Online appendix for *The long and short of financing government debt*

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Appendices

This appendix contains 3 sections. Section A complements our empirical analysis by providing data variables definition and sources and carrying out additional empirical exercises and robustness checks. Section B derives the analytical formulae shown in Section 3 of the main text and extends the results to alternative calibrations of the model. Finally, Section C solves the optimal policy model.

A Empirical Analysis: Definitions and Robustness Checks

A.1 Variable definitions and data sources

We obtain quarterly data on GDP, private consumption, private investment, government expenditures, wages, the short-term and long-term rates, federal government debt aggregates (short, long and total), the GDP deflator and taxes. All variables are seasonally adjusted except for interest rates and the debt aggregates (the latter are used as ratios, i.e. short over long, in the empirical analysis). The national account variables are obtained from NIPA statistics. The gross domestic product, private consumption and investment and government expenditures are all in billions of US dollars. Moreover, our measure of wages is Real Compensation Per Hour in the Nonfarm Business Sector. Taxes corresponds to total federal government tax receipts.

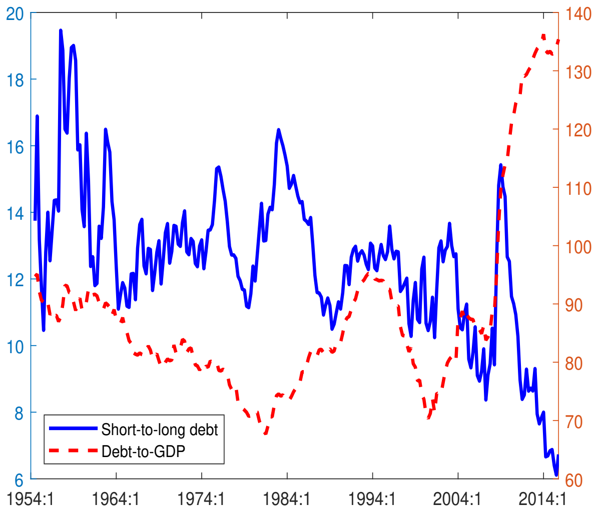
Interest rates and debt aggregates are extracted from the OECD database. As discussed in the main text, we defined as short-term debt, all government debt with maturity less than or equal to one year. Long-term debt is the remaining federal debt outstanding. Moreover, in our empirical analysis we used two different definitions for the short-term interest rate: The overnight interbank rate and the 3-month interbank rate. The long-term interest rate corresponds to the yield of 10 year US government bonds.

The data variables along with the precise definitions and the sources are listed in Table 1 for completeness. The first column of the table lists the variables with the names they will appear in the labels of the various figures. Our sample covers the period 1955:Q1- 2015:Q3.

Variable	Description	Source
output	Gross domestic product in billions of dollars	NIPA
consumption	Private consumption, in billion dollars	NIPA
investment	Private Investment, in billion dollars	NIPA
government expd.	Government total spending, billion dollars	NIPA
wage	Nonfarm Business Sector: Real Compensation Per Hour, Index 2012=100	NIPA
tax	Total federal government tax receipts	NIPA
gdp_deflator	Implicit gdp price deflator	NIPA
r_overnight	Overnight interbank rate, no seasonally adjusted	OECD
r_three	3-month interbank rate, not seasonally adjusted	OECD
r_long	Long-term interest rate, not seasonally adjusted	OECD
debt	General government total debts, billion dollars	OECD
debt_short	General government short-term debts, billion dollars	OECD
debt_long	General government long-term debts, billion dollars	OECD
News	Military news shock	Ramey and Zubairy (2018)

Figure 1 traces the evolution of the debt to GDP ratio (right axis, dashed red line) along with ratio of short-term over long-term debt (right axis, blue solid line). Though the short to long-term ratio displays some volatility over time, it should be noted that it is highly persistent, the first order autocorrelation coefficient is 0.89. Moreover, the standard deviation of the ratio is 0.024 and the correlation with debt over GDP is -0.43.

Figure 1: Debt-to-GDP and the share of short-to-long term debt

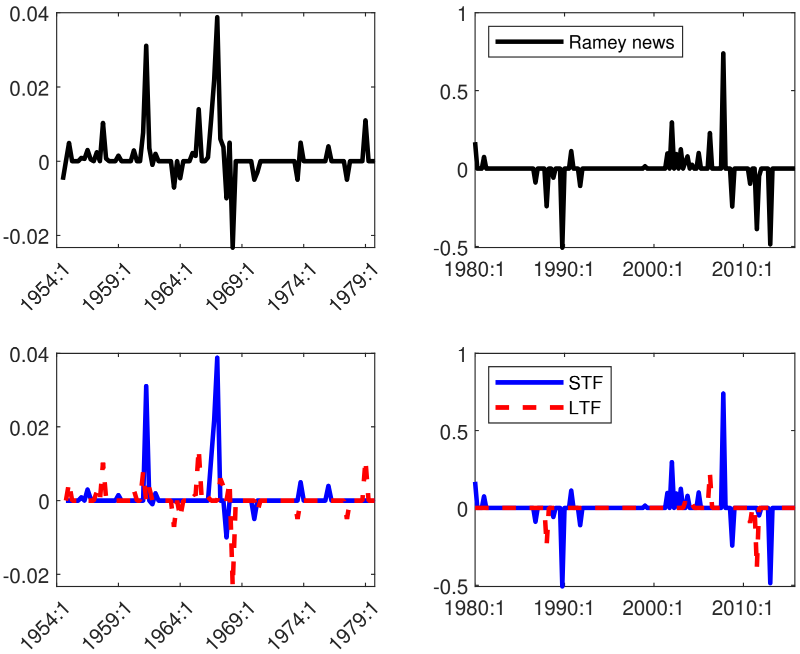


Red dotted line: US debt-to-GDP (in percentage terms, left y-axis); Solid blue line: US short-to-long government debt (in percentage terms, right y-axis). Data obtained from NIPA, OECD. Definitions provided in online appendix.

Figure 2 plots the Ramsey defense new series. The graphs in the top panels span the whole sample period. To make all shocks clearly visible we split the sample in two subsamples, 1954:Q1 to 1979:Q4 on the left and 1980:Q1 onwards on the right graph. The larger volatility of news shocks about government spending in the second subsample, is driven by the wars in Afganistan and Iraq in the 2000s and the subsequent cuts in spending in the late 2000s and early 2010s. Large cuts also took place in the early 1990s when the Cold War ended.

The bottom panels of the figure show separately short-term financed (STF) and long-term financed (LTF) shocks. As can be seen from the plots, the STF and LTF shocks concern both subperiods of our sample.

Figure 2: Identified fiscal shocks using Ramey defense news



Notes: Top row: Military spending news series from Ramey and Zubairy (2018) over the period 1954Q1 to 1979Q4 (left panel) and over 1980Q1 to 2015Q4 (right panel). Bottom row: Identified short-term (STF) and long-term (LTF) debt financed spending shocks over the period 1954Q1 to 1979Q4 (left panel) and over 1980Q1 to 2015Q4 (right panel). Series are scaled by the trend of GDP.

A.2 Additional Exercises and Robustness in the Proxy VAR

We now perform additional exercises to show the robustness of our main finding that short-term financing leads to a larger fiscal multiplier. The results that we show in this subsection mainly correspond to the robustness checks we had mentioned in text. We also show additional output from the baseline specifications of the empirical model studying the impulse responses of wages, interest rates etc.

Impulse responses of government spending. In Figure 3 we plot the cumulative impulse response functions of government expenditures to the spending shock under short-term financing (blue) and long-term financing (red). As is evident from the figure, the IRFS are similar across the two financing schemes. This evidence leads us to conclude that the differences in the cumulative multipliers we reported in the main text are not driven by differences in the spending processes. The US government does not issue short-term debt to finance a different type of shock than it does when it finances with long-term debt.

Impulse responses of additional controls. Table 2 in the main text reported the fiscal multipliers when we run the model including additional variables (interest rates, wages, the GDP deflator). These variables were included one at a time. In Figure 4 we plot the impulse response functions of these variables to the spending shock. The top left panel shows the responses of real wages. As can be seen from the figure both types of shocks induce a small drop in wages and the response is more negative in the case of short-term financing. These reactions of wages are indeed small (even though significant) and so we are not troubled by the fact that wages drop following the spending shock.[1] The finding that wages do not react more positively in the case of short-term financing is a more important finding for our main result. It reassures us that the larger multiplier we found under STF was not driven by a stronger reaction of wages to the shock.[2]

The middle and right top panels and the bottom left panel show the IRFS of the short and long-term rates and the term premium respectively, when these variables are included in the model. STF increases the short-term interest rate (top right) and decreases the term premium (bottom left). LTF increases the long-term rate and increases the term premium. Notice that these patterns are easy to rationalize within the context of theoretical models in which the relative supply of short and long bonds affects yields (as is the case in our theory). STF increases the relative supply of short bonds and increases yields at the short end of the yield curve; LTF increases the supply to long bonds and impacts the long end of the curve. These findings are at odds with the canonical macro model in which only the path of spending impacts interest rates.

Finally, the bottom right panel of Figure 4 shows the impulse response of the GDP deflator to LTF and STF shocks. As can be seen from the graph, the price level increases after a short-term financed shock and decreases (or responds insignificantly) in the case of long-term financing. This pattern is consistent with the finding that STF induces a larger expansion of output and consumption and is consistent with our New Keynesian model (see below).

Figure 5 shows the impulse response functions from a structural VAR when we include all controls together. This robustness check serves to illustrate that controlling simultaneously for all possible endogeneity concerns does not change our results. As can be seen the responses are consistent with our finding that STF induces a larger expansion of output and consumption than LTF. Moreover, the patterns of adjustment of wages, short and long-term rates and the GDP deflator are similar to those we previously found in Figure 4.

[1]This response is also easy to explain given the responses of aggregate prices to the shocks (see below). Since inflation increases considerably in the STF shock case, but not under LTF, a mild rigidity in nominal wages coupled with the responses of the aggregate price level, can indeed explain the pattern we find in the data.

[2]We continue finding similar responses of spending across STF and LTF in all models considered in this subsection. For brevity we do not show the IRFS of government expenditures.

A VAR with taxes.

We separately run a VAR with taxes (tax revenues as a % of GDP) as an additional control variable. The result that STF leads to a larger multiplier continued to hold. Interestingly, this exercise showed that taxes responded positively to an LTF shock and negatively to an STF shock. Though the responses were small in both cases, we wanted to address the possible concern that LTF shocks are partly tax financed whereas STF shocks are only debt financed. (In theory, tax financed shocks lead to smaller multipliers than debt financed shocks.)

We therefore run a VAR in which we constrained the responses of taxes to be zero for 4 quarters for each of the two financing schemes and treat this as our benchmark. The output is shown in Figure 6 where we plot the cumulative multipliers under STF and LTF. As is evident from the figure, the main finding that STF leads to a crowding in of aggregate consumption and a larger fiscal multiplier continues to hold.

Pre and Post 1980s samples, High and Low debt and the Zero Lower bound.

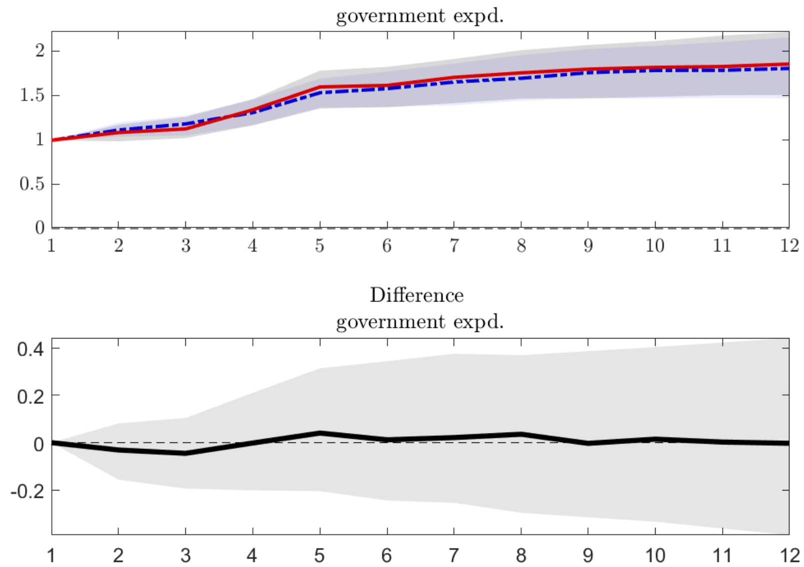
We now consider three additional robustness checks. First we run the model separately using the subsample of observations in which debt is above the median to investigate whether our results were driven by the fact that at high debt levels, the US Treasury typically issues more long-term debt (see Greenwood, Hanson, and Stein, 2015). In Figure 7 we plot the cumulative multipliers for the high debt subsample. Qualitatively the patterns that we identified with the full sample do not change. (If anything the gap in the STF and LTF multipliers is even slightly larger now). Therefore, the debt level is not important to explain our findings.

Next, we run our empirical model using observations post 1980. This enables us to identify whether the well documented structural break in the interest rate policy of the Federal reserve during the Volcker chairmanship, has an effect on our estimates. Figure 8 shows the cumulative multipliers under STF and LTF. The estimates are similar in magnitude to the analogous objects reported in the main text for the full sample.[3]

Finally, we run our empirical model, using observations up to 2007Q4 (dropping the Great recession and all quarters where the nominal interest rate was at its effective lower bound). During these years, the US economy suffered a severe recession, and government debt levels increased considerably. We also observed an increase in the new issuance of long-term debt by the US Treasury. Figure 9 however shows that omitting the post 2008 observations plays essentially no role in our estimates of the cumulative multipliers. We therefore conclude that our findings are not driven by the financial crisis period.

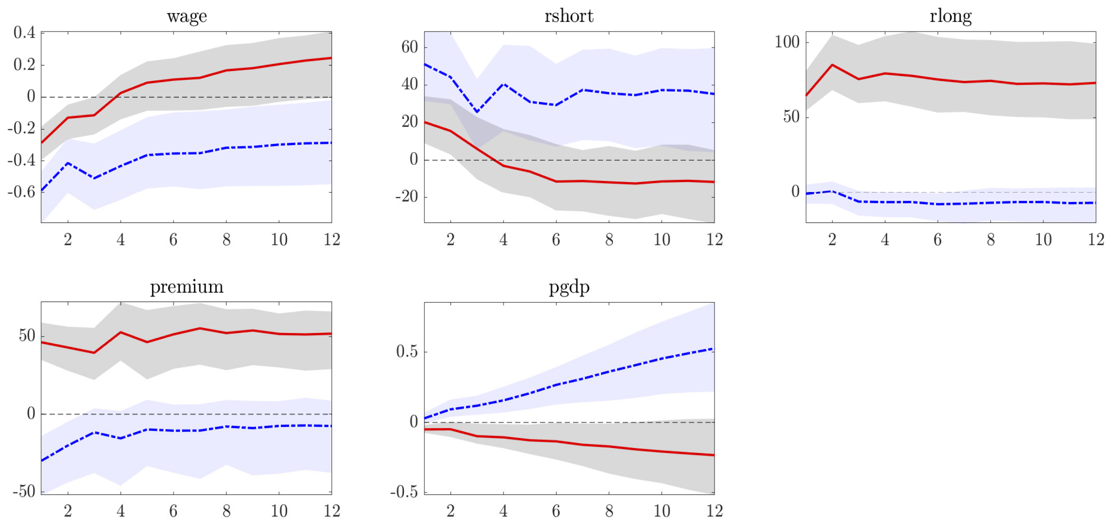
[3]For brevity we did not include a separate graph for the pre 1980s subsample, but the results were again similar. We also run several empirical models including variables from the list discussed previously. Again these additional exercises showed no significant difference with our full sample estimates.

Figure 3: Proxy-SVAR: Baseline specification. Cumulative impulse responses of government expenditures



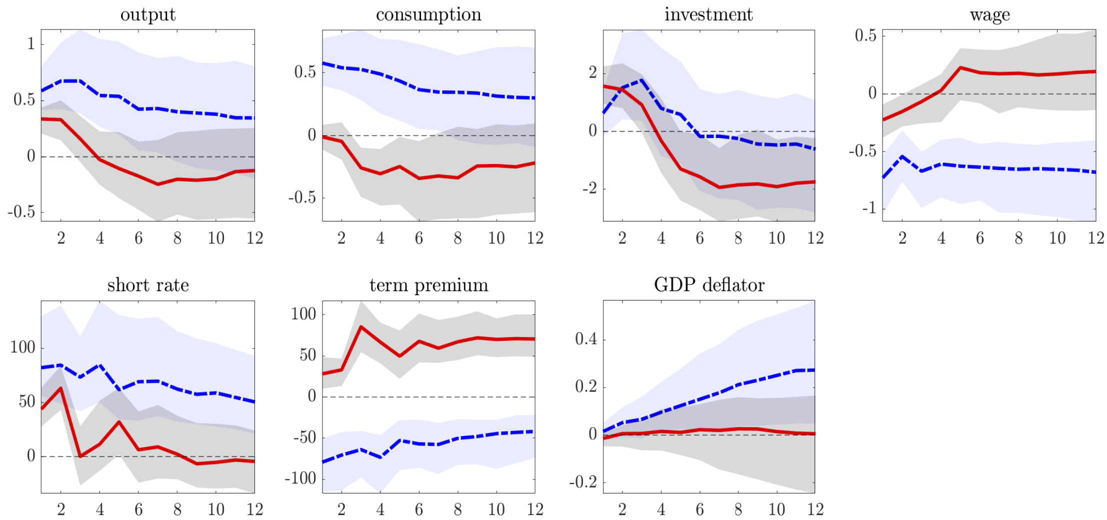
Notes: Top panel: Cumulative impulse response functions of government expenditures following a shock to short-term (blue, dash-dotted) and long-term debt-financed (red, solid) government expenditures. Lines correspond to median responses. Shaded areas correspond to confidence bands of one standard deviation. The bottom panel shows the difference in the estimated IRFS and the shaded area corresponds to one standard deviation confidence bands.

Figure 4: Proxy-SVAR: Robustness with additional controls included separately. Cumulative impulse response functions



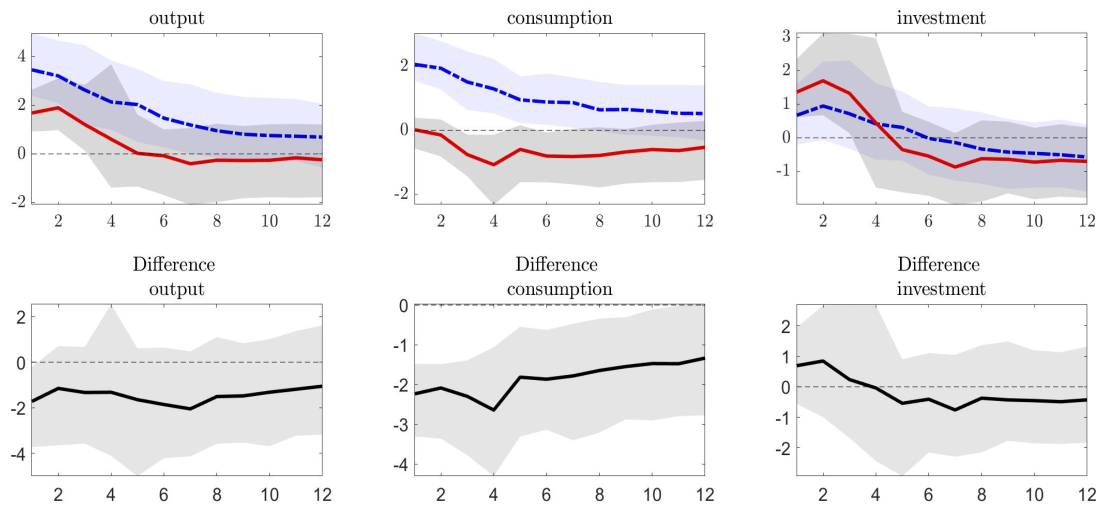
Notes: Cumulative impulse response functions following a shock to short-term (blue, dash-dotted) and long-term debt-financed (red, solid) government expenditures. Lines correspond to median responses. Shaded areas correspond to confidence bands of one standard deviation. Variables are reported in per cent deviations. Short-term and long-term rates and the term premium are in basis points.

