac{1-F(\widehat{A}_S)}{1-F(\widehat{A}_B)}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha\int_{\widehat{A}_B}^{\infty} AdF(A)}{1+\bar{r}}\right)^{\frac{1}{1-\alpha}}\,, \tag{3}$$

which is an implicit function for ℓ^* since $\widehat{A}_B \equiv \widehat{A}_B(\ell^*)$. From the bank's perspective, government bonds are safe, i.e., as long as the bank is able to repay depositors, the government is always able to repay bondholders and there is no sovereign default risk to contend with. Hence the only risk for the bank stems from the aggregate shock, A, to its investments. Thus, following standard lines of reasoning, the bank exploits its deposit insurance subsidy by risk-shifting and increasing its investments, i.e., $\ell^* > \ell^{AUT}$.

An increase in sovereign risk reduces the bank's investment. This result stems from how the return on government bonds, r_g, is shaped by sovereign risk. From Equation (1), we readily note that $\frac{\partial r_g}{\partial \widehat{A}_S} > 0$, i.e., the return on government bonds is increasing with sovereign risk. This is a direct consequence of the risk-neutrality of foreign investors. And so as the return on government bonds increases, the bank shifts its portfolio allocation towards holding more government bonds and investing less in the economy.

This result is consistent with the empirical observations that during the European sovereign debt crisis, banks increased their holdings of government bonds of stressed countries (Acharya and Steffen, 2015). While our analysis so far suggests that such behavior was privately optimal for banks, the question remains as to whether it was socially optimal? To answer this, we must understand how the increase in banks' holdings of domestic sovereign bonds influenced the government's incentives to default or not. In other words, sovereign risk must be endogenous, which is what we consider next.

3.2 Competitive equilibrium

Government default. Following the realisation of the aggregate shock, A, at $t = 1$, suppose that the government chooses to repay bond holders. The equity value of the representative bank is given by $\bar{e} \equiv \max\{0, A\,\ell^{\alpha} + (1+r_g)\,b - d\}$, and so the bank defaults whenever $A < \widehat{A}_B \equiv \bar{A} = \frac{d-(1+r_g)b}{\ell^{\alpha}}$. Thus, after covering losses suffered by depositors, the government's remaining revenue is $\overline{R} \equiv T - \max\{0, d - A\,\ell^{\alpha} - (1+r_g)\,b\}$.

If $\overline{R} \geq S(1+r_g)$, then the government pays bond holders using the revenue and provides $\overline{G} = \overline{R} - S(1+r_g)$ of the public good. The representative banker and household obtain utilities $U^B = \overline{G}/2 + \bar{e}$ and $U^H = \overline{G}/2 + d$, respectively. Alternatively, if $\overline{R} < S(1+r_g)$, then the government taxes households at the rate $\tau = \frac{S(1+r_g)-\overline{R}}{d}$ and pays bond holders using the combined revenue and taxes. Moreover, the government is unable to provide the public good. The utilities of the representative banker and household are $U^B = \bar{e}$ and $U^H = d(1-\tau) = d - \big(S(1+r_g) - \overline{R}\big)$. Irrespective of how the repayment of bond holders is financed, we obtain that aggregate utility of domestic bankers and households is given by

$$V^R(A) \equiv T + A\ell^{\alpha} - \big(S - b\big)(1+r_g)\,. \tag{4}$$

Suppose, instead, that the government decides to default on bond holders. In this case, the bank's equity value is $\widetilde{e} = \max\{0, (1-\delta)\,A\,\ell^{\alpha} - d\}$ and the bank defaults whenever $A < \widehat{A}_B \equiv \widetilde{A} = \frac{d}{(1-\delta)\ell^{\alpha}}$. Since the government default leads to losses on both bonds purchased and investments, the bank is more likely to fail whenever the government defaults. This implies an ordering of the two bank default thresholds whereby $\bar{A} < \widetilde{A}$. As we subsequently show, the relationship between these thresholds and that for the government play an important role in determining the equilibrium.

Government revenue, after paying deposit insurance, is given by $\widetilde{R} = T - \max\{0, d - (1 - \delta) A \ell^{\alpha}\}$. Since the government has no further obligations, this amount is used in its entirety to provide $\widetilde{G} = \widetilde{R}$ worth of the public good. The utilities of the representative banker and household are $U^B = \widetilde{G}/2 + \widetilde{e}$ an $U^H = \widetilde{G}/2 + d$, respectively. Aggregate utility of domestic bankers and households is

$$V^D(A) \equiv T + (1 - \delta) A \ell^{\alpha} . \tag{5}$$

Comparing the levels of aggregate domestic utility between defaulting and repaying, the government repays whenever

$$A \geq \widehat{A}_S \equiv \frac{(S - b)(1 + r_g)}{\delta \ell^{\alpha}} . \tag{6}$$

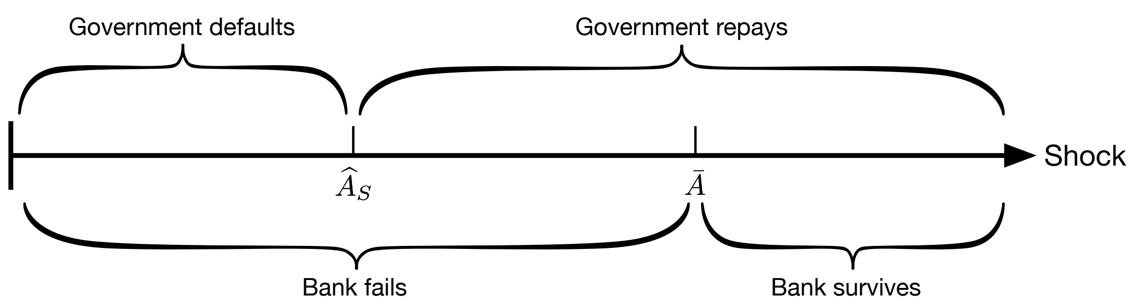
By choosing to repay, the government splits $S(1 + r_g)$ worth of domestic resources proportionally between domestic banks and foreign investors based on their holdings of government bonds. As the amount that accrues to the foreign investors, i.e., the numerator in Equation (6), increases, aggregate domestic domestic utility is reduced.

By defaulting, however, the government does not need to raise taxes to repay foreign investors and domestic banks. But banks suffer further losses due to the deadweight loss induced by the government defaulting. These losses to banks' investments are captured by the denominator in Equation (6). Thus, the government repays bond holders whenever the reduction to aggregate domestic welfare from having to pay foreign investors is smaller than the dead-weight losses if the government defaults.

Next, we solve for the representative bank's portfolio allocation and determine the interest rate on government bonds. We treat each in turn.

Bank's optimal portfolio. At $t = 0$, the representative bank chooses how much to invest in the real economy and how many government bonds to purchase. Crucially, the bank is a price taker in the market for government bonds and therefore does not internalise how changes in its bond holdings influences the government's default incentives. Nevertheless, sovereign default risk shapes the bank's incentives via the position of the government's default threshold, $\widehat{A}_S$, relative to those for the bank. We distinguish between two cases.

Figure 1: Asymmetric nexus.



This figure shows the case of an asymmetric nexus, where the bank always fails when the government defaults but not vice versa.

Case 1. Asymmetric nexus $\left(\widehat{A}_S < \bar{A} < \widetilde{A}\right)$**.** If the government defaults, $A < \widehat{A}_S$, then the bank also defaults since $\widehat{A}_S < \bar{A}$. But, if the government repays, $A \geq \widehat{A}_S$, then the bank is able to repay depositors in full and retain a positive equity value as long as $A \geq \bar{A}$. Thus, if the aggregate shock lies in the interval $(\widehat{A}_S, \bar{A})$, then the bank defaults even though the government repays all bond holders. And so, since $\widehat{A}_S < \bar{A}$, ex-ante bank default risk is greater than that for the government. Figure 1 depicts the classification of default thresholds under the asymmetric nexus. Consequently, the bank's portfolio problem is

$$\max_{\ell,b} \quad \int_0^\infty \bar{e}(A)\, dF(A) = \int_{\bar{A}(\ell,b)}^\infty \Big(A\,\ell^\alpha \; + \; (1+r_g)\, b \; - \; d \Big)\, dF(A)\,,$$

subject to the balance sheet constraint $\ell + b = d + k$.