Figure 5: Proxy-SVAR: Robustness with all additional controls together Cumulative impulse response functions



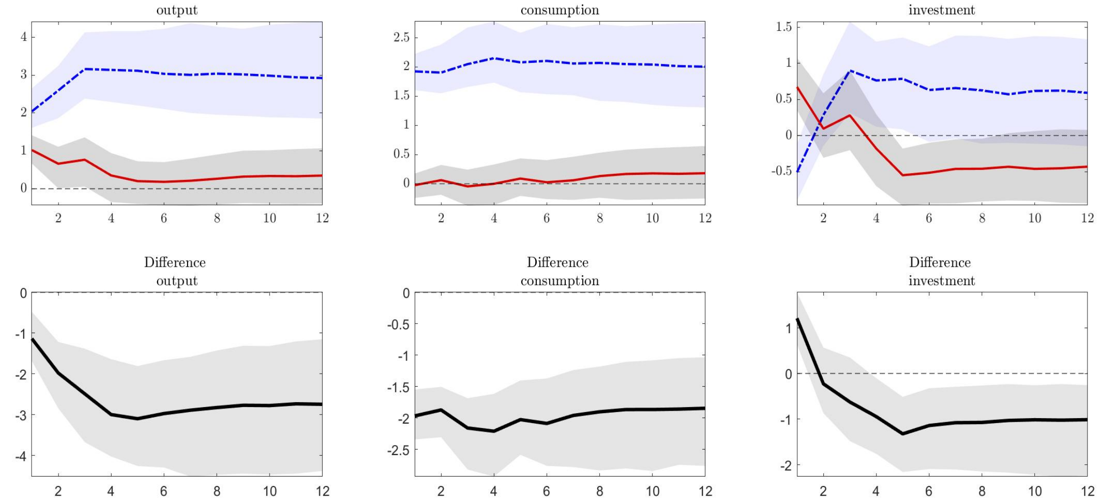
Notes: Cumulative impulse response functions following a shock to short-term (blue, dash-dotted) and long-term debt-financed (red, solid) government expenditures. Lines correspond to median responses. Shaded areas correspond to confidence bands of one standard deviation. Variables are reported in per cent deviations. Short-term and long-term rates and the term premium are in basis points.

Figure 6: Proxy-SVAR: Robustness with zero restrictions on tax revenues. Cumulative multipliers



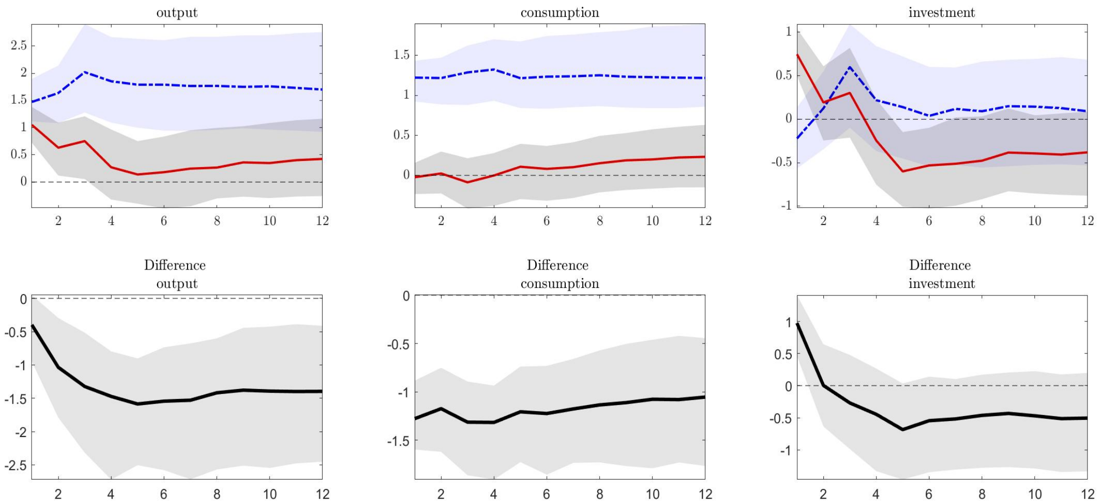
Notes: Cumulative multipliers following a shock to short-term (blue, dash-dotted) and long-term debt-financed (red, solid) government expenditures. Lines correspond to median responses. Shaded areas correspond to confidence bands of one standard deviation. Variables are reported in per cent deviations.

Figure 7: Proxy-SVAR: High debt subsample. Cumulative multipliers



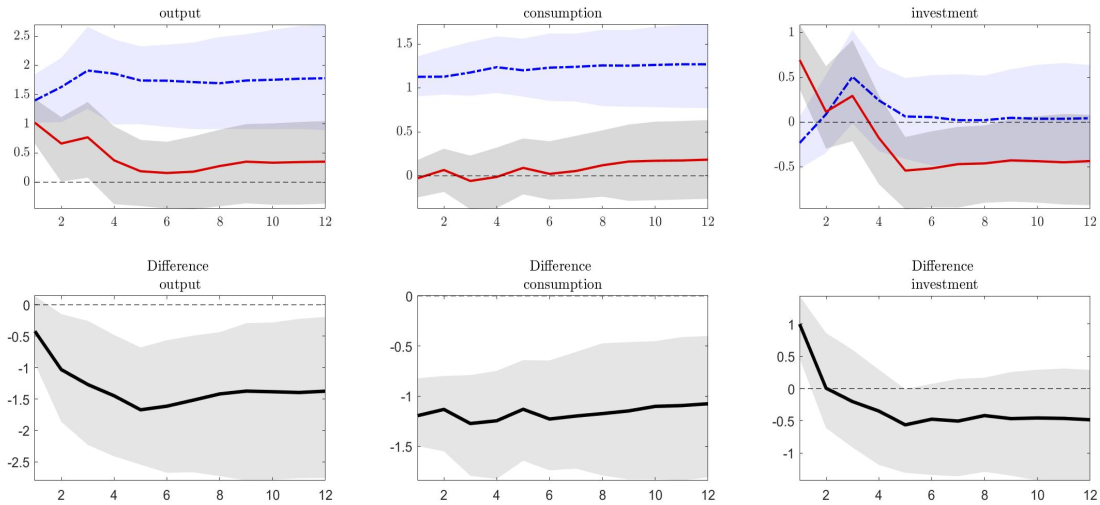
Notes: Cumulative multipliers following a shock to short-term (blue, dash-dotted) and long-term debt-financed (red, solid) government expenditures. Lines correspond to median responses. Shaded areas correspond to confidence bands of one standard deviation. Variables are reported in per cent deviations.

Figure 8: Proxy-SVAR: post-1980 subsample. Cumulative multipliers



Notes: Cumulative multipliers following a shock to short-term (blue, dash-dotted) and long-term debt-financed (red, solid) government expenditures. Lines correspond to median responses. Shaded areas correspond to confidence bands of one standard deviation.

Figure 9: Proxy-SVAR: pre-2008 subsample. Cumulative multipliers



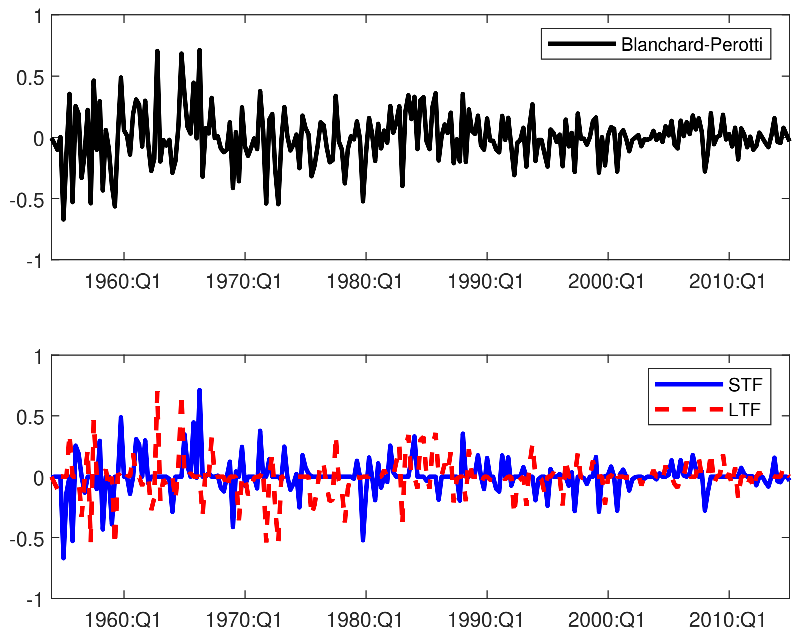
Notes: Cumulative multipliers following a shock to short-term (blue, dash-dotted) and long-term debt-financed (red, solid) government expenditures. Lines correspond to median responses. Shaded areas correspond to confidence bands of one standard deviation.

Identifying spending shocks using Blanchard - Perotti.

We now consider an alternative identification assumption of spending shocks. In particular we derive a series of shocks from a VAR with spending and aggregate output (as well as other macroeconomic variables) and using the Cholesky decomposition, imposing that shocks to output or other variables do not impact government spending within a quarter. This is the well known Blanchard - Perotti identification assumption.[4] After identifying the shocks, we use them in our local projection framework, described in subsection 2.3 of the paper.

Figure 10 plots the spending shocks series as they are identified from the VAR. The bottom panel distinguishes between STF and LTF shocks.

Figure 10: Identified fiscal shocks using Blanchard and Perotti (2002)



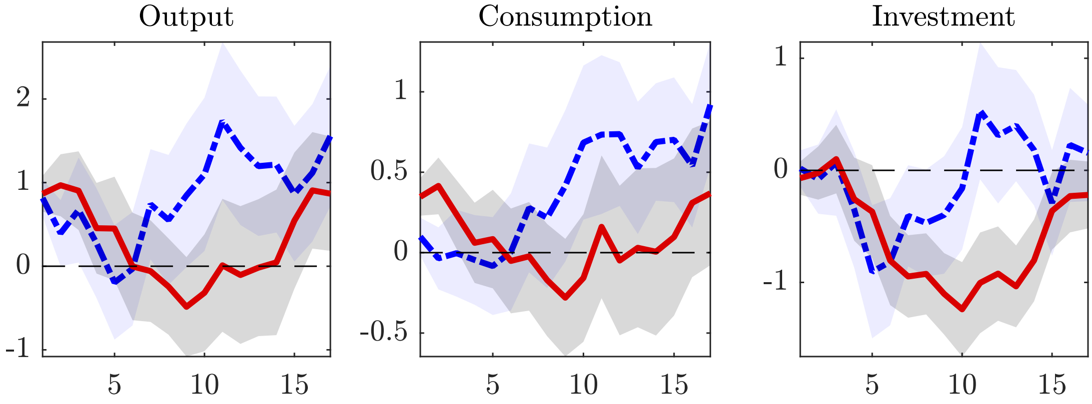
Notes: Top row: Aggregate fiscal shock identified as in Blanchard and Perotti (2002). Bottom row: Identified short-term (STF) and long-term (LTF) debt financed spending shocks. Series are scaled by the trend of GDP.

Figure 11 plots the cumulative multipliers we calculate using local projections. As was discussed in the paper, we continue finding significant differences between STF and LTF multipliers, especially at medium or long horizons. Specifically, the consumption and output multipliers for the STF shock are statistically significant, and the output multiplier exceeds unity. The output multiplier in the case of the LTF shocks is significant only for the first 3 quarters.

[4] A few recent papers have drawn caution on the ability of the Blanchard and Perotti identification scheme to identify exogenous shocks spending in structural VAR models. Ramey (2011); Leeper, Traum, and Walker (2017) have highlighted the limitations of these models in accounting for 'fiscal foresight', i.e. when fiscal measures not observable by the econometrician are known in advance by private agents. One solution that has been proposed is the literature to tackle this problem is to augment the VAR with forward looking variables that may react to news about spending, for example, bond returns, stock prices, spreads etc (e.g. Barsky and Sims, 2011). For this reason, we experimented with VARs featuring a large vector of macroeconomic variables (gdp, prices, spending etc) as well as long and short term interest rates. Our results were not significantly affected and we will not show the output of each of these exercises here.

Since in our baseline empirical estimates we relied on the Ramey news series, we treat the Blanchard-Perotti shocks estimates, as only as a supplement to our main estimates, in light of the issues described above.

Figure 11: State-dependent local projections: Baseline specification. Fiscal multipliers. Blanchard-Perotti shocks.



Notes: Fiscal multipliers following a shock to short-term (blue) and long-term debt-financed (red) government expenditures. Lines correspond to median responses.

Interestingly, some of the differences in the output multipliers we find, can be attributed to the responses of investment to the STF and LTF shocks. In the case of STF we find no significant crowding out of investment, whereas under LTF, and a few quarters after the shock, there is a significant reduction in investment. We speculate that this maybe due to the increase in the long term rates that accompanies the rise of the long bond supply and which can crowd out firm or housing investment that is usually financed with long term borrowing. Though this is an interesting finding, the different responses of investment are not a consistent finding for all the empirical models we run in this paper.

B Model Supplements, Analytic Formulae and Further Numerical Experiments.

This subsection derives the analytic results we showed in Section 3 of the paper and presents additional results from alternative calibrations of the baseline model. We also setup the program of the household in the baseline model and derive the Euler equations. Finally, we consider an extension of the baseline framework, in which we assume that long-term bonds provide partial liquidity services to the private sector.

B.1 Analytical Results in Section 3

Consider the log-linear model of Section 3. Assume that monetary policy sets $\frac{\overline{q}_S}{\overline{C}}\hat{q}_{t,S} + \beta\frac{F_{\bar{\bar{\theta}}}}{\overline{C}} = 0$. We derive the coefficient κ_1 shown in the main text.

First, noting that $\bar{\bar{\theta}}^2 f_{\bar{\bar{\theta}}}\overline{C} - f_{\bar{\bar{\theta}}}\bar{\bar{\theta}}\overline{b}_S = 0$ we can write the resource constraint as:

$$\overline{TC}\hat{TC}_t = \overline{C}\hat{C}_t + \int_0^{\bar{\bar{\theta}}} \theta dF_\theta \overline{C}\hat{C}_t + \overline{b}_S(1 - F_{\bar{\bar{\theta}}})\hat{b}_{t,S}$$

where from the steady state definition of total consumption it holds that:

$$\overline{TC} = \overline{C}(1 + \int_0^{\bar{\bar{\theta}}} \theta dF_\theta) + \overline{b}_S(1 - F_{\bar{\bar{\theta}}}).$$

Using formula (25) in the main text and the policy $\hat{b}_{t,S} = \rho_G^t \varrho \hat{G}_0$ we can write

$$\overline{TC}\hat{TC}_t = \left[\frac{\alpha_2}{\alpha_1}\frac{\overline{C}(1 + \int_0^{\bar{\bar{\theta}}} \theta dF_\theta)}{1 - F_{\bar{\bar{\theta}}}\frac{\beta}{\alpha_1\overline{C}}\rho_G} + \overline{b}_S(1 - F_{\bar{\bar{\theta}}})\right]\rho_G^t \varrho \hat{G}_0.$$

Combining the above it is easy to show that:

$$\hat{TC}_t = \kappa_1 \varrho \rho_G^t \hat{G}_0 \qquad \text{where}$$

$$\kappa_1 = \frac{1}{\overline{C}(1 + \int_0^{\bar{\bar{\theta}}} \theta dF_\theta) + \overline{b}_S(1 - F_{\bar{\bar{\theta}}})}\left[\frac{\alpha_2}{\alpha_1}\frac{\overline{C}(1 + \int_0^{\bar{\bar{\theta}}} \theta dF_\theta)}{1 - F_{\bar{\bar{\theta}}}\frac{\beta}{\alpha_1\overline{C}}\rho_G} + \overline{b}_S(1 - F_{\bar{\bar{\theta}}})\right].$$

[5] Given these formulae it is easy to derive the expression for the fiscal multiplier we showed in text.

Now let us turn to the model where monetary policy follows an inflation targeting rule. We apply the method of undetermined coefficients to find coefficients χ_1, χ_2, χ_3 (in $\hat{\pi}_t = \chi_1\hat{G}_t$, $\hat{C}_t = \chi_2\hat{G}_t$

[5] Notice that depending on the persistence of the shocks κ_1 could exceed 1. For i.i.d spending however κ_1 is strictly smaller than 1. To see this notice that

$$\alpha_1 = \frac{\overline{q}_S}{\overline{C}} + (1-\beta)\frac{1}{\overline{C}}f_{\bar{\bar{\theta}}}\bar{\bar{\theta}} = \beta\frac{F_{\bar{\bar{\theta}}}}{\overline{C}} + \frac{1}{\overline{b}_S}\int_{\bar{\bar{\theta}}}^{\infty} \theta dF_\theta + (1-\beta)\frac{1}{\overline{C}}f_{\bar{\bar{\theta}}}\bar{\bar{\theta}} > \frac{1}{\overline{b}_S}\int_{\bar{\bar{\theta}}}^{\infty} \theta dF_\theta + (1-\beta)\frac{1}{\overline{C}}f_{\bar{\bar{\theta}}}\bar{\bar{\theta}} = \alpha_2$$

and therefore the ratio $\frac{\alpha_2}{\alpha_1}$ is strictly smaller than 1. Then if $\rho_G = 0$, obviously, $\kappa_1 < 1$. For a sufficiently persistent shock we may have $\frac{\alpha_2}{\alpha_1}\frac{1}{1-F_{\bar{\bar{\theta}}}\frac{\beta}{\alpha_1\overline{C}}\rho_G} > 1$ and κ_1 exceeds unity. Clearly, shock persistence exerts an influence due to the assumption that the short bond follows G (implying a bigger increase in the short asset supply inter-temporally when $\rho_G > 0$) and due to the forward looking nature of total consumption.

$\hat{Y}_t = \chi_3 \hat{G}_t$ the expressions in the main text) to satisfy the Phillips curve, the resource constraint and the Euler equation. Recalling also that shocks are i.i.d (so expected future consumption and inflation are 0) we get:

$$\chi_2 = \frac{\alpha_2}{\alpha_1}\varrho - \chi_1 \frac{1}{\alpha_1}\frac{\overline{q}_S}{\overline{\overline{C}}}\phi_\pi$$

from the Euler equation,

$$\chi_1 = \frac{1+\eta}{\omega}\overline{Y}(\gamma_h \chi_3 + \frac{\alpha_2}{\alpha_1}\varrho - \chi_1 \frac{1}{\alpha_1}\frac{\overline{q}_S}{\overline{\overline{C}}}\phi_\pi)$$

from the Phillips curve. This expression can be rearranged to:

$$\chi_1 = \frac{1}{1 + \frac{1+\eta}{\omega}\frac{1}{\alpha_1}\frac{\overline{q}_S}{\overline{\overline{C}}}\phi_\pi}\frac{1+\eta}{\omega}\overline{Y}(\gamma_h \chi_3 + \frac{\alpha_2}{\alpha_1}\varrho)$$

Finally, the resource constraint gives:

$$\overline{C}\left(1 + \int_0^{\bar{\bar{\theta}}} \theta dF_\theta\right)\chi_2 + \bar{b}_S(1 - F_{\bar{\bar{\theta}}})\varrho + \overline{G} = \overline{Y}\hat{Y}_t\chi_3 \rightarrow$$

$$\overline{C}\left(1 + \int_0^{\bar{\bar{\theta}}} \theta dF_\theta\right)[\frac{\alpha_2}{\alpha_1}\varrho - \chi_1 \frac{1}{\alpha_1}\frac{\overline{q}_S}{\overline{\overline{C}}}\phi_\pi] + \bar{b}_S(1 - F_{\bar{\bar{\theta}}})\varrho + \overline{G} = \overline{Y}\chi_3 \rightarrow$$

$$\frac{\alpha_2}{\alpha_1}\varrho[\frac{\overline{C}\left(1 + \int_0^{\bar{\bar{\theta}}} \theta dF_\theta\right)}{1 + \frac{1+\eta}{\omega}\frac{1}{\alpha_1}\frac{\overline{q}_S}{\overline{\overline{C}}}\phi_\pi}] + \bar{b}_S(1 - F_{\bar{\bar{\theta}}})\varrho + \overline{G} = \left[1 + \frac{\overline{C}\left(1 + \int_0^{\bar{\bar{\theta}}} \theta dF_\theta\right)\frac{1}{\alpha_1}\frac{\overline{q}_S}{\overline{\overline{C}}}\phi_\pi}{1 + \frac{1+\eta}{\omega}\frac{1}{\alpha_1}\frac{\overline{q}_S}{\overline{\overline{C}}}\phi_\pi}\frac{1+\eta}{\omega}\gamma_h\right]\overline{Y}\chi_3$$

The final equation can be solved for χ_3. Since $\frac{d\hat{Y}_t}{d\hat{G}_t} = \chi_3$ it becomes easy to show that

$$m_0 = \frac{\overline{Y}d\hat{Y}_0}{\overline{G}d\hat{G}_0} = \frac{1}{\left[1 + \frac{\overline{C}\left(1 + \int_0^{\bar{\bar{\theta}}} \theta dF_\theta\right)\frac{1}{\alpha_1}\frac{\overline{q}_S}{\overline{\overline{C}}}\phi_\pi}{1 + \frac{1+\eta}{\omega}\frac{1}{\alpha_1}\frac{\overline{q}_S}{\overline{\overline{C}}}\phi_\pi}\frac{1+\eta}{\omega}\gamma_h\right]}\left[1 + (\frac{1}{\overline{\overline{G}}}\frac{\alpha_2}{\alpha_1}[\frac{\overline{C}\left(1 + \int_0^{\bar{\bar{\theta}}} \theta dF_\theta\right)}{1 + \frac{1+\eta}{\omega}\frac{1}{\alpha_1}\frac{\overline{q}_S}{\overline{\overline{C}}}\phi_\pi}] + \bar{b}_S(1 - F_{\bar{\bar{\theta}}}))\varrho\right]$$

Finally notice that in the notation we used in text we defined

$$a_3(\phi_\pi) = \frac{1}{\left[1 + \frac{\overline{C}\left(1 + \int_0^{\bar{\bar{\theta}}} \theta dF_\theta\right)\frac{1}{\alpha_1}\frac{\overline{q}_S}{\overline{\overline{C}}}\phi_\pi}{1 + \frac{1+\eta}{\omega}\frac{1}{\alpha_1}\frac{\overline{q}_S}{\overline{\overline{C}}}\phi_\pi}\frac{1+\eta}{\omega}\gamma_h\right]}$$

B.2 Household Optimality in the Baseline Model

We now derive the first order optimality conditions from the household's program in the baseline model. The dynamic program of household i is the following:

(B.1)

$$V_t(B^i_{L,t-1}, B^i_{S,t-1,2}, X_t) = \max_{B^i_{L,t}, B^i_{S,t}, C^i_t, c^i_t, h^i_t}\left\{u(C^i_t) + E_\theta \theta v(c^i_t) - \chi\frac{{h_t}^{i,1+\gamma}}{1+\gamma} + \beta E_t\left[V_{t+1}(B^i_{L,t}, B^i_{S,t,2}, X_{t+1})\right]\right\}$$

subject to:

(B.2)

$$P_tC_t^i + q_{L,t}B_{L,t}^i + q_{S,t}B_{S,t}^i = P_t(1-\tau_t)w_th_t^i + (1+q_{L,t}\delta)B_{L,t-1}^i + B_{S,t-1,2}^i + D_tP_t - T_tP_t - P_t\bar{C}_t^i,$$

(B.3) $$B_{S,t,2}^i = E_\theta(B_{S,t}^i - P_tc_t(\theta)) + P_t\bar{C}_{t,S}^i,$$

(B.4) $$P_tc_t^i(\theta) \leq B_{S,t}^i.$$

Let λ_t denote the multiplier on the budget constraint, $\omega_{S,t}$ and $\psi_t(\theta)$ the analogous objects on constraints (B.3) and (B.4); the first order conditions for the variables defining the optimal portfolio are the following:

$$\begin{aligned}
B_{S,t}^i &: \lambda_t q_{S,t} - \omega_{S,t} + \int \psi_t(\theta)dF_\theta = 0 \\
B_{L,t} &: \lambda_t q_{L,t} - \beta E_t V_{B_L,t+1} = 0 \rightarrow \lambda_t q_{L,t} = \beta E_t\lambda_{t+1} \\
B_{S,t,2}^i &: \omega_{S,t} = -\beta E_t V_{B_S,t+1} = \beta E_t\lambda_{t+1} \\
c_t^i(\theta) &: \theta v_c^i f_\theta + \omega_{S,t}P_tf_\theta - \psi_t(\theta)P_tf_\theta = 0
\end{aligned}$$

where we also made use of the envelope conditions $V_{B_S,t} = -\lambda_t$ and $V_{B_L,t} = -\lambda_t(1+\delta q_{L,t})$. Complementary slackness gives: $\psi_t(\theta) \geq 0$, $\psi_t(\theta)(B_{S,t}^i - P_tc_t^i(\theta)) = 0$.

The solution is characterized by $\widetilde{\theta}_t$ such that if $\theta \geq \widetilde{\theta}_t$ then (B.4) binds. Realizing also that $\lambda_t = -\frac{u_{C,t}}{P_t}$ (from the FONC of C_t) we can then show that:

$$\begin{aligned}
\lambda_t q_{S,t} - \omega_{S,t} + \int_{\widetilde{\theta}_t} \psi_t(\theta)dF_\theta &= \lambda_t q_{S,t} - \omega_{S,t} + \int_{\widetilde{\theta}_t} \frac{\theta v_c^i}{P_t}dF_\theta + \omega_{S,t}(1-F_{\widetilde{\theta}_t}) = 0 \\
\rightarrow q_{S,t}\frac{u_{C,t}}{P_t} &= \int_{\widetilde{\theta}_t} \frac{\theta v_c^i}{P_t}dF_\theta + \beta F_{\widetilde{\theta}_t}E_t\frac{u_{C,t+1}}{P_{t+1}}
\end{aligned}$$

the Euler equation for short-term debt as in the main text. Subsituting $\lambda_t = -\frac{u_{C,t}}{P_t}$ is the FONC for $B_{L,t}$ we can easily get the Euler equation for long-term bonds.

Finally, note that it is trivial to derive the static labour supply condition from the above dynamic program. We therefore omit the derivations.

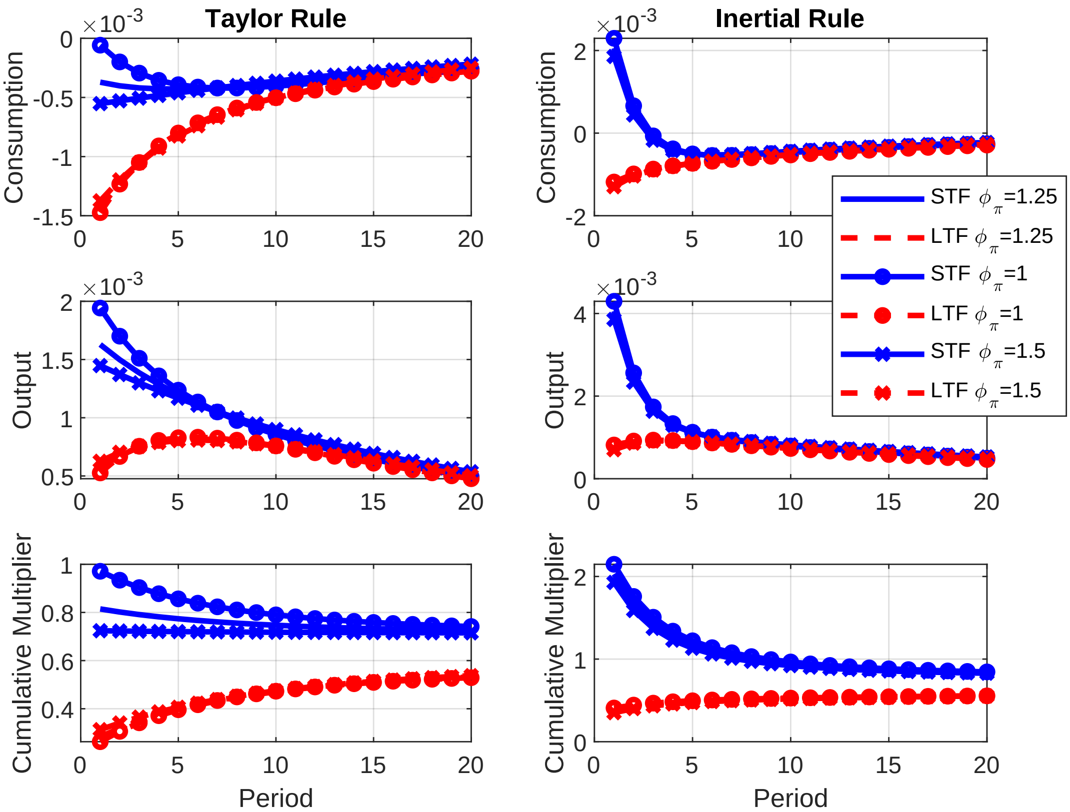
B.3 Alternative Interest rate rules, distortionary taxes.

We now show additional output from our baseline model. In the main text our numerical results relied on interest rate rules in which the nominal rate tracks the inflation rate and the lagged interest rate. We now perform additional experiments with broader calibrations of the inflation coefficients and also consider rules in which the output gap is targeted by the monetary authority along with inflation and the lagged interest rate.

$$\hat{i}_t = (1-\rho_i)(\phi_\pi\hat{\pi}_t + \phi_Y\hat{Y}_t) + \rho_i\hat{i}_{t-1}$$

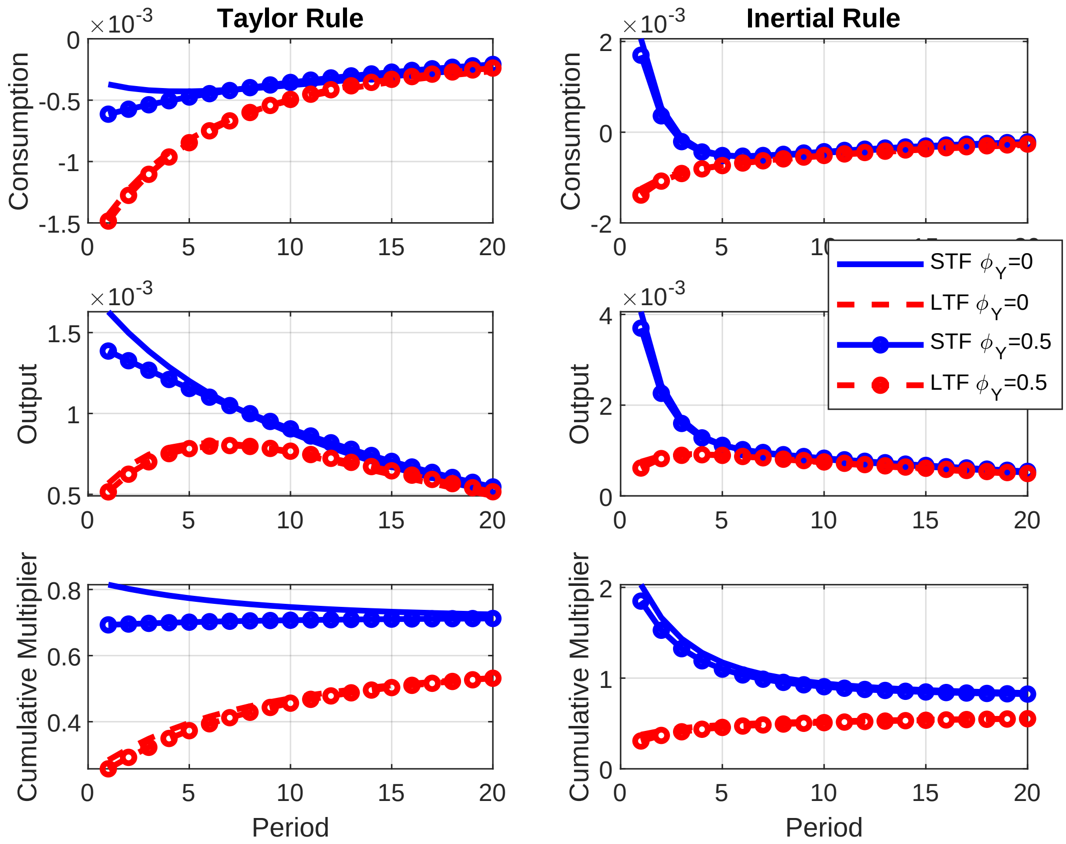
In Figure 12 we constrain ϕ_Y to be zero (our baseline calibration for this parameter) and show the impulse responses for $\phi_\pi = 1, 1.25$ (the baseline values) and 1.5. As can be seen from the figure, assuming a higher inflation coefficient does reduce somewhat the response of the economy to the STF shock, but the gap with LTF remains. Moreover, the inflation coefficient effectively does not matter for the responses of output, consumption and the multiplier in the case of LTF, since inflation reacts very little to the shock in that case.

Figure 12: Responses to a spending shock: Inflation coefficients



Notes: We plot the paths of consumption, output and the cumulative fiscal multiplier following a shock that increases spending by 1 percent on impact. The interest rate rule is $\hat{i}_t = (1-\rho_i)\phi_\pi \hat{\pi}_t + \rho_i \hat{i}_{t-1}$ *The 'Taylor rule' assumes* $\rho_i = 0$. *The 'Inertial Rule' sets* $\rho_i = 0.9$. *We assume that* $\phi_\pi \in \{1, 1.25, 1.5\}$

Figure 13: Responses to a spending shock: Output gap target



Notes: We plot the paths of consumption, output and the cumulative fiscal multiplier following a shock that increases spending by 1 percent on impact. The interest rate rule is $\hat{i}_t = (1-\rho_i)\phi_\pi\hat{\pi}_t + \phi_Y\hat{Y}_t + \rho_i\hat{i}_{t-1}$ *The 'Taylor rule' assumes* $\rho_i = 0$. *The 'Inertial Rule' sets* $\rho_i = 0.9$. *We assume that* $\phi_\pi = 1.25$ *and* $\phi_Y \in \{0, 0.5\}$

In Figure 13 we contrast the responses in the case $\phi_Y = 0$ with the analogous objects when $\phi_Y = 0.5$ (output gap target). We focus on the 'active' monetary policy scenario, assuming that taxes adjust to make government debt solvent. We set $\phi_\pi = 1.25$ as in the baseline calibration of the model.

The results show that setting a positive output target coefficient does not change our conclusions under both the 'Taylor rule' and the inertial monetary policy rule. We continue finding a large difference between STF and LTF.

Next, we study the impulse response functions in an economy with distortionary taxation. Under distortionary taxes the Euler equations we derived in the main text continue to hold, the only changes to the system of equilibrium conditions concern the government's budget constraint and the Phillips curve. The government's revenue now becomes

$$\overline{\tau}\overline{Y}\frac{1+\eta}{\eta}\left((1+\gamma_h)\hat{Y}_t + \hat{C}_t + \frac{1}{1-\overline{\tau}}\hat{\tau}_t\right)$$

where $\overline{\tau}$ denotes the steady state distortionary tax. Notice that now revenue depends also on aggregate output and on consumption, and hence of the path of these variables following a spending shock. Moreover, the Phillips curve now is:

$$\hat{\pi}_t = \frac{1+\eta}{\omega}\overline{Y}(\gamma\hat{Y}_t + \hat{C}_t + \frac{\overline{\tau}}{1-\overline{\tau}}\hat{\tau}_t) + \beta E_t\hat{\pi}_{t+1}$$

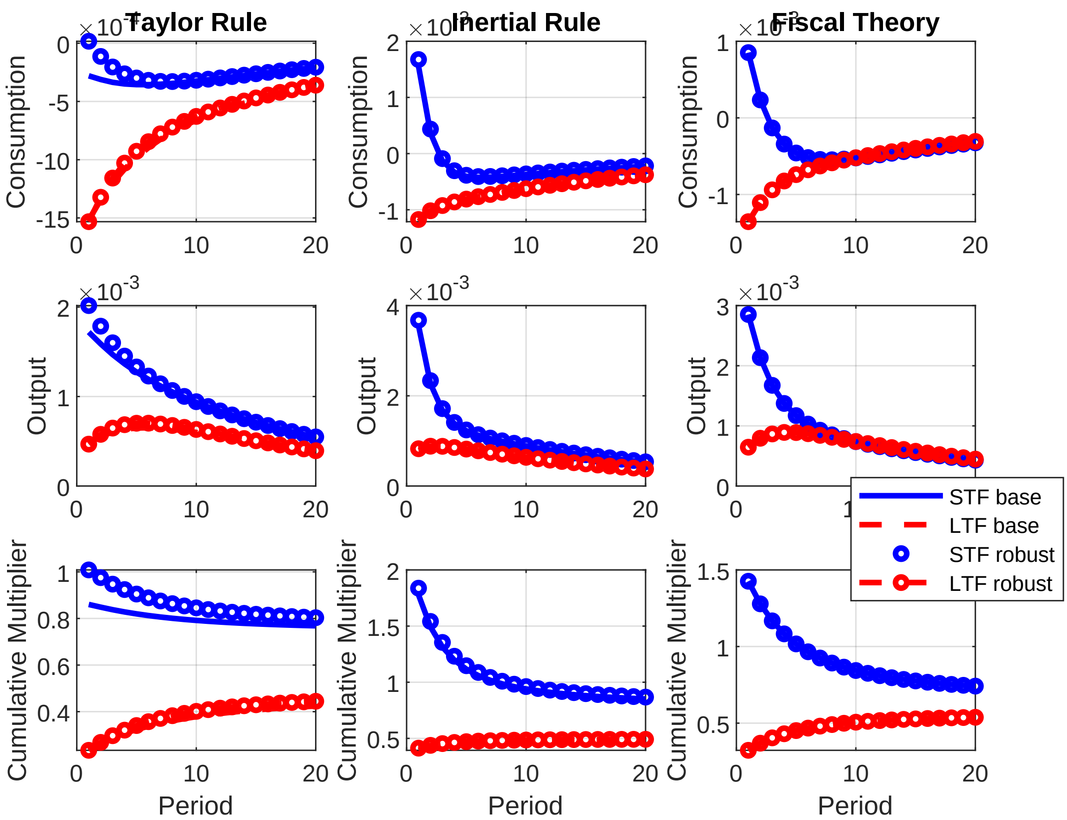
To solve the model we specify fiscal policy using the following tax rule

$$\hat{\tau}_t = \phi_\tau \hat{D}_{t-1}$$

As in the case of lump sum taxes we studied in the main text, we consider separately the case where monetary policy is 'active' (assuming that ϕ_τ is close to the threshold defining the determinacy region, so that government debt displays a near unit root) and the case where monetary policy is 'passive' (then setting $\phi_\tau = 0$.)

Figure 14 shows the impulse response functions for the same parameter values we considered ϕ_π, ρ_i we considered in the main text. Clearly, the model responses are (essentially) the same as in the model with lump sum taxation.