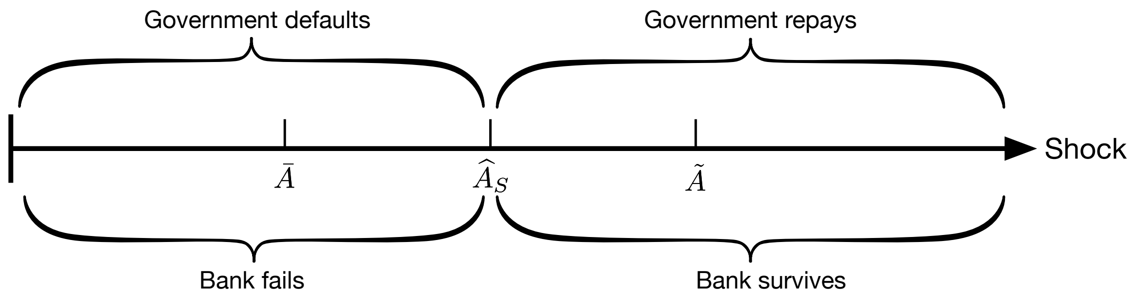
Case 2. Symmetric nexus $\big(\bar{A} < \widehat{A}_S < \widetilde{A}\big)$**.** As in the asymmetric case, if the government defaults, $A < \widehat{A}_S$, then the bank also defaults because $\widehat{A}_S < \widetilde{A}$. But, whenever the government repays, $A \geq \widehat{A}_S$, it follows that the bank has a strictly positive equity value and repays households since $\bar{A} < \widehat{A}_S$. Figure 2 depicts the default thresholds under the symmetric nexus where bank and government default are now perfectly synchronised. In its optimisation problem, the bank, effectively, replaces its own default threshold with that of the government and the bank's portfolio problem is

$$\max_{\ell,b} \quad \int_0^\infty \mathbb{1}_{A>\widehat{A}_S}\, \bar{e}(A)\, dF(A) = \int_{\widehat{A}_S}^\infty \Big(A\,\ell^\alpha \; + \; (1+r_g)\, b \; - \; d \Big)\, dF(A)\,,$$

subject to the balance sheet constraint.

Figure 3 plots bank equity value under the two cases. For the asymmetric nexus, the equity value is convex in the aggregate shock wherein the limited liability constraints binds for $A < \bar{A}(\ell, b)$. As such, a small change in the aggregate shock always leads to small changes in bank equity value. Moreover, by changing its investment decision, the bank can shift its failure threshold. Thus, in equilibrium, the bank's optimal investment choice trades-off attaining higher returns versus reducing fragility.

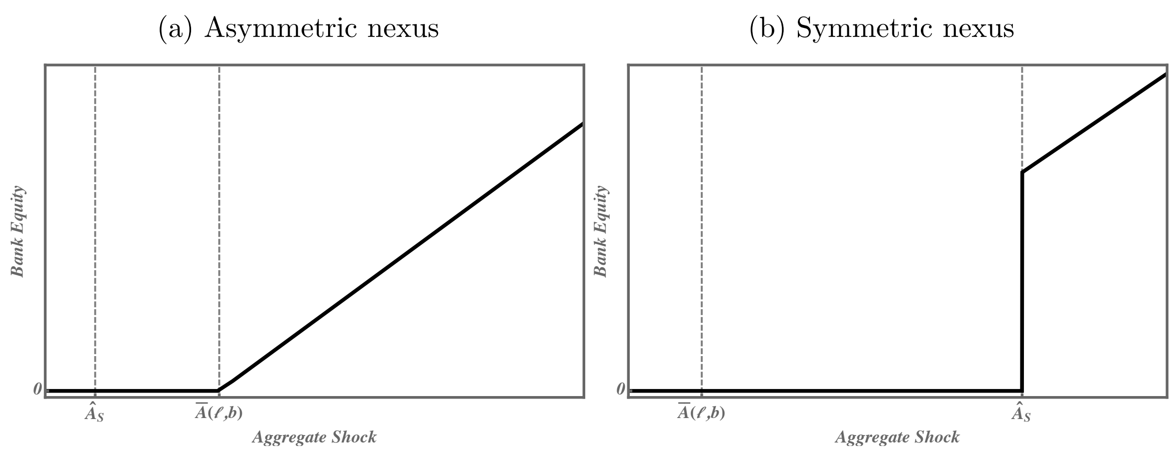
Figure 2: Symmetric nexus.



This figure shows the case of a symmetric nexus, where bank and government always fail at the same time.

In the symmetric nexus case, however, equity value is strictly positive for $A \geq \widehat{A}_S$ and zero otherwise. There is a discontinuous jump at the government's default threshold, which is the de facto failure threshold for the bank. As such, the bank is unable to influence its failure threshold via its investment decision.

Figure 3: Bank equity value under the asymmetric nexus and symmetric nexus.



For the asymmetric (symmetric) nexus case, we have $S = 0.35$ ($S = 0.7$). All other parameters are the same in both cases: $d = 0.5$, $k = 0.5$, $\delta = 0.9$, $\alpha = 0.4$ and $\bar{r} = 0$. The aggregate shock follows an exponential distribution with hazard rate $\lambda = 0.2$.

In principle, there is also a third case to consider where bank and government default are asynchronous and the ordering of thresholds satisfies $\bar{A} < \tilde{A} < \widehat{A}_S$. If the government repays, $A \geq \widehat{A}_S$, then the bank would always repay since $A \geq \bar{A}$. But, when the government default, $A < \widehat{A}_S$, there are two possibilities depending on the size of the shock. If $A < \tilde{A}$, then the bank would also default. But, if $A > \tilde{A}$, then the bank would repay, Moreover, its equity value would jump from $\delta A\ell^\alpha - d$ to $A\ell^\alpha + (1 + r_g)b - d$ at the government's default threshold, $\widehat{A}_S$. However, as we shall argue, this government default threshold is associated with a far too high interest rate charged by foreign investors that the debt is never re-financed at $t = 0$ and there is market breakdown. Thus, this case is not material in equilibrium.

Interest rate on government bonds. Focusing on equilibria where foreign investors are marginal buyers of government bonds, the interest rate, r_g, is determined according to their binding participation constraint, i.e.,

$$\big(1 - F(\widehat{A}_S)\big)\big(1 + r_g\big) = 1 + \bar{r}\,. \tag{7}$$

To characterise the equilibrium, we make the following two assumptions.

Assumption 1. *The hazard rate of the aggregate shock distribution λ is constant such that the government's propensity to default is relatively large, i.e $\lambda > \widehat{\lambda}$ where the threshold is formally defined in Appendix B.*

With a constant hazard rate, we are better able to isolate how changes in the bank's portfolio influence the government's default incentives, and how this translates into the pricing of government bonds.[7] And assuming a lower bound for the

[7]If the hazard rate is not constant, then a marginal change in the bank's portfolio that influence's the government's incentives to repay also induces a marginal change in the hazard rate. Insofar that the hazard rate is increasing – as is the case for a Normal distribution as well as for a Log-normal

hazard rate ensures that, once government debt grows beyond a level that sustains either the asymmetric or symmetric nexus, foreign investors charge exorbitantly high interest rates, which the government can never hope to repay. And so there is market breakdown.

Assumption 2. *The bank is awash in funding, i.e.,* $S < d + k$.