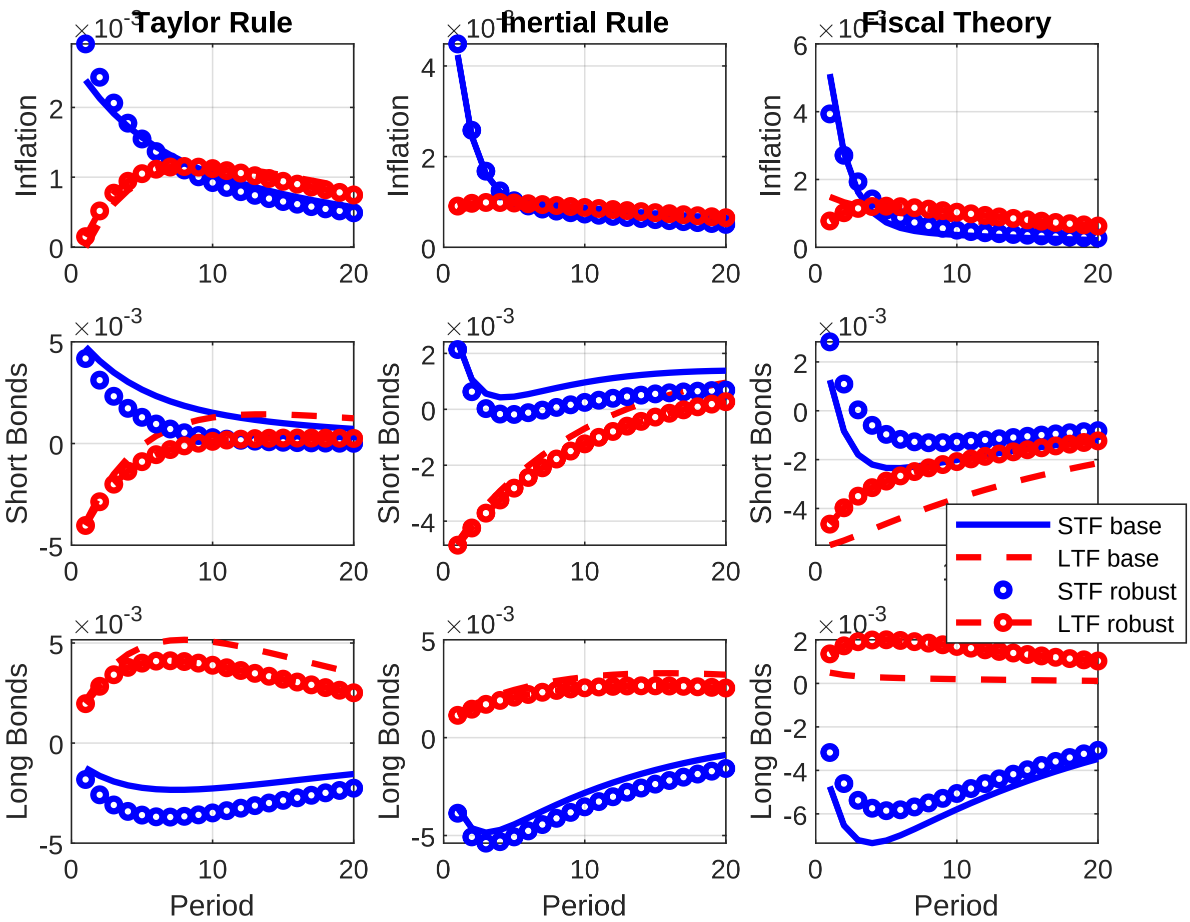
Figure 14: Responses to a spending shock: Distortionary Taxes



Notes: We plot the paths of consumption, output and the cumulative fiscal multiplier following a shock that increases spending by 1 percent on impact. The interest rate rule is $\hat{i}_t = (1-\rho_i)\phi_\pi\hat{\pi}_t + \rho_i\hat{i}_{t-1}$ *The 'Taylor rule' assumes* $\rho_i = 0$. *The 'Inertial Rule' sets* $\rho_i = 0.9$. *We assume in both cases that* $\phi_\pi \in \{1, 1.25\}$. *The Fiscal Theory scenario sets the baseline inflation coefficient to zero and the 'robust' graphs assume* $\phi_\pi = 0.5$.

B.4 Responses of Inflation, Short and Long Bonds.

Figure 15: Responses to a spending shock: inflation, short and long term bonds



Notes: The figure plots the responses of inflation, short bonds and long bonds, under the different specifications of monetary policy, which we considered in the main text. The left panel shows results for a standard Taylor rule, the middle panel for an inertial monetary policy rule, and the right panel for a passive monetary policy rule.

We now study the responses of inflation and the quantities of short and long term bonds to a spending shock under the different specifications of monetary policy we considered in the main text. The left panels in Figure 15 show the case of a standard Taylor rule, the middle panels an inertial monetary policy rule, and the right panels a 'passive' monetary policy as in the Fiscal Theory of the Price Level.

As shown in the top panels, under the STF shock, inflation increases substantially in all versions of the model. In contrast, the LTF shocks lead to much more moderate increases in inflation across all models. This is not surprising. As explained in the main text, financing the spending shock short-term, is equivalent to a positive demand shock (a shock to the Euler equation). Since the demand shock results in positive inflation, it reinforces the inflationary effect of the spending shock.

Interestingly, under the Taylor rule (left panels) the quantity of short term bonds in the LTF case turns higher than under STF after eight quarters. This is not as surprising as it initially sounds. Since initial inflation in the STF case is higher and more frontloaded, total debt increases by less than in the LTF scenario. Initially, the relative effect, the mechanical decline in the share of short bonds dominates the quantity effect stemming from the increase in total debt. Over time, the net effect changes sign in the case of a Taylor rule.

In the case of the Fiscal Theory (right panel) the increase in LTF inflation above STF inflation, is mainly explained by the paths of taxes and spending. Inflation in the LTF regime has to eventually rise above STF to ensure intertemporal budget solvency. In other words, the fiscal shock in the STF case is financed with higher and more frontloaded inflation and in the LTF case with lower and more persistent inflation that reduces the real value of debt. But, it is worth noting that the two models will not result in the same cumulative increase in the price level to finance the spending shock. The reason is that with liquid debt, debt is not only financed by surpluses but also by 'liquidity rents' (See Section C of this appendix); and these rents will tend to decrease when the government expands the short bond supply under STF.

The figure also shows that total debt under the Fiscal theory is, as it ought to be, lower than in the other two scenarios. This reflects that the fiscal deficits in this model are unbacked and that, therefore, inflation rises by more than in the other scenarios to ensure intertemporal debt solvency.

The middle graphs in Figure 15 illustrate that for all STF shocks, the quantity of short term bonds increases. In contrast, the real quantity of long term debt (in log deviation from the steady state level) may decrease (the LTF case produces the opposite patterns). Real long bonds decrease for two reasons. First, due to the rise in inflation (holding constant the nominal value of debt). Second, because of portfolio rebalancing (some of the long term debt outstanding has matured) and the government refinances with short term bonds when the ratio of short over long is higher.

B.5 Assuming Long Bonds provide partial liquidity services

We now consider an extension of the baseline model in which long bonds can provide *partial liquidity services* to the private sector. More specifically, we now assume the following constraint on subperiod 2 consumption:

$$P_t c_t^i(\theta) \leq B_{S,t}^i + \kappa B_{L,t}^i$$

where κ is the fraction of long-term asset that can be used to finance consumption in subperiod 2.[6] $\kappa = 0$ is our baseline. For $\kappa > 0$ long bonds can be liquidated along with short bonds to finance c_t^i.

The program of household i now is:

$$\text{(B.5)}\quad V_t(B_{L,t-1}^i, B_{S,t-1,2}^i, X_t) = \max_{B_{L,t}^i, B_{S,t}^i, C_t^i, c_t^i, h_t^i} \left\{ u(C_t^i) + E_\theta \theta v(c_t^i) - \chi \frac{{h_t}^{i,1+\gamma}}{1+\gamma} + \beta E_t \left[V_{t+1}(B_{L,t}^i, B_{S,t,2}^i, X_{t+1}) \right] \right\}$$

subject to:

$$\text{(B.6)}\quad P_t C_t^i + q_{L,t} B_{L,t}^i + q_{S,t} B_{S,t}^i = P_t(1-\tau_t) w_t h_t^i + (1 + q_{L,t}\delta) B_{L,t-1}^i + B_{S,t-1,2}^i + D_t P_t - T_t P_t - P_t(\bar{C}_{t,S}^i + \bar{C}_{t,L}^i)$$

$$\begin{aligned}
&\text{(B.7)} & B_{S,t,2}^i &= E_\theta(B_{S,t}^i - P_t(c_t(\theta) - \kappa d_{L,t}^i(\theta)) + P_t \bar{C}_{t,S}^i, \\
&\text{(B.8)} & B_{L,t,2}^i &= E_\theta(B_{L,t}^i - d_{L,t}^i(\theta) P_t) - P_t \bar{C}_{t,L}^i \\
&\text{(B.9)} & P_t c_t^i(\theta) &\leq B_{S,t}^i + \kappa d_{L,t}^i(\theta) P_t \\
&\text{(B.10)} & d_{L,t}^i(\theta) P_t &\leq B_{L,t}^i
\end{aligned}$$

[6] For simplicity, we assume a constant fraction of the quantity of bonds can be liquidated (or κ is a constant times the steady state bond price). As in the case of short-term debt we use only the bond quantity in the constraint (not quantity times price) to get an analogous Euler equation for long-term bonds.

We let variable $d^i_{L,t}$ denote the withdrawals from long-term asset account for convencience. $\bar{C}^i_{t,S}$ and $\bar{C}^i_{t,L}$ are the appropriate sales of household goods in subperiod 2 corresponding to short and long bonds respectively.

Let λ_t denote the multiplier on the budget constraint, $\omega_{S,t}$ $\omega_{L,t}$ $\psi_t(\theta)$ and $\epsilon_t(\theta)$ the analogous objects on constraints (B.7) to (B.10); the first order conditions for the variables defining the optimal portfolio are the following:

$$
\begin{aligned}
B^i_{S,t}: & \qquad \lambda_t q_{S,t} - \omega_{S,t} + \int \psi_t(\theta) dF_\theta = 0 \\
B^i_{L,t}: & \qquad \lambda_t q_{L,t} - \omega_{L,t} + \int \epsilon_t(\theta) dF_\theta = 0 \\
d^i_{L,t}(\theta): & \qquad -\kappa\omega_{S,t} P_t f_\theta + \omega_{L,t} P_t f_\theta + \kappa\psi_t(\theta) P_t f_\theta - \epsilon_t(\theta) P_t f_\theta = 0 \\
B^i_{S,t,2}: & \qquad \omega_{S,t} = -\beta E_t V_{B_S,t+1} = \beta E_t \lambda_{t+1} \\
B^i_{L,t,2}: & \qquad \omega_{L,t} = -\beta E_t V_{B_L,t+1} = \beta E_t \lambda_{t+1}(1+\delta q_{L,t+1}) \\
c^i_t(\theta): & \qquad \theta v^i_c f_\theta + \omega_{S,t} P_t f_\theta - \psi_t(\theta) P_t f_\theta = 0
\end{aligned}
$$

where we also made use of the envelope conditions $V_{B_S,t} = -\lambda_t$ and $V_{B_L,t} = -\lambda_t(1+\delta q_{L,t})$. Complementary slackness gives: $\psi_t(\theta) \geq 0$, $\epsilon_t(\theta) \geq 0$ $\psi_t(\theta)(B^i_{S,t} + \kappa d^i_{L,t}(\theta) P_t - P_t c^i_t(\theta)) = 0$ and $\epsilon_t(\theta)(B^i_{L,t} - d^i_{L,t}(\theta) P_t) = 0$.

As before, the solution is characterized by $\widetilde{\theta}_t$ such that if $\theta \geq \widetilde{\theta}_t$ then (B.9) and (B.10) bind. Realizing also that $\lambda_t = -\frac{u_{C,t}}{P_t}$ we can then show that

$$
\begin{aligned}
\lambda_t q_{S,t} - \omega_{S,t} + \int_{\widetilde{\theta}_t} \psi_t(\theta) dF_\theta &= \lambda_t q_{S,t} - \omega_{S,t} + \int_{\widetilde{\theta}_t} \frac{\theta v^i_c}{P_t} dF_\theta + \omega_{S,t}(1 - F_{\widetilde{\theta}_t}) = 0 \\
\rightarrow q_{S,t} \frac{u_{C,t}}{P_t} &= \int_{\widetilde{\theta}_t} \frac{\theta v^i_c}{P_t} dF_\theta + \beta F_{\widetilde{\theta}_t} E_t \frac{u_{C,t+1}}{P_{t+1}}
\end{aligned}
$$

the same Euler equation for short-term debt as in the main text. For long-term bonds we have:

$$
\begin{aligned}
\lambda_t q_{L,t} - \omega_{L,t} + \int_{\widetilde{\theta}_t} \epsilon_t(\theta) dF_\theta &= \lambda_t q_{L,t} - \omega_{L,t} + \int_{\widetilde{\theta}_t} [-\kappa\omega_{S,t} + \omega_{L,t} + \kappa\psi_t(\theta)] dF_\theta \\
&\rightarrow \lambda_t q_{L,t} - \omega_{L,t} F_{\widetilde{\theta}_t} + \kappa \int_{\widetilde{\theta}_t} \frac{\theta v^i_c}{P_t} dF_\theta = 0 \\
&\rightarrow \frac{u_{C,t}}{P_t} q_{L,t} = \kappa \int_{\widetilde{\theta}_t} \frac{\theta v^i_c}{P_t} dF_\theta + \beta F_{\widetilde{\theta}_t} E_t \frac{u_{C,t+1}}{P_{t+1}}(1+\delta q_{L,t+1})
\end{aligned}
$$

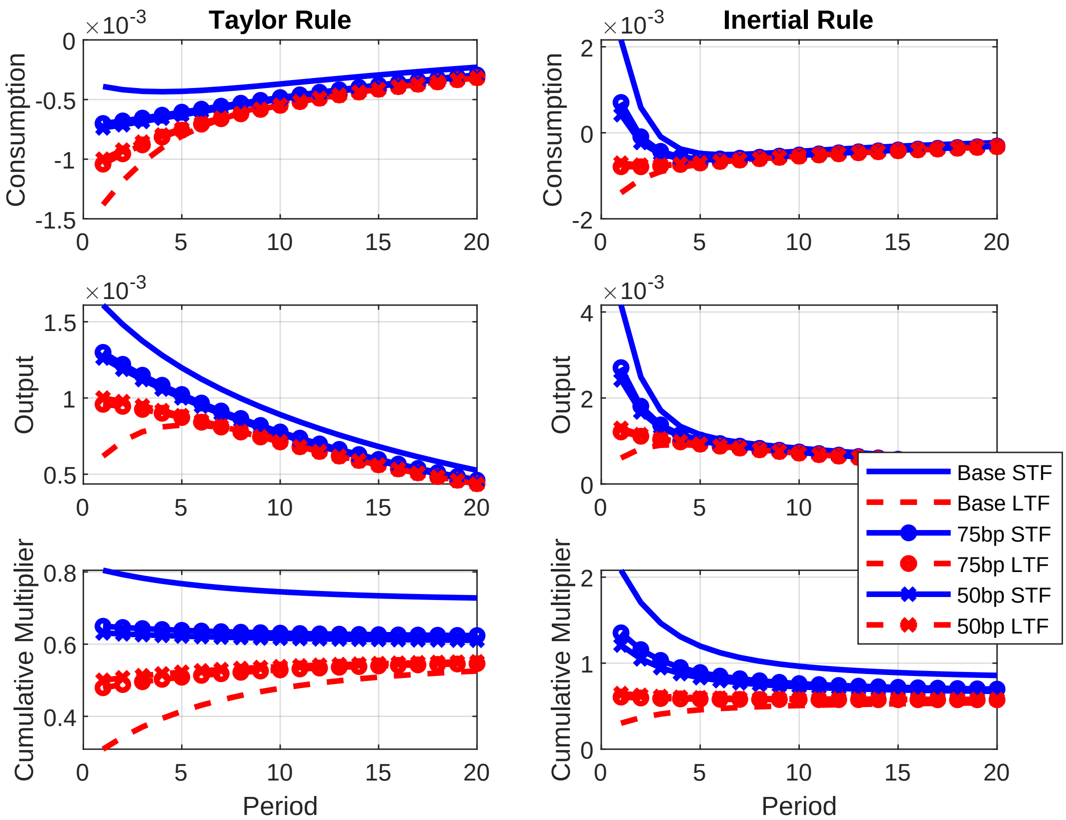
The resource constraint can be modified to reflect that now the consumption of constrained agents is given by $b_{t,S} + \kappa b_{t,L}$. For brevity, we omit the derivation since it is trivial.

We run the model for different calibrations of parameter κ. We discipline our exercise by choosing different κs to target different levels of the term spread. Our baseline calibration in the main text assumes a term spread that is equal to 1 percent per annum when $\kappa = 0$. We consider two alternative calibrations of κ to have an annual term premium equal to 75 and 50 basis points.[7]

[7] In each case we adjust the parameters of the distribution f_θ to match the estimates of Greenwood et al. (2015). Generically, positive κ implies a stronger reaction of the term premium to an increase in the Bills to GDP ratio (the variable used by Greenwood et al. (2015) in their empirical exercise) and so we need to increase the variance of f_θ to the empirical evidence. If we keep the variance constant as in our baseline calibration we get a much stronger reaction of the spending multiplier to financing.

The impulse responses are plotted in Figure 16. The left panels assume a non-inertial rule with inflation coefficient equal to 1.25 and in the right panels we set $\rho_i = 0.9$. As can be seen from the figure assuming partial liquidity services of long-term debt does mute the STF multipliers and increase the LTF multipliers. However, the differences continue being substantial, even when the term premium is as small as 50bps per annum, and especially in the case of the more empirically relevant inertial interest rate rule. We therefore conclude that our results do not hinge on the assumption that long bonds are not liquid and can be used to transfers resources across periods.

Figure 16: Responses to a spending shock when long bonds provide partial liquidity



Notes: We plot the paths of consumption, output and the cumulative fiscal multiplier following a shock that increases spending by 1 percent on impact. The various plots correspond to alternative calibrations of the model when long bonds can provide partial liquidity. Solid lines without markers are the baseline calibration (no liquidity). Lines with circles calibrate the liquidity parameter κ so that the term spread is 75 bps per annum. Lines with crosses set the term premium equal to 50 bps. As usual STF is blue lines and LTF is red lines.

B.6 An alternative calibration of the share of short over long.

As we discussed in text, short maturity debt in our model is of one quarter duration, however, in the empirical exercise we defined short term debt to be any debt that is of maturity less than a year. We now experiment with an alternative definition of the share of short over long in our model which includes debt which is of maturity 2, 3 and 4 quarters.

To do so we define as short term debt, the coupon payments of the long term asset that have duration less than or equal to one year. Recall that a long term bond issued in t pays coupons that decay at rate δ. Then the payments 1, δ, δ^2, δ^3 which are to be paid in $t+1, t+2, ..., t+4$ can essentially be counted as short bonds at the end of period t. Consequently, the face value of short debt becomes $b_{S,t} + b_{L,t}(1+\delta+\delta^2+\delta^3) = b_{S,t} + b_{L,t}\frac{1-\delta^4}{1-\delta}$ and analogously the value of long debt is $b_{L,t}\frac{\delta^4}{1-\delta}$.

The share (in levels) of short term over long term debt is:

$$\widetilde{s}_t^{\text{Short/Long}} = \frac{b_{S,t} + b_{L,t}\frac{1-\delta^4}{1-\delta}}{b_{L,t}\frac{\delta^4}{1-\delta}}$$

In log deviations we obtain:

$$\hat{\widetilde{s}}_t^{\text{Short/Long}} = \frac{1}{\overline{\widetilde{s}}^{\text{Short/Long}}}\frac{\overline{b}_S}{\overline{b}_L\frac{\delta^4}{1-\delta}}\left(\hat{b}_{S,t} - \hat{b}_{L,t}\right).$$

We now solve the model seting $\hat{\widetilde{s}}_t^{\text{Short/Long}} = \varrho\hat{G}_t$ and ϱ equal to 0.6 (-0.6) for a short term (long term) financed spending shock.

We calibrate the model as follows: First, we keep $\delta = 0.96$ as in the baseline calibration. Then we set the average share $\overline{\widetilde{s}}^{\text{Short/Long}}$ such the model produces an average debt maturity roughly equal to our baseline (5 years). This implies that the share of short over long term debt is roughly 0.30.[8] Furthermore, to calibrate the parameters of F_θ we repeated the steps reported in text, that is requiring that the model matches the empirical evidence of Greenwood et al. (2015). The remaining parameters of the model assume the values we reported in text.

Figure 17 repeats the main exercises we considered in text, in this new calibration of the model. Notice that now the differences in the fiscal multipliers across STF and LTF are even larger than in our baseline experiments. For example, we obtain a strong positive effect of the fiscal shock on consumption under STF even when we assume a simple Taylor rule (left panels). The corresponding cumulative multiplier then exceeds one for all values ϕ_π considered. The STF multipliers for the inertial rule (middle panels) and the passive monetary policy (right panels) are also larger than their baseline counterparts.

It is of course not difficult to explain these differences. Under the new calibration the quantity of short bonds needs to increase more sharply in the STF regime, when the elasticity of the share with respect to the spending shock is 0.6. Thus financing short term, induces a bigger consumption boom now than in the baseline calibration. Analogously, the quantity of short debt drops more sharply in the LTF scenario when the elastisticity is -0.6, leading to a tightening of the liquidity constraint.

C Optimal Policy.

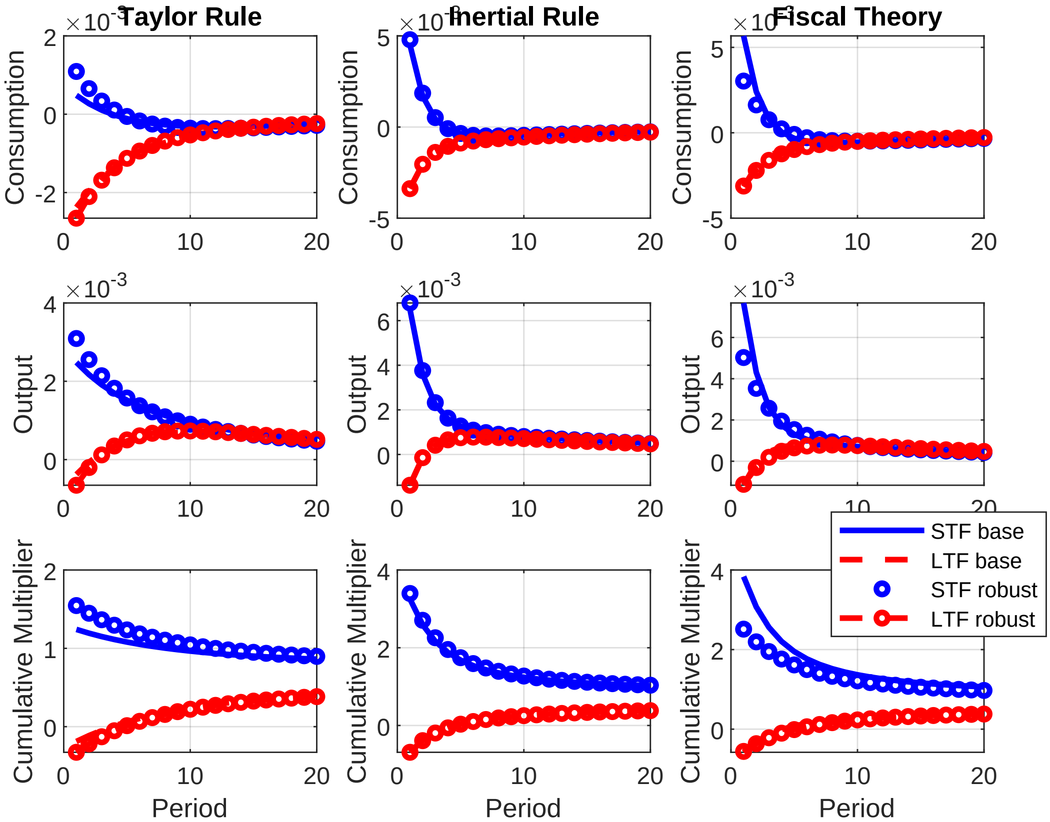
In this section we setup and solve the optimal policy presented in Section 4 of the main text. Our framework follows closely numerous papers in the related literature studying optimal fiscal, monetary and debt policies under a Ramsey planner (e.g. Aiyagari, Marcet, Sargent, and Seppälä (2002); Schmitt-Grohé and Uribe (2004); Lustig, Sleet, and Yeltekin (2008); Faraglia, Marcet, Oikonomou, and Scott (2016); Faraglia et al. (2019); Angeletos, Collard, and Dellas (2022) among many others). As in these papers we assume that the benevolent planner maximizes household utility by choosing

[8] Unfortunately, with this alternative modelling of the share, a target of 12.5 percent is too low, and fixing $\delta = 0.96$ implies that the quantity of quarterly bonds in steady state turns negative!

A 12.5 percent target is also not consistent with the data. Faraglia, Marcet, Oikonomou, and Scott (2019) report that in post world war II US data, the average share of short term debt (in their case also defined as all debt of maturity less than or equal to one year, including the coupons of long term bonds) over total debt was roughly 40 percent. If we calibrate $\overline{b}_S, \overline{b}_L$ to match this number we get a share of short over long term debt equal to 70 percent. But this calibration also does not correspond to our empirical exercise since in the empirical model we did not count coupon payments as short term debt.

Therefore, we compromise with $\overline{\widetilde{s}}^{\text{Short/Long}} = 0.3$ in our calibration to target average maturity.

Figure 17: Responses to a spending shock under an alternative definition of the share Short/Long.



Notes: We recalibrate the share of short over long as discussed in paragraph B.6. The Figure shows the impulse responses of consumption, output and the output multiplier for the same numerical experiments considered in the baseline model in text.

policy variables subject to the set of sufficient implementability conditions for a competitive equilibrium. We assume throughout that taxes can only be distortionary, the government cannot finance debt and spending using lump sum taxes, as was the case in some of the versions of the model we considered in the main text.

We begin by setting up the planning program and the implementability constraints in Proposition 1. We then setup the dynamic optimization problem using a Lagrangian and derive the first order conditions. Moreover, we also describe in this section the numerical algorithm that we use to solve the optimal policy allocation. As we discuss in detail, ours is a non-trivial optimization problem since in the presence of liquidity services provided by short-term government bonds, there may be multiple solutions to the system of first order conditions, corresponding to different local optima. Our numerical procedure therefore has to rank multiple stationary points according to the expected utility they yield, and for this purpose we wed the stochastic simulations PEA used in previous work to solve Ramsey models with global methods (Aiyagari et al., 2002; Faraglia et al., 2019, 2016), with a numerical approximation of value function.

C.1 Policy objective and constraints.

The benevolent planner maximizes expected household utility choosing the sequence of variables $\left\{\pi_t, Y_t, \theta_t, w_t, \tau_t, q_{S,t}, q_{L,t}, b_{L,t}, b_{S,t}, \widetilde{\theta}_t, C_t\right\}_{t\geq 0}$ subject to the equations that define the competitive equilibrium in our model. This system of equations comprises of the household and government budget constraints, the households' bond pricing/ Euler equations and the labour supply conditions, the resource constraint of the economy and the New Keynesian Phillips curve.

As it is common in the context of Ramsey policy programs, we can dispense with some of the equations focusing on a set of sufficient implementability constraints for a competitive equilibrium. The following proposition states the constraints that need to be acknowledged in the policy program and derives the policy objective function.

Proposition 1 (Ramsey program): *The benevolent Ramsey planner solves the following program:*

$$\max E_0 \sum_{t\geq 0} \beta^t V_t \tag{C.11}$$

$$\text{where} \quad V_t \equiv \left(1 + \int_0^{\widetilde{\theta}_t} \theta dF_\theta\right)\log(C_t) + \int_0^{\widetilde{\theta}_t} \theta\log(\theta)dF_\theta + \int_{\widetilde{\theta}_t}^{\infty} \theta\log(b_{S,t})dF_\theta - \chi\frac{Y_t^{1+\gamma_h}}{1+\gamma_h}$$

subject to constraints:

$$C_t + C_t\int_0^{\widetilde{\theta}_t} \theta dF_\theta + \int_{\widetilde{\theta}_t}^{\infty} b_{t,S}dF_\theta + G_t + \frac{\omega}{2}(\pi_t - 1)^2 = Y_t \tag{C.12}$$

$$b_{t,S}\left[\int_{\widetilde{\theta}_t}^{\infty} \theta v'(b_{t,S}^i)dF_\theta + F(\widetilde{\theta}_t)E_t\frac{u'(C_{t+1})}{\pi_{t+1}}\beta\right] + b_{t,L}\sum_{j\geq 1}\beta^j\delta^{j-1}E_t\frac{u'(C_{t+1})}{\Pi_{k=1}^j\pi_{t+k}}$$
$$= \frac{b_{t-1,S}}{\pi_t}u'(C_t) + \frac{b_{t-1,L}}{\pi_t}(u'(C_t) + \delta\sum_{j\geq 1}\beta^j\delta^{j-1}E_t\frac{u'(C_{t+j})}{\Pi_{k=1}^j\pi_{t+k}}) + G_tu'(C_t) - \frac{\tau_t}{1-\tau_t}\chi Y_t^{1+\gamma_h} \tag{C.13}$$

$$\pi_t(\pi_t - 1)U'(C_t) = \frac{\eta}{\omega}(\frac{1+\eta}{\eta}U'(C_t) - \frac{1}{1-\tau_t}\chi Y_t^{\gamma_h})Y_t + \beta E_tU'(C_{t+1})\pi_{t+1}(\pi_{t+1} - 1) \tag{C.14}$$

and condition $C_t\widetilde{\theta}_t = b_{S,t}$.

For brevity, we will not provide a full proof of the Proposition. We will only prove formally some of the elements and argue intuitively for the rest.

According to Proposition 1 the resource constraint (C.12, see below for a derivation) , the government budget constraint, the Phillips curve and condition $C_t\widetilde{\theta}_t = b_{S,t}$ are sufficient for a competitive equilibrium. We can therefore dispense with the household budget constraints. Given that in our model the family pools the resources of all agents and uses transfers to redistribute at the end of every period, agent specific budget constraints will be slack. That is, given an allocation we can always find transfers to satisfy the individual constraints and these can therefore be dropped from the Ramsey program. Furthermore, the family wide budget constraint can be obtained by adding the resource and the government budget constraints. Hence, any allocation that satisfies the latter constraints will also satisfy the family budget.

In equation (C.12) we have made use of the optimality condition $c_t = \theta C_t$ when $\theta < \widetilde{\theta}_t$ and $c_t = b_{t,S}$ when $\theta \geq \widetilde{\theta}_t$ (dropping superscripts i for simplicity). This gives

$$\int c_t(\theta)dF_\theta = C_t \int_0^{\widetilde{\theta}_t} \theta dF_\theta + \int_{\widetilde{\theta}_t}^{\infty} b_{t,S}dF_\theta$$

The resource constraint (C.12) equates aggregate consumption, $C_t + \int c_t(\theta)dF_\theta$, government spending and the resource costs of inflation to total output, Y.

Moreover, to derive (C.13) we proceed as follows: First, we use the Euler equations to substitute prices out of the government budget constraint. The two bond pricing conditions for short and long bonds are:

$$q_{t,S}u'(C_t) = \int_{\widetilde{\theta}_t}^{\infty} \theta v'(b^i_{t,S})dF_\theta + \beta F(\widetilde{\theta}_t)E_t\frac{u'(C_{t+1})}{\pi_{t+1}}$$
$$q_{t,L}u'(C_t) = \sum_{j\geq 1}\beta^j\delta^{j-1}E_t\frac{u'(C_{t+j})}{\Pi_{k=1}^{j}\pi_{t+k}}$$

Given the optimal policy allocation we can back out prices to satisfy these conditions. Second, we use the labour supply condition to write government revenue $w_t\tau_tY_t$ as $\frac{\tau_t}{1-\tau_t}\chi\frac{Y_t^{1+\gamma_h}}{u'(C_t)}$. Third, we multiply both sides of the government budget constraint by marginal utility of consumption, $u'(C_t)$.

Finally, to derive utility V_t we use the following:

$$\int_0^{\infty} \theta \log c_t dF_\theta = \int \theta \log c_t dF_\theta = \int_0^{\widetilde{\theta}_t} C_t\theta dF_\theta + \int_{\widetilde{\theta}_t}^{\infty} \theta \log(b_{S,t})dF_\theta$$

C.2 Lagrangian and optimality.

As is standard in the literature we solve the optimal policy program using a Lagrangian. Letting $\psi_{gov,t}, \psi_{RC,t}, \psi_{PC,t}, \psi_{\widetilde{\theta},t}$ denote the multipliers attached to the government budget, the resource constraint, the Phillips curve and the constraint $C_t\widetilde{\theta}_t = b_{S,t}$ respectively, the Lagrangian function can be written as:

$$E_0 \sum_{t\geq 0} \beta^t \Big\{ V_t(C_t, b_{S,t}, Y_t, \widetilde{\theta}_t) + \psi_{\widetilde{\theta},t}\Big(\widetilde{\theta}_t C_t - b_{S,t}\Big)$$

$$+\psi_{RC,t}\Big(C_t + C_t \int_0^{\widetilde{\theta}_t} \theta dF_\theta + \int_{\widetilde{\theta}_t}^{\infty} b_{S,t} dF_\theta + G_t + \frac{\omega}{2}(\pi_t - 1)^2 - Y_t\Big)+$$

$$+\psi_{PC,t}\Big(\pi_t(\pi_t-1)U'(C_t) - \frac{\eta}{\omega}(\frac{1+\eta}{\eta}U'(C_t) - \frac{1}{1-\tau_t}\chi Y_t^{\gamma_h})Y_t - \beta E_t U'(C_{t+1})\pi_{t+1}(\pi_{t+1}-1)\Big)+$$

$$\psi_{gov,t}\Big(b_{t,S}\Big[\int_{\widetilde{\theta}_t}^{\infty} \theta v'(b^i_{t,S})dF_\theta + F(\widetilde{\theta}_t)E_t\frac{u'(C_{t+1})}{\pi_{t+1}}\beta\Big] + b_{L,t}\sum_{j\geq 1}\beta^j\delta^{j-1}E_t\frac{u'(C_{t+1})}{\Pi_{k=1}^j \pi_{t+k}}$$

$$-\frac{b_{t-1,S}}{\pi_t}u'(C_t) - \frac{b_{L,t-1}}{\pi_t}(u'(C_t) + \delta\sum_{j\geq 1}\beta^j\delta^{j-1}E_t\frac{u'(C_{t+j})}{\Pi_{k=1}^j \pi_{t+k}}) - G_t u'(C_t) + \frac{\tau_t}{1-\tau_t}\chi Y_t^{1+\gamma_h}\Big)\Big\}$$

The first order conditions for the optimum are:

(C.15)

$$\widetilde{\theta}_t: \quad \frac{dV_t}{d\widetilde{\theta}_t} + \psi_{RC,t}\Big(\widetilde{\theta}_t f_{\widetilde{\theta}_t} C_t - b_{S,t} f_{\widetilde{\theta}_t}\Big) + \psi_{\widetilde{\theta},t}C_t + \psi_{gov,t}b_{t,S}\Big(\beta f_{\widetilde{\theta}_t}E_t\frac{u'(C_{t+1})}{\pi_{t+1}} - \widetilde{\theta}_t f_{\widetilde{\theta}_t}\frac{1}{b_{t,S}}\Big) = 0$$

(C.16) $$Y_t: \quad \frac{dV_t}{dY_t} - \psi_{RC,t} + \psi_{PC,t}\Big(\frac{1+\gamma_h}{1-\tau_t}\frac{\eta}{\omega}\chi Y_t^{\gamma_h} - \frac{1+\eta}{\omega}u'(C_t)\Big) + \psi_{gov,t}\frac{\tau_t}{1-\tau_t}(1+\gamma_h)\chi Y_t^{\gamma_h} = 0$$

(C.17) $$\tau_t: \quad \psi_{PC,t}\frac{\eta}{\omega}\chi\frac{Y_t^{1+\gamma_h}}{(1-\tau_t)^2} + \psi_{gov,t}\chi\frac{Y_t^{1+\gamma_h}}{(1-\tau_t)^2} = 0$$

$$C_t: \quad \frac{dV_t}{dC_t} + \psi_{RC,t}\Big(1 + \int_0^{\widetilde{\theta}_t}\theta dF_\theta\Big) + u''(C_t)\pi_t(\pi_t - 1)\Delta\psi_{PC,t} - \psi_{PC,t}\frac{1+\eta}{\omega}u''(C_t)Y_t + \psi_{\widetilde{\theta},t}\widetilde{\theta}_t$$

(C.18)

$$-\psi_{gov,t}u''(C_t)G_t - \frac{b_{t-1,S}}{\pi_t}u''(C_t)\Big(\psi_{gov,t} - F_{\widetilde{\theta}_{t-1}}\psi_{gov,t-1}\Big) - u''(C_t)\sum_{j\geq 0}\delta^j(\psi_{gov,t-j} - \psi_{gov,t-j-1})\frac{b_{L,t-j-1}}{\Pi_{k=0}^j \pi_{t-k}} = 0$$

$$\pi_t: \quad \omega\psi_{RC,t}(\pi_t - 1) + u'(C_t)(2\pi_t - 1)\Delta\psi_{PC,t} + \frac{b_{S,t-1}}{\pi_t^2}u'(C_t)(\psi_{gov,t} - \psi_{gov,t-1}F_{\widetilde{\theta}_{t-1}})$$

(C.19) $$+u'(C_t)(1+\delta q_{L,t})\sum_{j\geq 0}\delta^j\frac{b_{L,t-j-1}}{\pi_{t-j}...\pi_t^2}\Delta\psi_{gov,t-j} = 0$$

(C.20) $$b_{S,t}: \quad \frac{dV_t}{db_{S,t}} + \psi_{RC,t}\Big(1 - F_{\widetilde{\theta}_t}\Big) - \psi_{\widetilde{\theta},t} + \psi_{gov,t}\beta F_{\widetilde{\theta}_t}E_t\frac{u'(C_{t+1})}{\pi_{t+1}} - \beta E_t\frac{u'(C_{t+1})}{\pi_{t+1}}\psi_{gov,t+1} = 0$$

(C.21) $$b_{L,t}: \quad \psi_{gov,t}\sum_{j\geq 1}\beta^j\delta^{j-1}E_t\frac{u'(C_{t+j})}{\Pi_{k=1}^j \pi_{t+k}} = E_t\psi_{gov,t+1}\sum_{j\geq 1}\beta^j\delta^{j-1}\frac{u'(C_{t+j})}{\Pi_{k=1}^j \pi_{t+k}}$$

Optimal Debt Policies. Several comments are in order. Consider first equations (C.20) and (C.21), which represent the first order conditions for short and long bonds respectively. According to (C.21) the multiplier on the government budget constraint, $\psi_{gov,t}$, follows a risk adjusted random walk; we can write $\psi_{gov,t} = \frac{E_t \psi_{gov,t+1} \varpi_{t+1}}{E_t \varpi_{t+1}}$ where $\varpi_{t+1} \equiv \sum_{j \geq 1} \beta^j \delta^{j-1} E_{t+1} \frac{u'(C_{t+j})}{\Pi_{k=1}^{j} \pi_{t+k}}$.

This property is standard in models of optimal debt policy under incomplete financial markets (see Aiyagari et al., 2002). Government debt is chosen to smooth tax distortions across time. The multiplier $\psi_{gov,t}$ measures the burden of financing debt through distortionary taxation. The optimal policy makes $\psi_{gov,t}$ permanently rise (fall) in response to a positive (negative) spending shock which is to be financed with higher (lower) taxes, because the planner wants to spread evenly the distortions across time.