This ensures that an increase in the bank's holdings of government bonds reduces the government's incentives to default, i.e., $\frac{\partial \widehat{A}_S}{\partial \ell} > 0$.[8] Moreover, the impact that the bank's holdings of government bonds has on the government's default incentives remain robust to the introduction of a domestic non-bank financial sector (e.g., pension and insurance sector) that also holds government bonds. For example, if this sector holds a stock $N > 0$ of government bonds, the government's default threshold is given by $\widehat{A}_S = \frac{(S-N-b)(1+r_g)}{\delta \ell^{\alpha}}$. Accounting for the bank's balance sheet, $\frac{\partial \widehat{A}_S}{\partial \ell} > 0$ for all $N \geq 0$.

It is well established that in such models, where governments lack the ability to commit on a policy of always repaying bond holders, multiple equilibria arise and are driven by investors' beliefs (Calvo, 1988). If investors believe that the government will repay, the required return on bonds is low, which the government can readily service, reducing the incentives to default. While, if investors believe that the government will default, then the required return is high, which makes it more likely that the

distribution with a non-negative mean – this effect exacerbates the original incentive effect without qualitatively altering the mechanism.

[8]To ensure that government bonds continue to be priced at the margin by foreign investors, we can resort to an assumption of homogeneous mixing of bankers and foreign investors in the market for government bonds. Since there is a much larger mass of foreign investors than domestic bankers, the probability that the marginal investor purchasing the last infinitesimal amount of government bonds is a foreign investor, after the remainder has been bought up by domestic bankers, will be unity, almost surely.

government will default. The equilibrium where investors believe that the government will repay is Pareto efficient and the focus of our analysis. Proposition 1 describes the resulting equilibrium allocations.

Proposition 1. *There exist unique bounds, $\underline{S}$, and $\bar{S}$ on the level of government debt, where $\underline{S} < \bar{S}$, such that:*

- *For $S \leq \underline{S}$ the equilibrium is characterised by the asymmetric nexus where the bank's investment in real projects is given by*

$$\ell^* = \left(\frac{1 - F(\widehat{A}_S)}{1 - F(\bar{A}(\ell^*))} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha \int_{\bar{A}(\ell^*)}^{\infty} A \, \mathrm{d}F(A)}{1 + \bar{r}} \right)^{\frac{1}{1-\alpha}} . \tag{8}$$

Purchases of government bonds is given by $b^* = k + d - \ell$ and the sovereign's default threshold is implicitly defined by $\tau(\widehat{A}_S^*) = 0$, where

$$\tau(\widehat{A}_S) \equiv \widehat{A}_S - \frac{S - b}{\delta \ell^\alpha} \left(\frac{1 + \bar{r}}{1 - F(\widehat{A}_S^*)} \right) . \tag{9}$$

- *For $\underline{S} < S \leq \bar{S}$, the equilibrium is characterised by the symmetric nexus where the bank's investment in real projects is given by*

$$\ell^* = \left(\frac{\alpha \int_{\widehat{A}_S}^{\infty} A \, \mathrm{d}F(A)}{1 + \bar{r}} \right)^{\frac{1}{1-\alpha}} . \tag{10}$$

Government bond purchases and the sovereign's default threshold are given by $b^* = d + k - \ell$ and $\tau(\widehat{A}_S^*) = 0$, respectively.

- *Finally, for $S > \bar{S}$ there is no equilibrium.*

Proposition 1 shows how the relationship between bank risk-taking and sovereign default risk is shaped by the level of government debt. When this stock is low, then the tax burden on the domestic economy, if the government repays, is also low. This implies a low risk of a sovereign default and therefore a low interest rate r_g.

The bank, which is subject to limited liability, still has an incentive to 'gamble' – formally captured by the conditional expectation term in Equation (8) – and holds a relatively risky portfolio. Since the bank's likelihood to default is greater than the government's, we have $\widehat{A}_S < \bar{A}$ in the asymmetric nexus. Importantly, since the government always repays in states of the world where the bank survives, the bank perceives government bonds as 'risk-free' investments.

As the stock of government debt increases, so too does the risk of sovereign default. At the same time, the likelihood that the bank fails, conditional on the government repaying, remains relatively unchanged. For a sufficiently large stock of debt, we obtain that $\bar{A} < \widehat{A}_S$, and so bank and government default are perfectly synchronised around $\widehat{A}_S$ in the symmetric nexus regime. Again, since the government always repays in states of the world where the bank survives, government bonds are viewed as safe investments by the bank.

Under both the asymmetric and symmetric nexus, the bank ignores states of the world where the government defaults. The reason for this is that the bank always defaults in those states as well and is protected against further losses by limited liability. Sovereign default risks matter only indirectly through their effect on the equilibrium rate of return on bonds. This result will be important in the discussion below.

Finally, if the stock of debt is very high, then the rational expectations equilibrium does not exist. Such a situation can be interpreted as a market breakdown where the government always defaults for sure and the interest rate it is charged is infinitely large.

3.3 Comparative statics

Next, we show how the Pareto efficient equilibrium outcomes for bank's investment, sovereign default risk and bank default risk change with changes in bank capital κ, the stock of government debt S, and the refinancing cost $\bar{r}$.

Proposition 2. *Bank investment, ℓ^*, is increasing in capital, k under the symmetric nexus. In the asymmetric nexus, however, the effect is ambiguous.*

Mechanism. Under the asymmetric nexus, the effect from an increase in bank capital can be decomposed into a direct effect on the bank's profits, subject to limited liability, and an indirect – general equilibrium – effect on the government's incentives to default. Accordingly, the direct effect of having more capital is that the bank is better able to withstand adverse shocks and retain positive equity value. But, since the bank has more 'skin in the game' it seeks to reduce the riskiness of its portfolio. To this end, the bank increases its holdings of government bonds, which the bank views as risk-free since sovereign default only occurs for realisations of the shock where the bank fails as well.

The indirect effect from having more capital is a reduction in the extent to which investment is crowded out when the bank purchases government bonds. This improves the government's incentives to repay, which reduces the return that the bank earns on government bonds. The better capitalised bank responds, in turn, by reducing its holdings of government bonds, which counteracts the direct effect leading to an ambiguous total effect.

For the symmetric nexus, by contrast, the direct effect on the bank's profits is not present since the bank adopts the government's default threshold as its own and cannot influence this via its portfolio choice. Only the indirect effect via the government's default incentives is present, implying that following an increase in its capital, the bank reduces its holdings of government bonds and increases its investments instead.

Proposition 3. *Bank investment, ℓ^*, is decreasing in both stock of government debt, S, and the refinancing cost, $\bar{r}$, under both the asymmetric and symmetric nexus.*

Mechanism. Under both the asymmetric nexus and symmetric nexus, the amount of government debt, S, does not directly impact the bank's incentives to invest or hold government bonds. Instead, the increase in S implies a higher tax burden if the government repays. This, in turn, reduces the government's incentives to repay, which leads to an increase in the interest rate, r_g^*, required by bond holders to refinance the government's debt. This indirect equilibrium effect leads to the bank rebalancing its portfolio towards holding more government bonds. Thus, this result generalizes the partial equilibrium result obtained in our simple benchmark with exogenous sovereign risk.

While an increase in $\bar{r}$ also induces a similar indirect effect, there is also the direct effect of increasing the opportunity cost of investing in projects. This reduces the bank's incentive from investing in favour of holding more government bonds.

In sum, both the direct and indirect reinforce each other leading to a decline in investment.

Proposition 4. *The ex ante probability of government default,* $F(\widehat{A}_S^*)$*, is decreasing in bank capital,* k*, and increasing in the stock of debt to refinance,* S*, and the refinancing cost,* $\bar{r}$*.*

Mechanism. As bank capital increases, there is less crowding out of investment as the bank purchases government bonds. This improves the government's incentives to repay and reduces the interest rate, r_g^*. But this leads to a countervailing equilibrium effect, whereby the yield on government bonds is reduced. This weakens the bank's incentives to hold them. Thus, the increase in bank capital substitutes for the commitment effect that bank holdings of government bonds provide for the government.