Equation (C.20) however shows that an analogous property does not generally characterize the optimal policy for short-term debt. There are two reasons: Firstly, bond supply directly influences the welfare function, the resource constraint and the threshold $\widetilde{\theta}_t$. This is captured by the leading three terms in (C.20). Second, the last two terms in (C.20) may also imply that $\psi_{gov,t}$ does not follow a risk adjusted random walk, when $F_{\widetilde{\theta}_t} < 1$. Focusing momentarily on these two terms we can write:

$$\psi_{gov,t} \geq \frac{E_t \frac{u'(C_{t+1})}{\pi_{t+1}} \psi_{gov,t+1}}{E_t \frac{u'(C_{t+1})}{\pi_{t+1}}} \tag{C.22}$$

and thus $\psi_{gov,t}$ follows a supermantigale with respect to measure $\frac{u'(C_{t+1})/\pi_{t+1}}{E_t u'(C_{t+1})/\pi_{t+1}}$.

Notice that (C.22) is essentially a force decreasing the value of the multiplier over time. If, for example, $\psi_{gov,t}$ were to be bounded below by 0, then according to (C.22) the government budget constraint could eventually become 'slack' and thus irrelevant for the optimal allocation.

Intuitively, since short-term interest rates are (on average) lower than the discount rate, issuing short-term debt is cheap and enables the government to extract profits from liquidity provision to the private sector. When profits are maximized, debt can be rolled over at lower cost, and it is not necessary to rely heavily on distortionary taxes to finance it.

Let us denote the level of short-term debt which maximizes rents by $b_{S,t}^{rents}$ noting that it may be time-varying in an economy with aggregate shocks. Equation (C.20) says that generically it will be that $b_{S,t} > b_{S,t}^{rents}$ since rent maximization is not the only force governing optimal policy. The leading three terms in (C.20) measure the welfare benefit from liquidity provision to the private sector and will also contribute towards determining the optimal supply of the short bonds.

Of particular interest is the case where the optimizing government desires to issue a sufficiently large quantity of short-term debt so that, in effect, the friction facing agents in terms of financing c_t^i is no longer relevant.[9] We then have: $F_{\widetilde{\theta}_t} \approx 1$ and $\frac{dV_t}{db_{S,t}}, \psi_{RC,t}\Big(1 - F_{\widetilde{\theta}_t}\Big), \psi_{\widetilde{\theta},t} \approx 0$. Short bonds have essentially zero liquidity value for the private sector and (C.20) becomes a random walk, as in the canonical model. Let us denote

$$b_{S,t}^{canonical} = \sup\{b_{S,t} : F_{\widetilde{\theta}_t} < 1 - \epsilon\}$$

where $\epsilon \in \mathbf{R}_+$ is small. Then, for $b_{S,t} \geq b_{S,t}^{canonical}$ we have that $F_{\widetilde{\theta}_t} \approx 1$ (where approximately equal means numerically equal to 1 as defined by ϵ) our model is the canonical model.

Finally, the optimal short-term issuance may satisfy $b_{S,t}^{rents} < b_{S,t} < b_{S,t}^{canonical}$. In this case the optimal policy strikes a balance between rent maximization and private liquidity provision.

To summarize, long bonds in our model serve the purpose of smoothing tax distortions across time as in canonical models of optimal policy, whereas short bonds can be used to reduce tax distortions

[9] Technically, this is not possible since we assumed that the support of F is $[0, \infty)$ but it can still be relevant if $F_{\widetilde{\theta}_t}$ gets close to 1 for finite $\widetilde{\theta}$. This is the case in our calibrated model (see below).

though the profits that they generate for the government for its liquidity providing service. The optimal policy will trade off liquidity and rents to maximize welfare. This trade-off has been studied in a deterministic setting, and when the government issues debt in one liquid asset, by Angeletos et al. (2022). We instead focus on a model with aggregate shocks and study optimal government portfolios in the presence of both liquid and illiquid debt.

Optimal Inflation and Tax Policies. Next, consider equations (C.18) and (C.19) which represent the first order conditions for consumption and inflation respectively. Consider the terms $\sum_{j\geq 0} \delta^j \frac{b_{L,t-j-1}}{\Pi_{k=0}^{j}\pi_{t-k}} \Delta\psi_{gov,t-j}$ in (C.19) and $u''(C_t)\sum_{j\geq 0}\delta^j(\psi_{gov,t-j}-\psi_{gov,t-j-1})\frac{b_{L,t-j-1}}{\Pi_{k=0}^{j}\pi_{t-k}}$ in (C.18). Since these terms pertain to long-term debt, let us consider that the dynamics of $\psi_{gov,t}$ are determined only by (C.21), ignoring momentarily equation (C.20).

A positive spending shock tightens the government budget constraint and the multiplier ψ_{gov} increases permanently. To finance the shock, the Ramsey planner can increase inflation and, moreover, when debt is long-term it is optimal to rely both on current and future price growth to stabilize debt. The term $\sum_{j\geq 0} \delta^j \frac{b_{L,t-j-1}}{\Pi_{k=0}^{j}\pi_{t-k}} \Delta\psi_{gov,t-j}$ captures the adjustment of inflation in t that is due to shocks that hit the economy in past periods. The weight attached to the shock in $t-j$ is proportional to $\delta^j b_{L,t-j-1}$ because a higher bond issuance implies a larger impact of inflation on the real value of government debt. It decreases in j because the fraction of the debt issued in $t-j$ and that is outstanding in t, decays over time at rate δ (see, for example, Leeper and Zhou, 2021), Chafwehé, Priftis, Oikonomou, and Vogel (2022).

The term $u''(C_t)\sum_{j\geq 0}\delta^j(\Delta\psi_{gov,t-j})\frac{b_{L,t-j-1}}{\Pi_{k=0}^{j}\pi_{t-k}}$ captures an analogous margin of policy reacting to past shocks, however, it mainly concerns distortionary taxes as opposed to inflation. Consider a positive spending shock occurring in $t-j$. When the shock hits, the real value of long-term debt outstanding is

$$b_{L,t-j-1}(1+\overbrace{E_{t-j}\delta\sum_{l\geq 1}\beta^l\delta^{l-1}\frac{u'(C_{t-j+l})}{u'(C_{t-j})\Pi_{k=1}^{l}\pi_{t-j+k}}}^{\delta q_{t-j,L}:\ \text{Price of outstanding claims}})$$

Since this debt has to be compensated with higher distortionary taxes, it is beneficial, to reduce its real value by reducing the (expected) ratios $\frac{u'(C_{t-j+l})}{u'(C_{t-j})}$ for $l=1,2,....$, the real prices of the coupon claims δ^l. To do this, the government will promise to keep tax rates lower in $t+1, t+2, ...$ (relative to the rates needed to make debt solvent intertemporally) and get a higher consumption path than under a flat tax schedule. Because coupons decay over time, this policy progressively becomes less relevant and so taxes eventually increase to finance debt (see, for example, Faraglia et al., 2016)).

These standard channels of optimal Ramsey policy are also relevant for short-term debt. However, as equations (C.18) and (C.19) reveal, in this case, there are additional forces determining the optimal paths of inflation and taxes.

Consider first the term $\frac{1}{\pi_t^2}u'(C_t)(\psi_{gov,t}-\psi_{gov,t-1}F_{\widetilde{\theta}_{t-1}})$ in (C.19). When $F_{\widetilde{\theta}_{t-1}}\approx 1$ this is again the standard channel of using inflation to dilute the real value of debt in response to a positive spending shock. However, if $F_{\widetilde{\theta}_{t-1}}<1$, and even in the absence of any shock (or, even when $\psi_{gov,t}=\psi_{gov,t-1}$ as in the steady state) this term can exert a positive impact on inflation.

To understand this feature, recall that agents in the model purchase the short-term asset not only for its return properties but also for its liquidity services. A higher inflation rate, will then not increase the nominal interest rate (the cost of servicing debt) proportionally since households will not demand to be fully compensated for inflation. This enables the government to use the inflation tax and reduce the real value of debt. Lower real debt means lower distortionary taxation.

To further explain this channel, let us consider the intertemporal budget constraint of the government in period $t-1$. Assuming for simplicity that no long-term debt has been issued (focusing on short bonds), using the government budget constraint and iterating forward we can write:

(C.23)
$$E_{t-1}\sum_{j\geq 0}\beta^j\left\{s_{t+j-1}+b_{t+j-1,S}\underbrace{\left[\int_{\widetilde{\theta}_{t+j-1}}^{\infty}\theta v'(b^i_{t+j-1,S})dF_\theta+(F(\widetilde{\theta}_{t+j-1})-1)\frac{u'(C_{t+j})}{\pi_{t+j}}\beta\right]}_{\text{Liquidity Rent}}\right\}=\frac{b_{t-2,S}}{\pi_{t-1}}u'(C_{t-1})$$

where $s_{t+j-1}\equiv -G_{t+j-1}u'(C_{t+j-1})+\frac{\tau_{t+j-1}}{1-\tau_{t+j-1}}\chi Y_{t+j-1}^{1+\gamma_h}$ denotes the government surplus. According to (C.23) a higher debt outstanding in $t-1$ (RHS) does not necessarily need to be financed via higher surpluses. The 'Liquidity Rent' can be used to compensate for debt. This term is positive if short bonds provide liquidity and is zero otherwise.

To study the impact that this will have on inflation, let us focus on the optimal policy in period t. Suppose that $F(\widetilde{\theta}_{t-1})<1$. Then, committing to a higher inflation rate in t increases the rent for the government and ensures satisfaction of (C.23) for a lower surplus sequence. In the first order condition for π_t, this policy will show up as $\frac{b_{t-1,S}}{\pi_t^2}u'(C_t)(\psi_{gov,t}-\psi_{gov,t-1}F_{\widetilde{\theta}_{t-1}})$. This term effectively encapsulates the promise made by the planner for a higher inflation rate in t.

Turning to equation (C.18) we can see the analogous force in the tax schedule. The term $\frac{b_{t-1,S}}{\pi_t}u''(C_t)(\psi_{gov,t}-\psi_{gov,t-1}F_{\widetilde{\theta}_{t-1}})$ implies that if $F_{\widetilde{\theta}_{t-1}}<1$ then the tax rate will be lower in t. In (C.23) this channel shows up through the marginal utility $u'(C_t)$. A lower tax will lower the marginal utility in t, thus increasing the 'Liquidity Rent'.

C.3 Steady State

We now consider the solution of the model in the deterministic steady state. Dropping the time subscripts and the conditional expectations and dropping terms that cancel out at steady state, we can write the system of first order conditions as:

(C.24)
$$\omega\psi_{RC}(\overline{\pi}-1)+\psi_{gov}\frac{1}{\overline{\pi}^2}u'(\overline{C})(1-F_{\overline{\widetilde{\theta}}})=0$$

(C.25)
$$\psi_{\overline{\widetilde{\theta}}}\overline{C}+\psi_{gov}\overline{b}_S f_{\overline{\widetilde{\theta}}}u'(\overline{C})\left(\frac{\beta}{\overline{\pi}}-1\right)=0$$

(C.26)
$$-\chi\overline{Y}^{\gamma_h}-\psi_{RC}+\psi_{PC}\frac{\gamma_h}{1-\tau}\frac{\eta}{\omega}\chi\overline{Y}^{\gamma_h}+\psi_{gov}\frac{\overline{\tau}}{1-\overline{\tau}}(1+\gamma_h)\chi\overline{Y}^{\gamma_h}=0$$

(C.27)
$$\psi_{PC}\frac{\eta}{\omega}+\psi_{gov}=0$$

$$\text{(C.28)} \qquad \frac{dV}{d\bar{b}_S} + \psi_{RC}\left(1 - F_{\bar{\tilde{\theta}}}\right) - \psi_{\bar{\tilde{\theta}}} + \beta\frac{u'(\overline{C})}{\overline{\pi}}\psi_{gov}\left(F_{\bar{\tilde{\theta}}} - 1\right) = 0$$

$$\text{(C.29)} \qquad \psi_{gov}\sum_{j\geq 1}\beta^j\delta^{j-1}\frac{u'(\overline{C})}{\overline{\pi}^j} = \psi_{gov}\sum_{j\geq 1}\beta^j\delta^{j-1}\frac{u'(\overline{C})}{\overline{\pi}^j}$$

$$\text{(C.30)} \qquad \omega\overline{C}\psi_{RC}(\overline{\pi} - 1) + \frac{\overline{b}_S}{\overline{\pi}^2}\psi_{gov}(1 - F_{\bar{\tilde{\theta}}}) = 0$$

(C.31)

$$\left(\frac{1}{\overline{C}} + \psi_{RC}\right)\left(1 + \int_0^{\bar{\tilde{\theta}}}\theta dF_\theta\right) - \psi_{PC}\frac{1+\eta}{\omega}u''(\overline{C}) + \psi_{\tilde{\theta}}\tilde{\theta} - \psi_{gov}u''(\overline{C})\overline{G} - \frac{\overline{b}_S}{\overline{\pi}}u''(\overline{C})\psi_{gov}\left(1 - F_{\bar{\tilde{\theta}}}\right) = 0$$

where $\overline{x}$ denotes the steady state value of variable x (for the Lagrange multipliers we simply drop time subscripts to denote the steady state).

Consider first the optimality conditions for bonds, (C.28) and (C.29). Notice that the condition for long bonds, (C.29), will trivially hold at steady state, for any debt level $\overline{b}_L$. Thus, (C.29) will not uniquely define an optimal level of long-term debt, a standard property of the canonical Ramsey model. However, the first order condition for short-term bonds, (C.28) may define an optimal level $\overline{b}_S$ in equilibrium. Using (C.25) and the expression for $\frac{dV}{d\bar{b}_S}$ from Proposition 1 we can write (C.28) as:

$$\text{(C.32)} \qquad \frac{1}{\overline{b}_S}\int_{\bar{\tilde{\theta}}}^{\infty}\theta dF_\theta + \psi_{RC}\left(1 - F_{\bar{\tilde{\theta}}}\right) - \psi_{gov}\frac{\bar{\tilde{\theta}}}{\overline{C}}f_{\bar{\tilde{\theta}}}\left(1 - \frac{\beta}{\pi}\right) + \beta\frac{u'(\overline{C})}{\overline{\pi}}\psi_{gov}\left(F_{\bar{\tilde{\theta}}} - 1\right) = 0$$

Assume first that $F_{\bar{\tilde{\theta}}} \approx 1$. Then, the above condition will trivially hold independently of $\overline{b}_S$, the usual indeterminacy of the debt level applies also to short-term debt. However, if $F_{\bar{\tilde{\theta}}} < 1$ then we can solve the above equation (jointly with the rest of the equations in the system) to obtain $\overline{b}_S$ at the optimum.

Analogously, to the stochastic model of the previous paragraph, we can define $\overline{b}_S^{canonical}$ such that if $\overline{b}_S \geq \overline{b}_S^{canonical}$ then $F_{\bar{\tilde{\theta}}} \approx 1$. The steady state system of equations will pin down a unique optimum $\overline{b}_S < \overline{b}_S^{canonical}$ but not for $\overline{b}_S \geq \overline{b}_S^{canonical}$.

Equation (C.30) shows how the steady state level of inflation depends on $F_{\bar{\tilde{\theta}}}$ and consequently on $\overline{b}_S$. When $F_{\bar{\tilde{\theta}}} < 1$ the equilibrium net inflation rate is positive. Too see this, focus on the relevant scenario $\psi_{gov} > 0$. Then, since $\psi_{RC} < 0$ it follows trivially that inflation is positive. The government will use the inflation tax in steady state.[10] In contrast, if the optimal $\overline{b}_S$ is high enough so that $F_{\bar{\tilde{\theta}}} \approx 1$ the standard Ramsey outcome of zero net inflation in steady state obtains.

To determine the solution, we need to solve the steady state model numerically. We assume the same values for the model parameters as in our baseline calibration reported in Section 3 of the paper and experiment with different levels of steady state long-term debt.

[10] As discussed previously, using the inflation tax, serves the purpose of lowering distortionary labour income taxes. In steady state, this can be proved analytically for certain parameter values.

Start from (C.31) which can be written as:

$$\left(\frac{1}{C} + \psi_{RC}\right)\left(1 + \int_0^{\tilde{\theta}}\theta dF_\theta\right) + \psi_{gov}u''(C)\left(\frac{1+\eta}{\eta} - G\right) + \psi_{\tilde{\theta}}\tilde{\theta} - \frac{b_S}{\overline{\pi}}u''(C)\psi_{gov}\left(1 - F_{\tilde{\theta}}\right) = 0$$

Table C1 summarizes our results for the optimal $\overline{b}_S$, $\overline{\pi}$, $\overline{\tau}$ and $F_{\overline{\widetilde{\theta}}}$. The first column is the baseline level of long-term bonds (the steady state calibration in Section 3 of the paper) whereas in columns 2 and 3 we set $\overline{b}_L$ equal to 0 and -0.5 respectively.

The top panel of the table reports the optimal policy outcomes. Consider the first column corresponding to the baseline with positive long-term debt. At the optimum we get (annualized net) inflation equal to 2.16%, $\overline{b}_S = 0.249$ and $F_{\overline{\widetilde{\theta}}}$ is (approximately) 0.

The optimal policy targets a low supply of short-term debt, and benefits from the rents of providing liquidity services. Consistent with the intuition laid out previously, the optimal inflation rate is positive in the steady state equilibrium.

The bottom panel of Table C1 solves the canonical Ramsey model, when short bonds do not provide liquidity services. We use the same short-term debt level we found at the optimum in the top panel, and the same quantity of long-term bonds. Compare the steady state levels of taxes across these two solutions. The optimal policy with liquid short bonds sets $\overline{\tau} = 21.1\%$ and in the Ramsey model we get $\overline{\tau} = 24.3\%$. Clearly, the rents accruing to the government from liquidity provision enable to finance the same debt level with lower taxes.

These findings carry over to the case of zero long-term debt (middle column) but do not hold when we assume a negative value of long bonds in steady state. As can be seen from the right column of the table, the optimal policy in this case sets inflation equal to 0 and $F_{\overline{\widetilde{\theta}}}$ is approximately 1. We essentially obtain the canonical Ramsey outcome.

It is easy to find the intuition behind these results. When the (long-term) debt level is high, tax distortions are higher. This is when the government can benefit most by reducing the supply of short-term debt and generating rents from liquidity. However, in the case of a sufficiently negative debt level, the cost of distortionary taxation is low, the government can finance spending through

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Moreover, using the FONC to substitute out ψ_{RC} and $\psi_{\widetilde{\theta}}\widetilde{\theta}$, assuming log utility we get:

$$\left(\frac{1}{C} - \chi Y^{\gamma_h} + \frac{\psi_{gov}\chi Y^{\gamma_h}}{1-\tau}\left[(1+\gamma_h)\tau - \gamma_h\right]\right)\left(1 + \int_0^{\widetilde{\theta}} \theta dF_\theta\right) =$$
$$-\psi_{gov}u''(C)(\frac{1+\eta}{\eta} - G) - \psi_{gov}b_S\widetilde{\theta}f_{\widetilde{\theta}}u''(C)\left(\frac{\beta}{\pi} - 1\right) + \frac{b_S}{\pi}u''(C)\psi_{gov}\left(1 - F_{\widetilde{\theta}}\right)$$

Notice that

$$\left(1 - \chi Y^{\gamma_h}C + \frac{\psi_{gov}\chi Y^{\gamma_h}C}{1-\tau}\left[(1+\gamma_h)\tau - \gamma_h\right]\right) = \left(1 - \overline{w}(1-\overline{\tau}) + \psi_{gov}\overline{w}\left[(1+\gamma_h)\tau - \gamma_h\right]\right)$$
$$= 1 - \overline{w}(1+\psi_{gov}\gamma_h) + \overline{\tau}\overline{w}(1+\psi_{gov}(1+\gamma_h))$$

whereby we used the labour supply condition $\chi Y^{\gamma_h}C = \overline{w}(1-\overline{\tau})$ and $\overline{w}$ denotes the steady state wage rate. From the Phillips curve this can be written as:

$$\overline{w} = \frac{1+\eta}{\eta} + (\beta - 1)\frac{\omega}{\eta\overline{Y}}(\overline{\pi} - 1)\overline{\pi} \approx \frac{1+\eta}{\eta} \quad \text{if} \quad \beta \approx 1$$

Making use of this result (and skipping tedious algebra for brevity) we get:

$$\overline{\tau} = \frac{1}{(1+\psi_{gov}(1+\gamma_h))}\left[\lambda - \frac{\overline{b}_S}{\overline{\pi}}\frac{\eta}{C(1+\eta)}\frac{\psi_{gov}}{\xi_{\overline{\widetilde{\theta}}}}\left(1 - F_{\widetilde{\theta}} + (\overline{\pi} - \beta)\widetilde{\theta}f_{\widetilde{\theta}}\right)\right] \tag{C.33}$$

where $\lambda = (1+\psi_{gov}\gamma_h) - \frac{\eta}{1+\eta} + \frac{\psi_{gov}}{\xi_{\overline{\widetilde{\theta}}}}\frac{\eta}{C(1+\eta)}(\frac{1+\eta}{\eta} - G)$ and $\xi_{\overline{\widetilde{\theta}}} = \left(1 + \int_0^{\widetilde{\theta}} \theta dF_\theta\right)$

Quite evidently, the last term on the RHS of (C.33) is negative if $F_{\widetilde{\theta}} < 1$. Then inflation is positive at steady state and taxes are lower.