The effects from an increase in either S or $\bar{r}$ can be similarly decomposed. First, an increase in either variable weakens the government's incentives to repay, which increase the interest rate, r_g^*. But, insofar that the bank reallocates its portfolio towards holding more government bonds, this will improve the government's incentives to repay, which is a countervailing effect on r_g^*.

Corollary 1. *The ex ante probability of bank default is decreasing in bank capital,* k*, but is increasing in the stock of debt,* S*, and the refinancing cost,* $\bar{r}$*, under the symmetric nexus. The effects for the asymmetric nexus are ambiguous.*

The results for the symmetric nexus follow directly from Proposition 4, where the bank adopts the sovereign's default threshold as its own. Thus, our results on sovereign risk-premia follow through to describe bank risk-premia, and how these are driven by macro and fiscal factors.

For the asymmetric nexus, however, the comparative static exercises on the bank's default threshold are ambiguous. As an illustration, consider the effect of an increase in bank capital on the bank's failure threshold. This can be decomposed into three effects: (i) a direct effect, (ii) an indirect effect via the bank's investment choice and (iii) an indirect effect via the sovereign's default threshold. The direct effect of an increase in bank capital is for the bank default threshold to decrease, thereby reducing the incidence of bank default.

But, at the same time, since an increase in bank capital also reduces sovereign default risk, the yield on government bonds is reduced, which reduces the net return that the bank earns. This increases the likelihood of bank default. Finally, as bank capital increases, the bank reduces its investments and favours holding more government bonds in the asymmetric nexus. This, in turn, also increases the likelihood of bank default. In sum, while the direct effect of an increase in bank capital is to reduce the likelihood of bank default, the indirect effects increase this likelihood instead.

4 When is financial repression socially optimal?

In our analysis thus far, banks failed to internalise how their holdings of government bonds influenced the government's decision to repay and the return on government bonds. In this section, we derive the portfolio allocation chosen by a social planner who maximises expected aggregate domestic utility but still has to abide by the participation constraint of foreign investors.

By increasing banks' holdings of government bonds, the planner trades off increasing the government's incentives to repay versus the crowding-out of real investments. We subsequently show that the welfare effects of financial repression, i.e., formally requiring banks to hold more bonds than they would voluntary choose, depend on the cost of default and the type of the nexus.

4.1 Planner's problem

The planner seeks to maximise aggregate domestic utility of bankers and households subject to the government's commitment friction to repay. Our welfare benchmark is constrained efficiency, and the planner's problem is

$$\max_{b,\ell,r_g,\widehat{A}_S} \quad \int_0^{\widehat{A}_S} V^D(A)\mathrm{d}F(A) + \int_{\widehat{A}_S}^{\infty} V^R(A)\mathrm{d}F(A) \tag{11}$$

subject to

$$\ell + b = d + k$$

$$1 + \bar{r} - \big(1 + r_g\big)\big(1 - F(\widehat{A}_S)\big) = 0$$

$$\frac{\big(S - b\big)\big(1 + r_g\big)}{\delta \ell^{\alpha}} - \widehat{A}_S = 0$$

where $V^R(A)$ and $V^D(A)$ are aggregate domestic utility if the government repays and defaults and are defined by Equation (4) and Equation (5), respectively. The optimisation is subject to three constraints. The first is the balance sheet identity for banks. The second is the participation constraint for foreign investors, from which we determine the price of government bonds. The third constraint defines the government default threshold as a function of banks' portfolio choices.

Proposition 5. *The planner's choice for the optimal level of investment is given by*

$$\frac{\alpha(\ell^{SP})^{\alpha-1}}{1-F(\widehat{A}_S^{SP})}\left[\left(1-\delta\right)\int_0^{\widehat{A}_S^{SP}} A\,dF(A)+\int_{\widehat{A}_S^{SP}}^{\infty} A\,dF(A)\right]$$
$$-\left[S-\left(d+k-\ell^{SP}\right)\right]\times\left.\frac{\partial r_g^*}{\partial \ell}\right|_{\ell^{SP},\widehat{A}_S^{SP}}=\frac{1+\bar{r}}{1-F(\widehat{A}_S^{SP})}, \tag{12}$$

where the sovereign default threshold is given by

$$\widehat{A}_S^{SP}=\frac{\left(S\;-\;\left(d+k-\ell^{SP}\right)\right)}{\delta\left(\ell^{SP}\right)^{\alpha}}\left(\frac{1+\bar{r}}{1-F(\widehat{A}_S^{SP})}\right),$$

and the interest rate on government bonds, r_g^*, is derived from the foreign investors' binding participation constraint.

Compared with the allocation chosen by the representative bank, we note two striking differences. First, the planner also cares about aggregate domestic utility in states of the world where the government defaults. The bank, in contrast, ignores outcomes in these states. This is because the bank is protected by limited liability and the government only defaults in states where the bank also defaults. And second, the planner accounts for how changes in the bank's investment influences the interest rate charged on government bonds, and thereby the tax revenue transferred to foreign investors.

Proposition 6. *There exist two bounds for the cost of sovereign default on the bank's investment, $\bar{\delta}$ and $\underline{\delta}$, where $\bar{\delta} > \underline{\delta}$ such that:*

	Asymmetric Nexus ($S \leq \underline{S}$)	*Symmetric Nexus* ($\underline{S} < S \leq \bar{S}$)
$\delta < \underline{\delta}$	$\ell^{SP} > \ell^*$	$\ell^{SP} > \ell^*$
$\delta \in (\underline{\delta}, \bar{\delta})$	$\ell^{SP} < \ell^*$	$\ell^{SP} > \ell^*$
$\delta > \bar{\delta}$	$\ell^{SP} < \ell^*$	$\ell^{SP} < \ell^*$

The optimality of financial repression depends on the economic losses resulting from a sovereign default and the type of the nexus. In general, the benefit from banks holding more government bonds is to improve the incentives of the government to repay, which reduces the interest rate on government bonds and the tax burden on the domestic economy, insofar that the government chooses to repay. The cost from the bank holding more government bonds is the crowding-out of domestic investment and, therefore, output at $t = 1$.

Proposition 6 shows that if the real cost of a sovereign default is large, $\delta > \bar{\delta}$, the bank holds too few government bonds in the competitive equilibrium, relative to the planner's allocation. Since the bank does not directly internalize the relatively high cost of default, it invests too much in the real economy. This exacerbates the potential costs from a default. At time same time, however, the costs borne by the government from repaying are higher since more of its debt is held by foreign investors. By forcing the bank to hold more government bonds, the planner continues to maintain incentives for the government to repay, while reducing the net costs from doing so.

If, however, the real cost of a sovereign default is low, $\delta < \underline{\delta}$, avoiding default becomes relatively less important. In the competitive equilibrium banks hold too many government bonds. They do not internalise that their investment choice crowds-out too much real investment which in turn leads to a lower tax base in the next period. The planner, in contrast, chooses an allocation where banks hold less government bonds than in the competitive equilibrium.

For intermediate values, $\delta \in [\underline{\delta}, \bar{\delta}]$, the social planner engages in financial repression only in the asymmetric nexus. In this regime, limited liability and risk-taking incentives influence banks' portfolio choices and, thus, their default risk.[9] In particular, banks reduce their holdings of safe government bonds and increase their level of investment, which is risky. But such risk-taking by banks leads to foreign investors holding too much sovereign debt, which weakens the government's incentives to repay. To remedy this, the planner requires banks to reduce investments and hold more government bonds.

Our analysis does not directly address how such financial repression, which in our case, could also mean to force banks to hold less bonds, may be implemented in practice. However, there are several tools, some already existing, that could be used. One could have differential risk weights in the bank capital framework. One could also use the tax system to either tax bonds more or less than real investment projects. Another option would be to impose explicit limits and restrictions on banks purchases of government bonds if the government wanted to reduce banks' holdings of its bonds. If it wanted to increase it, it could, for example, increase liquidity requirements, which typically require banks to hold more domestic sovereign debt.

[9] In contrast, in the symmetric nexus, banks cannot influence their default risk by altering their portfolios.

4.2 Implications for recent policy proposals

The policy debate surrounding the European sovereign debt crisis and its aftermath has largely focused on the pernicious role risk-taking by domestic banks in the affected countries that lead them to increase their holdings of domestic sovereign bonds so as to benefit from bailout and deposit insurance subsidies (Brunnermeier et al., 2016). This, in turn, has lead to calls for rules that limit banks' holdings of domestic sovereign debt. One such proposal, for example, envisions introducing an upper bound on the ratio between a bank's holdings of domestic sovereign debt and the bank's capital (European Systemic Risk Board, 2015).[10]

Our analysis suggests that, accounting for endogenous decision of a government to default or repay bondholders, bank risk-taking can be virtuous, especially when the costs of a sovereign default are high. The key insight here is that bank risk-taking improves the government's incentives to repay. This, in turn, reduces the price the government has to pay to refinance its debts and thus the tax burden on the domestic economy. In sum, risk-taking by banks can improve domestic aggregate welfare.

In practice, the real economic cost of a sovereign default can be linked to whether the default proceeded orderly or disorderly (Asonuma et al., 2015). When sovereign default is disorderly, then the costs borne by the domestic economy tend to be high.[11] As Proposition 6 suggests, it is precisely in such circumstances, i.e., $\delta > \bar{\delta}$, that risk-

[10] While such a 'large exposure limit' already exists for other bank assets, sovereign exposures are currently exempt under the Basel III regulation. A related proposal suggests introducing risk-weights for banks' sovereign debt exposures in calculating capital requirements (Basel Committee on Banking Supervision, 2017). Finally, a recent market-based approach proposal suggests establishing special financial vehicles to buy up sovereign debt from euro area banks to be used for securitisation (European Commission, 2018).

[11] Hebert and Schreger (2017) estimate that between January 2011 and July 2014, when Argentina defaulted on bond holders who had previously accepted to restructure their debt, the value of Argentine firms reduced by about 30%.

taking by banks to increase their holdings of government bonds improves welfare. And so limiting banks' holdings of domestic sovereign bonds, as the recent proposals suggest, would result in higher interest rates being charged on government bonds and would, thus, be detrimental to the domestic economy.

If, however, sovereign default was orderly, for example, facilitated by a sovereign debt restructuring mechanism (e.g., Krueger, 2002; Brookings Committee on International Economic Policy and Reform (CIEPR), 2013 and Deutsche Bundesbank, 2016), then the costs to the real economy would be more mute.[12] In such circumstances, i.e., $\delta < \underline{\delta}$, bank risk-taking is detrimental since the costs from crowding out of investments in the real economy are too high. Limiting banks' holdings of domestic sovereign bonds in this case would actually improve welfare.

Our model, thus, suggests that the design of policy to regulate banks' holdings of domestic government bonds must take into account the cost of a sovereign default and the type of the nexus between sovereign risk and banking risk-taking, which in turn depends crucially on the level of debt.

5 Conclusion

We have developed a model of bank risk-taking with strategic sovereign default risk. Domestic banks can either invest in real projects or purchase government bonds. While an increase in purchases of government bonds crowds out profitable investment, it nevertheless improves the government's incentives to repay and therefore reduces

[12] Asonuma and Trebesch (2016) show that orderly preemptive restructurings imply lower output costs than defaults.

the bond price. We document three key results.

First, the connection between bank risk-taking and sovereign default risk depends on the level of government debt. An asymmetric nexus in which banks always default when the sovereign defaults, but not vice versa, arises for low levels of debt. While, when debt levels are high, we obtain a symmetric nexus where bank and sovereign default are perfectly synchronised. Second, the nexus shapes bank's investment incentives. Under the asymmetric nexus, a bank can influence its default probability by altering its portfolio. Under the symmetric nexus, however, this is no longer possible. Finally, depending on the nexus, banks either under-invest or over-invest in government bonds. In particular, we find that under the symmetric nexus banks under-invest. Thus, regulations aimed at limiting banks' holdings of domestic government bonds are likely to be detrimental when debt levels are high.

There are, at least, two important directions for future research. First, the output loss in our model occurs when the government defaults and not when banks default. In the symmetric nexus, bank default and government default are synchronised, so we may attribute the cost to a systemic banking crisis. In the asymmetric case, however, there are situations when only banks default. While introducing a cost of bank default into the government's problem complicates the analysis, it would yield additional insights that are relevant outside crises periods. Second, it would be interesting to extend our model to a dynamic setting in order to be able to quantitatively assess the mechanism.

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A Deriving the interest rate on bank deposits

The representative household chooses between bank deposits, purchasing government bonds and storage. The first two options are inherently risky, while storage is risk-free. Thus, in the absence of a credible deposit guarantee by the government, households place their entire endowments in storage. By ensuring that bank deposits are safe, the deposit guarantee ensures that households are indifferent between deposits and storage and that both these options are strictly preferred over purchasing government bonds.

For the government to credibly provide the deposit guarantee, we require that households are senior claimants on the government's resources. Since $T > 1$, the government can fully guarantee households' initial deposits.[13] Thus, households bear no risk from lending to banks. Finally, insofar that only the principal is insured and households are infinitely risk-averse and only value safety, banks offer deposit contracts with a zero interest rate due to perfect competition. Lemma 1 summarises.

Lemma 1. *With a credible government guarantee on households' deposits, the equilibrium deposit rate is $r_d^* = 0$. The representative household invests its entire endowment in bank deposits, i.e., $h^* = d$.*

It is worth noting that the result of Lemma 1 would also obtain in an environment where households are risk-neutral and banks are local monopolies over subsets of households. Thus, while banks cannot extract full monopoly rents, they would nevertheless continue to set $r_d^* = 0$ to extract wealth from local households.

[13] Implicitly, we assume that the government does not need to finance the guarantee by issuing additional external debt, as in Farhi and Tirole (2017), but can manage the payments using internal resources.

B Proof of Proposition 1

Let $\pi^A = \int_{\bar{A}(\ell)}^{\infty} \big(A\,\ell^\alpha + (1+r_g)\big(d + k - \ell\big) - d\big)\,dF(A)$ denote the bank's objective function under the asymmetric nexus, where $\bar{A}(\ell) = \frac{d-(1+r_g)(d+k-\ell)}{\ell^\alpha}$ is the bank's default threshold. The objective function under the symmetric nexus is $\pi^S = \int_{\widehat{A}_S}^{\infty} \big(A\,\ell^\alpha + (1+r_g)\big(d + k - \ell\big) - d\big)\,dF(A)$, where $\widehat{A}_S$ is the government's default threshold.

To determine the optimal levels of investment under the different nexus, we first take the derivatives of the objective functions with respect to ℓ. This yields

$$
\begin{aligned}
\pi_\ell^A &= \alpha\,\ell^{\alpha-1}\int_{\bar{A}(\ell)}^{\infty} A\,dF(A) - (1+r_g)\Big(1 - F\big(\bar{A}(\ell)\big)\Big)\,,\\
\pi_\ell^S &= \alpha\,\ell^{\alpha-1}\int_{\widehat{A}_S}^{\infty} A\,dF(A) - (1+r_g)\big(1 - F(\widehat{A}_S)\big)\,.
\end{aligned}
$$

Optimal investment under the different nexus regimes are given by the first-order conditions, $\pi_\ell^A(\ell^*) = 0$ and $\pi_\ell^S(\ell^*) = 0$. Under the symmetric nexus, an explicit solution for ℓ^* is obtained, which is unique. For the asymmetric nexus, under the condition that $1 + r_g < \frac{\alpha(d+k)^{\alpha-1}}{1-F(d/(d+k)^\alpha)} \int_{d/(d+k)^\alpha}^{\infty} AdF(A)$, we can appeal to the intermediate value theorem for ℓ^* to be unique.