Table C1: Optimal policies in steady state

	Baseline Calibration	Zero Long Debt	Negative Long Debt
$\overline{b}_L$	?	0	-0.5
A: Liquid Short Model			
$4(\overline{\pi}-1)$	2.16	2.08	0.00
$\overline{\tau}$	0.211	0.208	0.196
$F_{\overline{\overline{\theta}}}$	0	0	0.999
$\overline{b}_S$	0.249	0.252	0.308
B: Canonical Ramsey			
$4(\overline{\pi}-1)$	0.00	0.00	0.00
$\overline{\tau}$	0.243	0.238	0.196
$F_{\overline{\overline{\theta}}}$	1	1	1
$\overline{b}_S$	0.249	0.252	0.308

Notes: The table shows the optimal inflation and tax rates, the short bond supply and the cumulative distribution. The top panel is baseline model where short bonds provide liquidity. The bottom panel is the canonical Ramsey model where debt does not provide liquidity.

its asset stock. In this case the need of the rents of liquidity provision is less, and the optimal policy targets to loosen the friction for the private sector.

Multiplicity. The results in Table C1 report the (global) optima, but the system of first order conditions at steady state is (generally) satisfied at more than one stationary points. To illustrate further the properties of the system and the various local optima we may obtain, in Figure 18 we plot welfare as a function of the short-term debt issuance in the three calibrations considered in Table C1. To calculate welfare, we fixed the supply of short-term bonds and computed the optimal $\overline{C}, \overline{\pi}, \overline{\tau}$ etc, solving the corresponding first order conditions. Using the graph we can easily determine at which points equation (C.28), the FONC for short bonds, will also hold.

Welfare is plotted on the left axis. $F_{\overline{\overline{\theta}}}$ is plotted on the right axis. The top panel is the baseline calibration. Note that the welfare function is concave for $F_{\overline{\overline{\theta}}} < 1$ and is (nearly) flat at high short-term debt levels when $F_{\overline{\overline{\theta}}} \approx 1$ (past point B in the graph).[11] The global optimum (corresponding to the entries of column 1 in Table C1) is at point A in the graph.

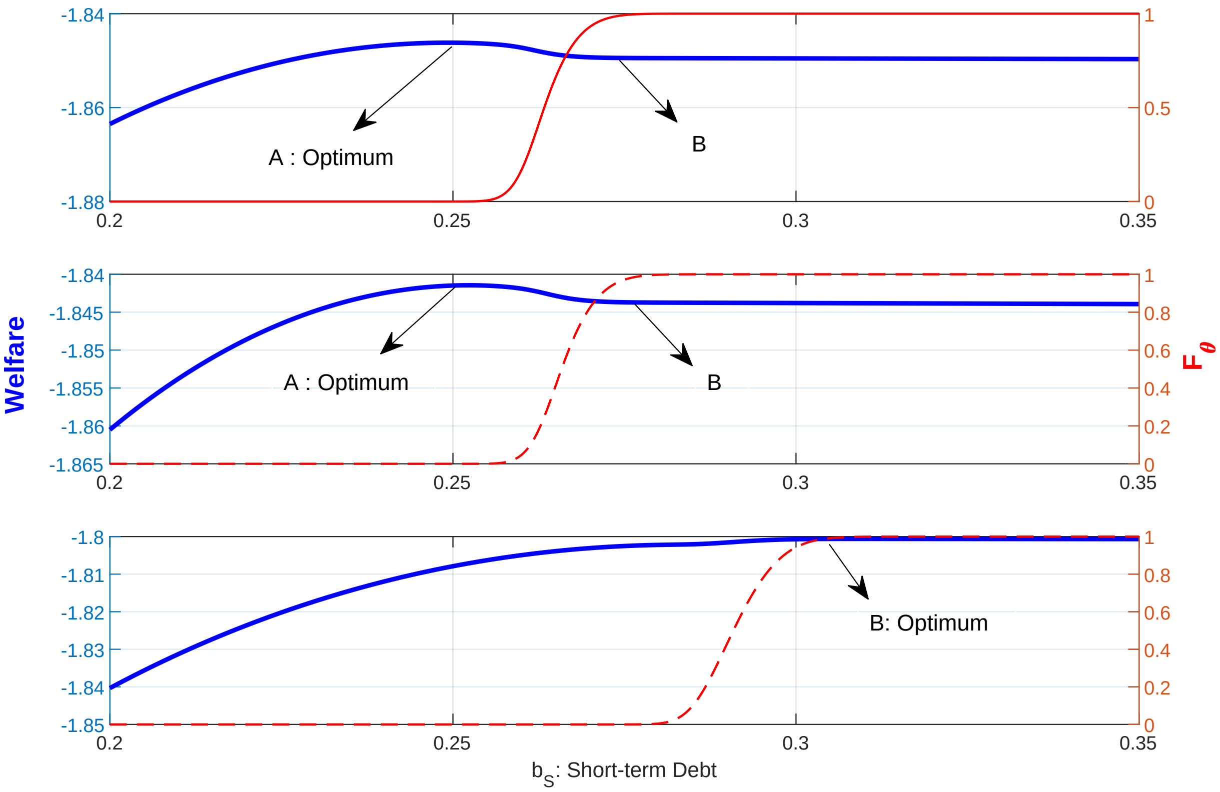
Given the flatness of the welfare function for $\overline{b}_S$ exceeding point B, and the fact that $F_{\overline{\overline{\theta}}} \approx 1$, the first order condition for short-term bonds (C.32) is satisfied. Thus, by solving the system of the optimality conditions we may find several stationary points, all $\overline{b}_S$ >B and one global maximum at A.

The middle and right panels show the cases of zero and negative long-term debt. As is evident, zero long debt is analogous to the baseline scenario. Assuming negative long debt, however, changes drastically the plot of the welfare function. Now, the global optimum is at point B but as noted previously, for any $\overline{b}_S$ >B the graph is nearly flat and the system of first order conditions is satisfied.

We can derive two crucial findings from this steady state subsection. First, we have shown that the quantity of long-term debt issued by the government affects the optimal supply of short bonds

[11] The near flatness of the welfare function reflects the well known property of the Ramsey model that the welfare impact of higher taxes is rather small.

Figure 18: Welfare and cumulative distributions functions.



Notes: We plot the welfare objective defined in Proposition 1 as a function of the short-term debt level (left axis). On the right axis we plot the cumulative distribution function $F_{\tilde{\overline{\theta}}}$ corresponding to each level of short debt. The top panel fixes long-term debt at the level assumed in the baseline calibration of the model. The middle panel assumes zero level of long-term debt. The bottom panel sets $\overline{b}_L < 0$ assuming that the government has accumulated a large stock of long-term assets (roughly 5 times steady state annual GDP). The welfare and the cumulative distributions functions have been computed solving the system of first order conditions for all model variables except $\overline{b}_S$.

in the model. Second, we found that solving the system of first order conditions is generically not sufficient to determine the global optimum. We thus needed to complement the FONC with a welfare function evaluation. This finding is particularly relevant for the algorithm that we need to setup in order to solve the model with aggregate shocks. We turn to this in the next section.

C.4 The Stochastic Optimal Policy Equilibrium

Though useful to investigate the channels of optimal policy, the deterministic steady state may not be visited by the simulations of the model when we have solved for the optimal policy with aggregate shocks. This property is well known for the canonical model with distortortionary taxation. Aiyagari et al. (2002) and Faraglia et al. (2016) solve the optimal policy problem when the government can issue debt in one maturity. Though the deterministic steady state debt level is undefined, the optimal quantity of bonds in the stochastic equilibrium is defined and is a negative stock that the government will use as a buffer against spending shocks. Analogously, in Angeletos (2002) and Buera and Nicolini (2004), the optimal government portfolios are also not defined in the absence of shocks. With aggregate shocks there is a unique optimal portfolio featuring long-term debt and short-term savings.

These properties are relevant here and in particular since our modelling of the long-term bonds is similar to these papers. If the government desires to accumulate a large stock of savings in the long term asset, then our results from the previous subsection indicate that supplying also a large quantity of short-term bonds (to the point where the private sector's preferences for liquidity are effectively satiated) becomes optimal. When the overall debt level is low, the costs of distortionary taxes is low and the government can utilize the returns from the savings in the long-term asset to finance spending shocks and smooth taxes across time (see, for example, Aiyagari et al., 2002).

However, this policy may entail large intertemporal distortions (frontloading taxes to accumulate assets) that the government may want to avoid and therefore not target a very negative supply of long bonds. Our previous results then suggest that a lower supply of short-term debt may be optimal. The benefit of such a policy is twofold. First, the optimizing government can benefit from the profits derived from liquidity provision and reduce the level of taxes needed to finance debt, an effect which we identified in the steady state of the model and which carries over to the economy with aggregate shocks. Second, in the model with aggregate fluctuations, financing spending shocks with liquid short-term debt will lead to a larger fiscal multiplier. With distortionary taxes government revenue depends on output and a higher multiplier will lead to lower fiscal deficits in times of high spending needs. This enables the government to smooth taxes across time.

On the other hand, issuing long-term debt may also entail potential tax smoothing benefits for the government. In canonical macroeconomic models an increase in the spending level leads to a drop in long bond prices. When consumption is crowded out following a positive shock, and is expected to revert back to steady state, the real long term rate increases and a government which has issued long-term debt, benefits from *fiscal insurance*, from the drop in the real value of its outstanding debt obligations when the deficit rises. To exploit this channel, in our model, the government would need to issue positive amounts of long-term debt and also continue financing spending shocks long-term.

It is therefore evident that when we move from the steady state allocation to the economy with business cycles, the government will not only be concerned about the trade-off between liquidity provision and generating rents to finance the debt, but also the tax smoothing benefits of long and short-term bonds will affect the debt issuance strategy.

It is to these issues that we now turn. We solve the nonlinear model with aggregate shocks and characterize the optimal debt and tax policies. We first outline the algorithm that we use to approximate the optimal policy equilibrium numerically and which is based on Parameterizing expectations (see ,for example, Den Haan and Marcet, 1990; Faraglia et al., 2019) and solving the system of the first order conditions. However, ours is a non-standard application of the PEA, since as indicated by the discussion of the previous paragraph, the first order conditions of the planning program are necessary but not sufficient and in general more than one stationary point can be found to satisfy the FONC. We therefore, complement our numerical algorithm with an approximation of the welfare objective (the value function), and use this object to compute the optimum. We also discuss how we leverage on the findings of the extant literature on debt management in canonical models (Aiyagari, 1994; Faraglia et al., 2016, 2019) to construct an efficient grid for bonds on which we solve the system of first order conditions.

Parameterizing Expectations. The system of equations that we need to solve is (C.15) to (C.21) together with the constraints of the planner's program. As usual the PEA class of solution methods entails approximating the conditional expectations in these equations with polynomial functions of state variables. Simple inspection of the first order optimality conditions is sufficient to determine that the state vector X in our model is:

$$X_t = \left(G_t, b_{S,t-1}, \widetilde{\theta}_{t-1}, \psi_{gov,t-1}, b_{L,t-1}, \Psi_{t-1} \right)$$

where we define Ψ_{t-1} as:[12]

$$\Psi_{t-1} = \frac{b_{L,t-2}\Delta\psi_{gov,t-1}}{\pi_{t-1}} + \frac{\delta}{\pi_{t-1}}\Psi_{t-2}$$

We thus can express the conditional expectations as polynomials of X. We assume the following approximations:

$$\begin{gathered}
E_t u'(C_{t+1})\pi_{t+1}(\pi_{t+1}-1) \approx \mathcal{H}_1(X_t, \lambda_1) \\
E_t \frac{u'(C_{t+1})}{\pi_{t+1}} \approx \mathcal{H}_2(X_t, \lambda_2) \\
E_t \frac{u'(C_{t+1})}{\pi_{t+1}}\psi_{gov,t+1} \approx \mathcal{H}_3(X_t, \lambda_3) \\
E_t \psi_{gov,t+1} \sum_{j\geq 1} \beta^j \delta^{j-1} \frac{u'(C_{t+j})}{\Pi_{k=1}^{j} \pi_{t+k}} \approx \mathcal{H}_4(X_t, \lambda_4) \\
E_t \sum_{j\geq 1} \beta^j \delta^{j-1} \frac{u'(C_{t+j})}{\Pi_{k=1}^{j} \pi_{t+k}} \approx \mathcal{H}_5(X_t, \lambda_5)
\end{gathered}$$

where $\mathcal{H}_i$ can be specified to include the levels, squares, cubes, etc of the variables in X depending on the order of the approximation.[13]

[12] Notice that variable $\psi_{PC,t-1}$ (the lagged multiplier on the Phillips curve) does not need to be added to X, since from the FONC, it is proportional to $\psi_{gov,t-1}$ at the optimum.

[13] Non-linearities may be important since for example $F_{\widetilde{\theta}_{t-1}}$ is a non-linear function of $\widetilde{\theta}_{t-1}$. Notice however that some non-linearity is already present in X through state variable Ψ.

Given the notation above, we can write the full system of equations that we need to solve as:

$$\psi_{\widetilde{\theta},t}C_t + \psi_{gov,t}b_{t,S}\left(\beta f_{\widetilde{\theta}_t}\mathcal{H}_2(X_t,\lambda_2) - \widetilde{\theta}_t f_{\widetilde{\theta}_t}\frac{1}{b_{t,S}}\right) = 0$$

$$\int_{\widetilde{\theta}_t}^{\infty}\frac{1}{b_{S,t}} + \psi_{RC,t}\left(1 - F_{\widetilde{\theta}_t}\right) - \psi_{\widetilde{\theta},t} + \psi_{gov,t}\beta F_{\widetilde{\theta}_t}\mathcal{H}_2(X_t,\lambda_2) - \beta\mathcal{H}_3(X_t,\lambda_3) = 0$$

$$-\chi Y_t^{\gamma_h} - \psi_{RC,t} + \psi_{PC,t}\left(\frac{1+\gamma_h}{1-\tau_t}\frac{\eta}{\omega}\chi Y_t^{\gamma_h} - \frac{1+\eta}{\omega}u'(C_t)\right) + \psi_{gov,t}\frac{\tau_t}{1-\tau_t}(1+\gamma_h)\chi Y_t^{\gamma_h} = 0$$

$$\psi_{PC,t}\frac{\eta}{\omega} + \psi_{gov,t} = 0$$

$$\left(\frac{1}{C_t} + \psi_{RC,t}\right)\left(1 + \int_0^{\widetilde{\theta}_t}\theta dF_\theta\right) + u''(C_t)\pi_t(\pi_t - 1)\Delta\psi_{PC,t} - \psi_{PC,t}\frac{1+\eta}{\omega}u''(C_t)Y_t + \psi_{\widetilde{\theta},t}\widetilde{\theta}_t$$

$$-\psi_{gov,t}u''(C_t)G_t - \frac{b_{t-1,S}}{\pi_t}u''(C_t)\left(\psi_{gov,t} - F_{\widetilde{\theta}_{t-1}}\psi_{gov,t-1}\right) - u''(C_t)\Psi_t = 0$$

$$u'(C_t)(2\pi_t - 1)\Delta\psi_{PC,t} + \psi_{gov,t}\frac{1}{\pi_t^2}u'(C_t) - \psi_{gov,t-1}u'(C_t)\frac{1}{\pi_t^2}F_{\widetilde{\theta}_{t-1}} + (u'(C_t) + \delta q_{L,t}u'(C_t))\frac{1}{\pi_t}\Psi_t = 0$$

$$\psi_{gov,t}\mathcal{H}_5(X_t,\lambda_5) = \mathcal{H}_4(X_t,\lambda_4)$$

together with the constraints:

$$\pi_t(\pi_t - 1) = \frac{\eta}{\omega}(\frac{1+\eta}{\eta} - \chi\frac{Y_t^{\gamma}}{U'(C_t)})Y_t + \beta\mathcal{H}_1(X_t,\lambda_1)$$

$$b_{t,S}\left[\int_{\widetilde{\theta}_t}^{\infty}\theta v'(b_{t,S}^i)dF_\theta + \beta F(\widetilde{\theta}_t)\mathcal{H}_2(X_t,\lambda_2)\right] + b_{L,t}\mathcal{H}_5(X_t,\lambda_5)$$

$$= \frac{b_{t-1,S}}{\pi_t}u'(C_t) + \frac{b_{L,t-1}}{\pi_t}(u'(C_t) + \delta\mathcal{H}_5(X_t,\lambda_5)) + G_t u'(C_t) - \frac{\tau_t}{1-\tau_t}\chi Y_t^{1+\gamma_h}$$

$$b_{t,S} = C_t\widetilde{\theta}_t$$

and the resource constraint (C.12).

As usual, the PEA algorithm initiates functions $\mathcal{H}_i^0$ with a guess for the values of the coefficients λ_i. Subsequently, the model is solved for a large number (T_S) of periods and the simulated output is utilized to updated the coefficients λ_i and obtain a new approximation $\mathcal{H}_i^1$. If $\mathcal{H}_i^0 \approx \mathcal{H}_i^1$ then the algorithm stops. Otherwise, we iterate using the updated polynomials as the initial guess until a fixed point of the coefficients is found.

Debt limits. In solving for the optimal bond portfolio we impose limits on the amount of short and long-term debt that the government can issue in any period t.

More specifically, we assume as Aiyagari (1994) and Faraglia et al. (2016, 2019) that the issuance of long-term bonds is subject to ad-hoc limits of the form:

$$b_{L,t} \in [\underline{M}_L, \overline{M}_L] \tag{C.34}$$

where $|\underline{M}_L|, |\overline{M}_L| < \infty$.

There are a couple of reasons for the inclusion of these additional constraints into the model. First, for numerical stability purposes, solving the model with the ad hoc debt constraints improves the performance of the numerical algorithm.[14] Second, these bounds are also meaningful economically. For example, the upper bound $\overline{M}_L$ can be seen as a constraint which rules out government

[14]See, for example, Maliar and Maliar (2003) and the discussion in the online appendix of Faraglia et al. (2019).

overborrowing, the possibility that debt is high enough so that the economy is at the wrong side of the Laffer curve. Analogously, the lower bound constraint has been motivated in the related literature to rule out overborrowing by the private sector,[15] or to rule out having negative positions of the government altogether (Lustig et al., 2008; Faraglia et al., 2019).