Next, since domestic banks are price takers, the price of sovereign bonds are determined by foreign investors according to Equation (7), which on substituting into the first-order conditions yields our results for optimal investment.

To derive the critical sovereign default threshold, we rewrite Equation (6) as

$$
\widehat{A}_S = \frac{S - (d + k - \ell)}{\delta\,\ell^\alpha}\big(1 + r_g\big)\,.
$$

Substituting out $1 + r_g$ using Equation (7) yields our result that the equilibrium sovereign default threshold is implicitly defined by $\tau(\widehat{A}_S^*) = 0$, where

$$\tau(\widehat{A}_S) \equiv \widehat{A}_S - \frac{S - (d + k - \ell)}{\delta\, \ell^\alpha} \left(\frac{1 + \bar{r}}{1 - F(\widehat{A}_S)} \right) . \tag{13}$$

Market failure. The function $\tau(\widehat{A}_S)$ is globally concave. We derive this by noting that

$$\tau''(\widehat{A}_S) = -\frac{\lambda^2 \big(S - (d + k - \ell)\big)}{\delta \ell^\alpha} \left(\frac{1 + \bar{r}}{1 - F(\widehat{A}_S)} \right) ,$$

which is strictly negative as long as $d + k - \ell < S$, i.e., the domestic bank does not hold all government bonds. This is always true since, at the margin, foreign investors must hold some government bonds to determine the price. We also note that $\lim_{\widehat{A}_S \to 0} \tau(\widehat{A}_S) < 0$ and $\lim_{\widehat{A}_S \to T} \tau(\widehat{A}_S) = -\infty < 0$. This implies that if $\tau(\widehat{A}_S)$ crosses the x-axis, then it does so twice, implying two distinct equilibria. But, it is also possible that $\tau(\widehat{A}_S)$ does not cross the x-axis, and hence there is market failure and no equilibrium solution. The market failure condition is derived as the point, $\widehat{A}^{MF}$ where the curve $\frac{\lambda^2 \left(S - (d+k-\ell)\right)}{\delta \ell^\alpha} \left(\frac{1+\bar{r}}{1 - F(\widehat{A}^{MF})} \right)$ is tangential to the 45-degree line, i.e., $\tau'(\widehat{A}^{MF}) = 1$. We obtain that

$$\widehat{A}^{MF} = F^{-1} \left(1 - \frac{\lambda \big(S - (d + k - \ell)\big)}{\delta \ell^\alpha} \big(1 + \bar{r}\big) \right) ,$$

where F^{-1} is the inverse cumulative distribution function for the TFP shock. As long as $\tau(\widehat{A}^{MF}) \geq 0$, there is no market failure, where

$$\tau(\widehat{A}^{MF}) = \widehat{A}^{MF} - \frac{S - (d + k - \ell)}{\delta \ell^\alpha} \left(\frac{1 + \bar{r}}{\frac{\lambda \left(S - (d+k-\ell)\right) \left(1+\bar{r}\right)}{\delta \ell^\alpha}} \right) = \widehat{A}^{MF} - \frac{1}{\lambda} .$$

Rearranging the condition, we obtain that as long as $S \leq S^{MF}$, there is no market failure, where S^{MF} is implicitly given by

$$F^{-1}\left(1-\frac{\lambda\big(S^{MF}-(d+k-\ell)\big)}{\delta\ell^{\alpha}}\big(1+\bar{r}\big)\right)-\frac{1}{\lambda}=0\,.$$

Bound for asymmetric nexus. For the asymmetric nexus, we require $\widehat{A}_S < \bar{A} < \tilde{A}$. In the vicinity of the Pareto efficient equilibrium, we have that $\tau_{\widehat{A}_S} > 0$. This implies that to be in the asymmetric nexus, we must have that $\tau(\bar{A}) > 0$. We can express the equilibrium condition as follows.

$$S < \frac{\delta d}{1+\bar{r}}\big(1-F(\bar{A}(\ell^*))\big)+\big(d+k-\ell^*\big)\left[1-\frac{\delta\big(1-F(\bar{A}(\ell^*))\big)}{1-F(\widehat{A}^*_S)}\right]\equiv \underline{S}\,.$$

Interval for symmetric nexus. In general, it is also possible to obtain the ordering of thresholds whereby $\bar{A} < \tilde{A} < \widehat{A}_S$. This occurs whenever $\tau(\tilde{A}) < 0$, and can be expressed as

$$S > \frac{\delta d}{1+\bar{r}}\left(\frac{1-F(\tilde{A}(\ell^*))}{1-\delta}\right)+(d+k-\ell^*)\equiv \tilde{S}\,.$$

However, if $\bar{S} < \tilde{S}$, then the market equilibrium breaks down before we reach the new regime. This requires $\tau(\widehat{A}^{MF}) < \tau(\tilde{A})$, which on rearranging yields

$$\lambda > \widehat{\lambda} \equiv \left[\widehat{A}^{MF}-\tilde{A}^*+\frac{S-(d+k-\ell^*)}{\delta\ell^{\alpha}}\left(\frac{1+\bar{r}}{1-F(\tilde{A}^*)}\right)\right]^{-1}. \qquad (14)$$

C Proof of Propositions 2 - 4 and Corollary 1

In this section we investigate how changes to the lending rate for foreign investors, $\bar{r}$, banker's endowment, k, and stock of debt to refinance for the sovereign, S, influence the equilibrium level of investment. In general, we can decompose the effects into direct effects via the bank's first-order condition, and an indirect effect via the pricing of government bonds. Since the pricing of government bonds is the same under both the asymmetric nexus and symmetric nexus, we first describe the partial effects of changes in the exogenous variables on $\widehat{A}_S^*$. We obtain the following.

$$\begin{aligned}
\tau_{\widehat{A}_S}(\widehat{A}_S^*) &= 1 - \lambda \widehat{A}_S^* > 0 \\
\tau_\ell &= \frac{1}{\delta \ell^\alpha} \left(\frac{1+\bar{r}}{1-F(\widehat{A}_S)} \right) \left[-1 + \frac{\alpha \ell^{\alpha-1}}{\ell^\alpha} \left(S - (d+k-\ell) \right) \right] < 0 \\
\tau_{\bar{r}} &= -\frac{S-(d+k-\ell)}{\delta \ell^\alpha \big(1-F(\widehat{A}_S)\big)} < 0 \\
\tau_k &= \frac{S}{\delta \ell^\alpha} \left(\frac{1+\bar{r}}{1-F(\widehat{A}_S)} \right) > 0 \\
\tau_S &= -\frac{1}{\delta N \ell^\alpha} \left(\frac{1+\bar{r}}{1-F(\widehat{A}_S)} \right) < 0 \,.
\end{aligned}$$

We now turn to the two nexus and first determine the partial effects of changes in the exogenous parameters on the bank's optimal choice and subsequently derive the total effects using Cramer's rule.

Asymmetric Nexus

First, we show that the optimal level of investment is a maximum. This is given by showing $\pi^A_{\ell\ell}(\ell^*) < 0$. We obtain that

$$\begin{aligned}\pi^A_{\ell\ell} &= \alpha(\alpha-1)(\ell)^{\alpha-2}\int_{\bar{A}(\ell)}^{\infty} A\,dF(A) - \alpha\ell^{\alpha-1}\bar{A}(\ell)f(\bar{A}(\ell))\frac{\partial\bar{A}}{\partial\ell} + \frac{1+\bar{r}}{1-F(\widehat{A}_S)}f(\bar{A}(\ell))\frac{\partial\bar{A}}{\partial\ell} \\ &= \alpha(\alpha-1)(\ell)^{\alpha-2}\int_{\bar{A}(\ell)}^{\infty} A\,dF(A) - f(\bar{A}(\ell))\frac{\partial\bar{A}}{\partial\ell}\left\{\alpha\ell^{\alpha-1}\bar{A}(\ell) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right\},\end{aligned}$$

where $\frac{\partial\bar{A}}{\partial\ell} = -\frac{1}{\ell^\alpha}\left[\alpha\ell^{\alpha-1}\bar{A}(\ell) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right]$. At the equilibrium, ℓ^*, we get

$$\begin{aligned}\pi^A_{\ell\ell}(\ell^*) &= \frac{\alpha(\alpha-1)(\ell^*)^{\alpha-2}}{\alpha(\ell^*)^{\alpha-1}}\big(1+\bar{r}\big)\frac{1-F(\bar{A}(\ell))}{1-F(\widehat{A}_S)} + \frac{f(\bar{A}(\ell^*))}{(\ell^*)^\alpha}\left\{\alpha(\ell^*)^{\alpha-1}\bar{A}(\ell^*) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right\}^2 \\ &= \big(1-F(\bar{A}(\ell^*))\big)\left[\frac{\alpha(\alpha-1)(\ell^*)^{\alpha-2}}{\alpha(\ell^*)^{\alpha-1}}\frac{1+\bar{r}}{1-F(\widehat{A}_S)} + \frac{\lambda}{(\ell^*)^\alpha}\left\{\alpha(\ell^*)^{\alpha-1}\bar{A}(\ell^*) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right\}^2\right].\end{aligned}$$

Since the first term in the square brackets is negative, while the second is positive, if the hazard rate satisfies, $\lambda < \bar{\lambda}$, then $\pi^A_{\ell\ell}(\ell^*) < 0$, where the upper bound is given by the solution to

$$\frac{\alpha(\alpha-1)(\ell^*)^{\alpha-2}}{\alpha(\ell^*)^{\alpha-1}}\frac{1+\bar{r}}{1-F(\widehat{A}_S)} + \frac{\bar{\lambda}}{(\ell^*)^\alpha}\left\{\alpha(\ell^*)^{\alpha-1}\bar{A}(\ell^*) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right\}^2 = 0\,.$$

Next, we derive the partial effects from increases in the sovereign default threshold, $\widehat{A}_S$, risk-free rate, $\bar{r}$, stock of debt, S, and bank capital, k, on the optimal investment.

We obtain that

$$\begin{aligned}
\pi^A_{\ell\widehat{A}_S} &= -f(\bar{A}(\ell))\frac{\partial \bar{A}}{\partial \widehat{A}_S}\left[\alpha\ell^{\alpha-1}\bar{A}(\ell) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right] - \lambda(1+\bar{r})\frac{1-F(\bar{A}(\ell))}{1-F(\widehat{A}_S)} \\
\pi^A_{\ell\bar{r}} &= -f(\bar{A}(\ell))\frac{\partial \bar{A}}{\partial \bar{r}}\left[\alpha\ell^{\alpha-1}\bar{A}(\ell) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right] - \frac{1-F(\bar{A}(\ell))}{1-F(\widehat{A}_S)} \\
\pi^A_{\ell S} &= 0 \\
\pi^A_{\ell k} &= -f(\bar{A}(\ell))\frac{\partial \bar{A}}{\partial k}\left[\alpha\ell^{\alpha-1}\bar{A}(\ell) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}\right]
\end{aligned}$$

Clearly, the signs for $\pi^A_{\ell\widehat{A}_S}$, $\pi^A_{\ell\bar{r}}$ and $\pi^A_{\ell k}$ depend on the sign of $\alpha\ell^{\alpha-1}\bar{A}(\ell) - \frac{1+\bar{r}}{1-F(\widehat{A}_S)}$, which at the optimum ℓ^* can we re-written as $\alpha(\ell^*)^{\alpha-1}\big[\bar{A}(\ell^*) - \frac{\int_{\bar{A}(\ell^*)}^{\infty} AdF(A)}{1-F(\bar{A}(\ell^*))}\big] < 0$. Hence, $\pi^A_{\ell\widehat{A}_S} < 0$, $\pi^A_{\ell\bar{r}} < 0$ and $\pi^A_{\ell k} < 0$.

The determinant of the Jacobian matrix is

$$|J^A| = \begin{vmatrix} \pi^A_{\ell\ell} & \pi^A_{\ell\widehat{A}_S} \\ \tau_\ell & \tau_{\widehat{A}_S} \end{vmatrix} < 0\,.$$

The comparative statics for the optimal level of investment are, thus, as follows.

$$\begin{aligned}
\frac{d\ell^*}{d\bar{r}} &= \frac{\begin{vmatrix} -\pi^A_{\ell\bar{r}} & \pi^A_{\ell\widehat{A}_S} \\ -\tau_{\bar{r}} & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^A|} < 0\,, \quad \frac{d\ell^*}{dk} = \frac{\begin{vmatrix} -\pi^A_{\ell k} & \pi^A_{\ell\widehat{A}_S} \\ -\tau_k & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^A|}\,, \\
\frac{d\ell^*}{dS} &= \frac{\begin{vmatrix} -\pi^A_{\ell S} & \pi^A_{\ell\widehat{A}_S} \\ -\tau_S & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^A|} < 0\,, \quad \frac{d\ell^*}{d\delta} = \frac{\begin{vmatrix} -\pi^A_{\ell\delta} & \pi^A_{\ell\widehat{A}_S} \\ -\tau_\delta & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^A|} > 0\,.
\end{aligned}$$

In general the effect of a change in bank capital on investment has an ambiguous sign.

Note, however, that

$$\omega(S) \equiv \begin{vmatrix} -\pi^A_{\ell k} & \pi^A_{\ell \widehat{A}_S} \\ -\tau_k & \tau_{\widehat{A}_S} \end{vmatrix} = -\pi^A_{\ell k}(1 - \lambda \widehat{A}^*_S) + \frac{S}{\delta(\ell^*)^\alpha}\left(\frac{1+\bar{r}}{1-F(\widehat{A}^*_S)}\right)\pi^A_{\ell \widehat{A}_S}$$

is decreasing is S and at $S = 0$ it is strictly positive. Thus If $\omega(\underline{S}) > 0$, then this establishes that under the asymmetric nexus, an increase in bank capital leads to a decrease in investment, i.e., $\frac{\partial \ell^*}{\partial k} < 0$. This is equivalent to requiring that

$$d > \underline{d} \equiv \frac{\pi^A_{\ell k}\left(1 - \lambda \widehat{A}^*_S\right) - \frac{k-\ell^*}{\delta(\ell^*)^\alpha}\left[1 - \delta\xi^*\right]\left(\frac{1+\bar{r}}{1-F(\widehat{A}^*_S)}\right)}{\pi^A_{\ell \widehat{A}_S}\left[\xi^* + \frac{1}{\delta(\ell^*)^\alpha}\left(1 - \delta\xi^*\right)\left(\frac{1+\bar{r}}{1-F(\widehat{A}^*_S)}\right)\right]},$$

where $\xi^* = \frac{1-F(\bar{A}^*)}{1-F(\widehat{A}^*_S)}$. For sufficiently small k, this condition is satisfied for all d.

The total effects on the sovereign's default threshold are

$$\frac{d\widehat{A}^*_S}{d\bar{r}} = \frac{\begin{vmatrix} \pi^A_{\ell\ell} & -\pi^A_{\ell\bar{r}} \\ \tau_\ell & -\tau_{\bar{r}} \end{vmatrix}}{|J^A|} \lesseqgtr 0\,, \quad \frac{d\widehat{A}^*_S}{dk} = \frac{\begin{vmatrix} \pi^A_{\ell\ell} & -\pi^A_{\ell k} \\ \tau_\ell & -\tau_k \end{vmatrix}}{|J^A|} < 0$$

$$\frac{d\widehat{A}^*_S}{dS} = \frac{\begin{vmatrix} \pi^A_{\ell\ell} & -\pi^A_{\ell S} \\ \tau_\ell & -\tau_S \end{vmatrix}}{|J^A|} > 0\,, \quad \frac{d\widehat{A}^*_S}{d\delta} = \frac{\begin{vmatrix} \pi^A_{\ell\ell} & -\pi^A_{\ell\delta} \\ \tau_\ell & -\tau_\delta \end{vmatrix}}{|J^A|} < 0\,.$$

Symmetric Nexus

As before, we first show that the optimal level is a maximum, which requires $\pi^S_{\ell\ell}(\ell^*) < 0$. We readily obtain

$$\pi^S_{\ell\ell} = \alpha(\alpha-1)(\ell)^{\alpha-2}\int_{\widehat{A}_S}^{\infty} A\,dF(A) < 0\,.$$

Next, for the partial effects of a change in $\widehat{A}_S$, $\bar{r}$, k and S, we obtain $\pi^S_{\ell\widehat{A}_S} = -\alpha\ell^{\alpha-1}\,\widehat{A}_S f(\widehat{A}_S) < 0$, $\pi^S_{\ell\bar{r}} = -1 < 0$, $\pi^S_{\ell k} = 0$, $\pi^S_{\ell S} = 0$, and $\pi^S_{\ell\delta} = 0$.

The determinant of the Jacobian matrix is

$$|J^S| = \begin{vmatrix} \pi^S_{\ell\ell} & \pi^S_{\ell\widehat{A}_S} \\ \tau_\ell & \tau_{\widehat{A}_S} \end{vmatrix} < 0\,.$$

The comparative statics for the optimal level of investment are, thus, as follows.

$$\frac{d\ell^*}{d\bar{r}} = \frac{\begin{vmatrix} -\pi^S_{\ell\bar{r}} & \pi^S_{\ell\widehat{A}_S} \\ -\tau_{\bar{r}} & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^S|} < 0\,, \quad \frac{d\ell^*}{dk} = \frac{\begin{vmatrix} -\pi^S_{\ell k} & \pi^S_{\ell\widehat{A}_S} \\ -\tau_k & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^S|} > 0$$

$$\frac{d\ell^*}{dS} = \frac{\begin{vmatrix} -\pi^S_{\ell S} & \pi^S_{\ell\widehat{A}_S} \\ -\tau_S & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^S|} < 0\,, \quad \frac{d\ell^*}{d\delta} = \frac{\begin{vmatrix} -\pi^S_{\ell\delta} & \pi^S_{\ell\widehat{A}_S} \\ -\tau_\delta & \tau_{\widehat{A}_S} \end{vmatrix}}{|J^S|} > 0\,.$$

The total effects on the sovereign's default threshold are

$$\frac{d\widehat{A}^*_S}{d\bar{r}} = \frac{\begin{vmatrix} \pi^S_{\ell\ell} & -\pi^S_{\ell\bar{r}} \\ \tau_\ell & -\tau_{\bar{r}} \end{vmatrix}}{|J^S|} \lesseqgtr 0\,, \quad \frac{d\widehat{A}^*_S}{dk} = \frac{\begin{vmatrix} \pi^S_{\ell\ell} & -\pi^S_{\ell k} \\ \tau_\ell & -\tau_k \end{vmatrix}}{|J^S|} < 0$$

$$\frac{d\widehat{A}^*_S}{dS} = \frac{\begin{vmatrix} \pi^S_{\ell\ell} & -\pi^S_{\ell S} \\ \tau_\ell & -\tau_S \end{vmatrix}}{|J^S|} > 0\,, \quad \frac{d\widehat{A}^*_S}{d\delta} = \frac{\begin{vmatrix} \pi^S_{\ell\ell} & -\pi^S_{\ell\delta} \\ \tau_\ell & -\tau_\delta \end{vmatrix}}{|J^S|} < 0\,.$$

Finally, since the bank default threshold is identical to the sovereign default threshold, the comparative statics are identical.

D Proof of Propositions 5 - 6

We can re-write the planner's problem as $\max_\ell W(\ell)$, where

$$\begin{aligned} W(\ell) \;\equiv\; & \ell^\alpha \left[(1-\delta)\int_0^{\widehat{A}_S(\ell)} A\,dF(A) + \int_{\widehat{A}_S(\ell)}^{\infty} A\,dF(A)\right] \\ & - \Big(1 + r^*_g(\ell)\Big)\Big(S - (d + k - \ell)\Big)\left(1 - F(\widehat{A}_S(\ell))\right)\,, \end{aligned}$$

where $r^*_g(\ell)$ is derived from the foreign investors' binding participation constraints such that $\frac{\partial r^*_g}{\partial \ell} > 0$. The result in Equation (12) following immediately from the first-order condition, $W_\ell(\ell^{SP}) = 0$, where all partial effects via the sovereign default threshold cancel out. We also assume that this optimum is a maximiser, i.e., $W_{\ell\ell}(\ell^{SP}) < 0$.

We next compare the level of investment from the competitive equilibrium, ℓ^*, versus the social planner's allocation, ℓ^{SP}. To this end, if there is too much investment in the real economy under the competitive solution, i.e., $\ell^* > \ell^{SP}$, then this would imply that $W_\ell(\ell^*) < 0$. We consider the competitive equilibrium investment under the asymmetric nexus and symmetric nexus in turn.

Asymmetric nexus. Evaluating the planner's first-order condition at the competitive equilibrium, we get

$$\begin{aligned} W_\ell(\ell^*) &= \frac{\alpha\,(\ell^*)^{\alpha-1}}{1-F(\widehat{A}_S^*)}(1-\delta)\int_0^{\widehat{A}_S^*} A\,dF(A) - \Big(S-(d+k-\ell^*)\Big)\,\frac{\partial r_g^*}{\partial \ell}\bigg|_{\ell=\ell^*} \\ &- \frac{\alpha\,(\ell^*)^{\alpha-1}}{1-F(\bar{A}^*)}\int_0^{\bar{A}^*} A\,dF(A)\,. \end{aligned}$$

Denoting by $\Omega \equiv \int_0^{\widehat{A}_S^*} A\,dF(A) - \frac{1-F(\widehat{A}_S^*)}{1-F(\bar{A}^*)}\int_0^{\bar{A}^*} A\,dF(A) < 0$, we have that the level of investment under the competitive equilibrium is too high whenever

$$\delta > \underline{\delta} \equiv 1 - \frac{\Big(S-(d+k-\ell^*)\Big)\Big(1-F(\widehat{A}_S^*)\Big)\,\frac{\partial r_g^*}{\partial \ell}\Big|_{\ell=\ell^*}}{\alpha\,(\ell^*)^{\alpha-1}\int_0^{\widehat{A}_S^*} A\,dF(A)} + \frac{\Omega}{\alpha\,(\ell^*)^{\alpha-1}\int_0^{\widehat{A}_S^*} A\,dF(A)}$$

Symmetric nexus. In this case, we have that

$$W_\ell(\ell^*) = \alpha\,(\ell^*)^{\alpha-1}\,(1-\delta)\int_0^{\widehat{A}_S^*} A\,dF(A) - \Big(S-(d+k-\ell^*)\Big)\Big(1-F(\widehat{A}_S^*)\Big)\,\frac{\partial r_g^*}{\partial \ell}\bigg|_{\ell=\ell^*}\,.$$

Thus, the level of investment under the competitive equilibrium is too high whenever

$$\delta > \bar{\delta} \equiv 1 - \frac{\Big(S-(d+k-\ell^*)\Big)\Big(1-F(\widehat{A}_S^*)\Big)\,\frac{\partial r_g^*}{\partial \ell}\Big|_{\ell=\ell^*}}{\alpha\,(\ell^*)^{\alpha-1}\int_0^{\widehat{A}_S^*} A\,dF(A)}$$

Finally, since $\Omega < 0$, it follows that $\underline{\delta} < \bar{\delta}$.