We assume loose bounds. We set $\overline{M}_L = 0.3$ to have a maximum quantity of long-term bonds equal to 30 percent of steady state GDP. Note that in terms of the market value of this debt (bond quantities times prices) we basically allow long debt to be as high as 167% of annual steady state GDP. For the lower bound we experiment with several calibrations. Our baseline is $\underline{M}_L = -0.3$. We refer to this scenario as the 'Lending' model (meaning that the government can lend to the private sector). We also consider $\underline{M}_L = 0$, a 'No Lending' scenario as in Faraglia et al. (2019); Lustig et al. (2008).[16] Finally, we summarize results from calibrations of $\underline{M}_L$ between these two values whenever it is helpful.[17]

For short bonds we do not need to assume a lower bound constraint; only the upper bound is required to solve the model accurately. Recall that for a high short bond supply we get $F_{\tilde{\theta}_t} \approx 1$ and the first order condition for the optimal quantity $b_{S,t}$ becomes:

$$\psi_{gov,t} = \frac{E_t \psi_{gov,t+1} u'(C_{t+1})}{E_t u'(C_{t+1})} \tag{C.35}$$

the usual risk adjusted random walk.

Under the PEA approximation (C.35) is not useful to solve for the optimal quantity of short-term debt; or, to be more precise, the system of first order conditions for bonds, which now is equations (C.21) and (C.35), cannot pin down the optimal portfolio. Both of these equations determine the current value of the multiplier, $\psi_{gov,t}$ as a function of the state variables X_t (whose elements are lagged quantities of debt and past multipliers) and the only object in the model which features $b_{S,t}, b_{L,t}$, the budget constraint, is not sufficient to pin down the portfolio.

Note that (in a way) this indeterminacy was also present in the steady state version of the model. For b_S above a certain level (in the range where $F_{\tilde{\theta}_t} \approx 1$) the first order conditions were always satisfied, we had multiple stationary points. The household welfare function was essentially flat and numerically it was not easy to find the optimum.

We claim that a similar property will apply to the model with aggregate shocks. It will not be optimal for the government to accumulate short-term debt, past the point where $F_{\tilde{\theta}_t} \approx 1$, at least not unless it is constrained in the issuance of long-term bonds. The rationale is simple: In canonical models with two assets (Faraglia et al., 2019), the Ramsey policy targets a portfolio of long debt and short savings, it is not optimal to issue large positive amounts of short bonds.[18] Since in the region $F_{\tilde{\theta}_t} \approx 1$ our model is essentially the canonical model we anticipate this property to hold, increasing $b_{S,t}$ when household preferences for liquidity are already satiated will not be optimal.

[15]See Aiyagari et al. (2002) for a formal definition of the *natural asset limit* in this class of models.

[16]This scenario has been motivated in the related literature based on the observations that governments in practice are not willing to save in private assets and bear the risk of private default (e.g. Lustig et al., 2008). A noteworthy exception is during the 2008-9 financial crisis, when the Fed purchased private assets. However, this had more to do with financial market disruptions rather than with financing spending shocks.

[17]To account for the debt limits the system of equations that needs to be resolved has to be modified. We have

$$\psi_{gov,t} \sum_{j \geq 1} \beta^j \delta^{j-1} E_t \frac{u'(C_{t+j})}{\Pi_{k=1}^j \pi_{t+k}} \neq E_t \psi_{gov,t+1} \sum_{j \geq 1} \beta^j \delta^{j-1} \frac{u'(C_{t+j})}{\Pi_{k=1}^j \pi_{t+k}}.$$

when a constraint binds. We utilize the PEA algorithm described in Marcet and Lorenzoni (1998) to solve the model with occasionally binding debt constraints.

[18]Intuitively, since issuing more long-term debt improves tax smoothing via fiscal insurance, but issuing short debt does not, the optimal policy focuses on issuing the long bond.

We solve the model with two different approaches. First we impose an upper bound constraint $b_{S,t} \leq \overline{M}_S$ where $\overline{M}_S$ is such that $F_{\widetilde{\theta}_t} \approx 1$ for all t.[19] Then, one of the stationary points that we find when we solve the FONC is $\overline{M}_S$.

Second, we solve the model adding a transaction cost of the form $\mathcal{T}_S = \nu(b_{S,t} - \widetilde{M}_S)^2$ when $b_{S,t} > \widetilde{M}_S$ and 0 otherwise. $\widetilde{M}$ is such that $F_{\widetilde{\theta}_t} < 1$ but close enough to 1. For a small cost parameter ν this enables us to avoid indeterminacy since the first order condition for $b_{S,t}$ now becomes:

$$\beta\psi_{gov,t}E_t u'(C_{t+1}) - \beta E_t \psi_{gov,t+1} u'(C_{t+1}) - 2\nu(b_{S,t} - \widetilde{M}_S)u'(C_t) = 0 \tag{C.36}$$

(when $F_{\widetilde{\theta}_t} \approx 1$).[20] Clearly, in (C.36) the short bond supply is determined since $b_{S,t}$ appears in the equation.

Both approaches yield similar results. For clarity we show below the model solution using the first approach only as in this case it is easier to verify when the government wants to satiate the economy with liquidity.

Welfare objective approximation. Ours is a non-convex optimization problem and solving the system of first order conditions gives multiple critical points. To discern the welfare maximizing solution we complement the PEA with an approximation of the objective function. Let $\mathcal{V}(X_t)$ be the lifetime utility as a function of the state variables. Our approximation is:

$$\mathcal{V}(X_t) \equiv E\left(\sum_{j\geq 0} \beta^j V_{t+j}\right)|X_t \approx \mathcal{H}^{\text{welfare}}(X_t, \lambda^w)$$

Function $\mathcal{H}^{\text{welfare}}$ is a polynomial of the state variables and the coefficients are λ^w (see below for exact specification of this function).

To find the optimum in period t we first find the stationary points by solving the system of first order conditions, drawing from different initial conditions. Then we evaluate the welfare function for each of the points we have found. The welfare associated with a generic stationary point (denoted using the asterisk here) is

$$V^* + \beta E_t \mathcal{V}(X^*_{t+1})$$

where $X^*_{t+1} = \left(G_{t+1}, b^*_{S,t}, \widetilde{\theta}^*_t, \psi^*_{gov,t}, b^*_{L,t}, \Psi^*_t\right)$

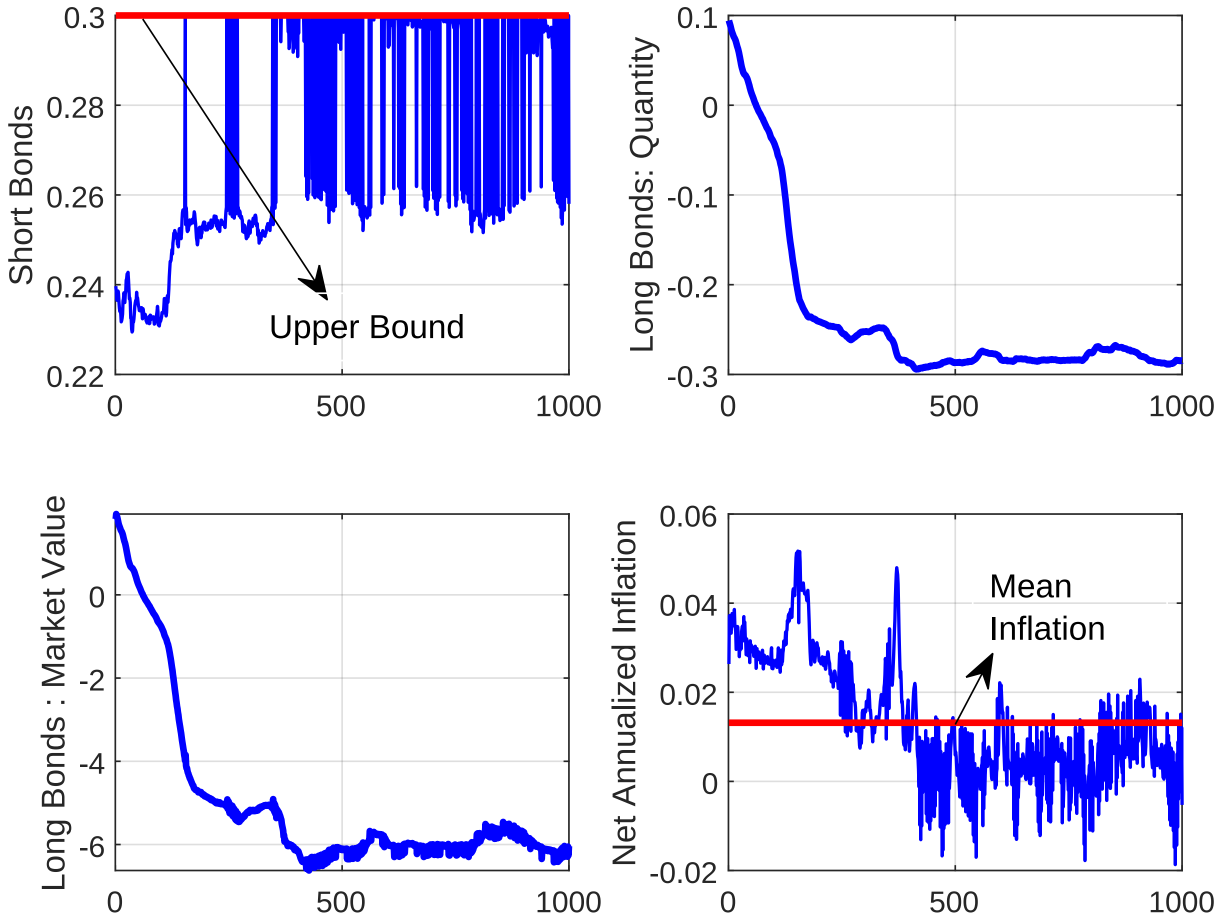
The λ^w coefficients are obtained assuming an initial guess, solving the model and iterating until the coefficients have converged.

Polynomial specification. The functions $\mathcal{H}^i$ are specified as orthogonal (Chebyshev) polynomials. We specify these to be of first order in the state variables $G_t, \psi_{gov,t-1}, \Psi_{t-1}$ and of 3rd order in $b_{S,t-1}, b_{L,t-1}$. Non-linearities are also present in the approximation through the state variable Ψ_{t-1} which is the weighted sum of lagged cross terms of bonds and multipliers.

[19] Obviously, in the context of the stochastic model $\overline{M}_S$ is not easy to determine a priori, since changes in spending levels can shift the point where liquidity preferences are satiated. We thus gradually adjust the upper bound until we find the appropriate level.

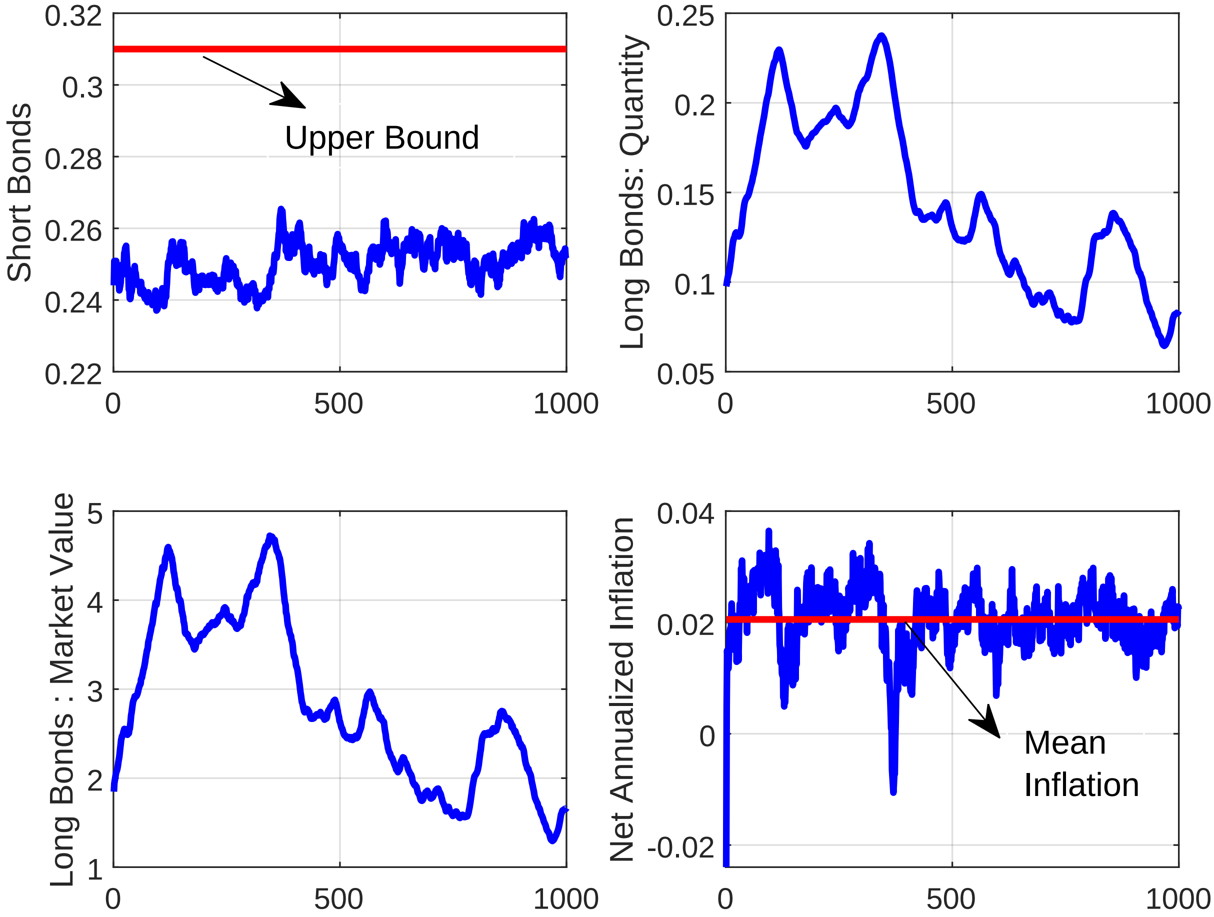
[20] This transaction cost is included in the government budget constraint and is assumed not to affect bond prices. See Faraglia et al. (2019) for derivations and for a motivation of this modelling assumption.

Figure 19: Model Simulation: Lending



Notes: The Figure shows 1000 simulated periods from the optimal policy model under 'Lending'. The top left panel plots the quantity of short bonds issued. The flat red line is the upper bound on short-term debt which corresponds to the issuance that fullfills the liquidity demand for short debt. The top right panel plots the quantity of long bonds. The bottom left is the market value of long-term debt (quantity times price level). The bottom right panel shows annualized net inflation rates in the model. The flat line is the sample average inflation.

Figure 20: Model Simulation: No Lending



Notes: The Figure shows 1000 simulated periods from the optimal policy model under 'No Lending'. The top left panel plots the quantity of short bonds issued. The flat red line is the upper bound on short-term debt which corresponds to the issuance that fullfills the liquidity demand for short debt. The top right panel plots the quantity of long bonds. The bottom left is the market value of long-term debt (quantity times price level). The bottom right panel shows annualized net inflation rates in the model. The flat line is the sample average inflation.

C.5 Numerical Results.

Lending Models. Consider first the case of the lending model. We summarize the output of this model in Figure 19 where we show a simulated path of short bonds (top left panel), long bond quantities (top right panel) the market value of long-term debt (quantity times price, bottom left) and the (annualized) inflation rate (bottom right). We plot these objects over 1000 model periods drawing a random sample of spending shocks from the distribution. Initially the economy is at the deterministic steady state.

Consider the bond quantities shown in the top panels. As is evident from the graphs, the quantity of long-term bonds quickly turns negative after a few model periods and continues dropping until bonds fluctuate around (and occasionally hit) the lower bound. Along the transition leading to negative long-term debt, the issuance of short-term debt fluctuates around 0.24. Clearly, the government seeks to maximize rents, limiting the supply of liquidity in the economy.

However, when long bonds are sufficiently negative, the supply of short debt increases and we find frequently in our simulations that short debt hits the upper bound constraint, where the private sector's preferences are effectively satiated with liquidity.

These patterns are analogous to our findings in the deterministic steady state version of the model. As we saw in the previous paragraph, having a large quantity of long bonds outstanding, implies a greater desire to exploit the rents from liquidity provision by the government, in order to reduce the burden of distortionary taxation. In contrast, when long bonds turn sufficiently negative, tax distortions are less of a burden to society and the optimal policy focuses on increasing the supply of liquidity to the economy. This reduces the rents extracted from supplying the liquid asset.

Turning to the bottom right panel, which plots the simulated path of inflation, we note that inflation is on average positive (sample average is the red horizontal line). Inflation is higher at the start of the sample when the supply of short bonds is lower and subsequently, fluctuates around 0 when long bonds converge to their stationary distribution, close to the lower bound constraint. These patterns are clearly in line with the properties of the optimal policy highlighted previously. With liquid short-term debt, the planner can use the inflation tax since agents will not need to be compensated fully for inflation. Higher inflation lowers the cost of servicing debt, or equivalently, maximizes rents for the government.

No Lending Models. The lending model of the previous subsection predicted that the government wants to accumulate a large stock of savings in the long-term asset, and target a positive quantity of short-term debt. Total debt was negative, the absolute position taken in the long bond (in market value) was considerably larger than the analogous value of the short-term debt supply.

Note that negative debt is a standard prediction of canonical models of optimal Ramsey policy with non-state contingent debt (e.g. Aiyagari et al., 2002). Accumulating wealth in private assets, enables the government to smooth taxes over time, by financing spending through the returns on its portfolio. However, finding that long bonds in particular are negative, is a surprising finding since as discussed previously, in canonical models of optimal debt management, governments issue positive amounts of long-term bonds. The rationale behind this, is *fiscal hedging*: Long bond prices comove negatively with government deficits and an optimizing government may want to exploit the negative comovement to smooth taxes across time. In canonical models of optimal DM under incomplete markets, total debt may be negative but this is due to short-term debt being negative rather than long-term debt.

Evidently, in our model, the government wants to put aside fiscal hedging, and rather use the stock of accumulated wealth to smooth taxes across time. This is an interesting finding that is worth to highlight.

We now consider an alternative setup in which we rule out negative long bond positions from the outset, that is we assume $\underline{M}_L = 0$. This assumption follows several papers on debt management (e.g. Lustig et al., 2008; Nosbusch, 2008; Faraglia et al., 2019). Since the prediction that governments issue negative amounts of debt is clearly at odds with the data, a 'No Lending' constraint' is basically an easy way to rule out this outcome, without needing to consider deeper micro-foundations that may explain why governments in practice are reluctant to invest in private assets.[21]

Based on our previous findings it may seem that tightening the lower bound will simply result in optimal policy setting $b_{L,t}$ close to 0 and continuing to issue positive short bonds. If in the unconstrained optimum, long-term debt is negative, the constrained outcome ought to be zero long debt. Yet, as we shall now show, this is not what our model predicts.

Figure 20 plots the optimal bond portfolio under 'No Lending'. As can be seen from the top panels, the optimal supply of short-term debt is positive and at the level in which the government benefits from liquidity rents. The quantity of long-term debt is strictly positive throughout the sample and it never hits the lower bound.

Notice further that short bond issuance is remarkably stable over the simulated sample. Effectively, the government uses long bonds to finance spending shocks, keeping the supply of short bonds stable around 0.25.

The property that short debt is below the upper bound (red line in top left panel) when long-term debt is positive, is easy to apprehend given our previous remarks. The property that there is no tendency for long-term debt to hit, even occasionally, the zero bound is more puzzling.[22] Our interpretation is the following: given that the government cannot accumulate assets in the no lending model, the best alternative is to benefit from fiscal hedging. When long-term debt levels are positive, an increase in spending induces a fall in the long bond price which can absorb part of the spending shock enabling to smooth tax distortions across time. With long debt close to zero, this channel is obviously mute, and though lower debt levels imply more fiscal space, it is still preferable to maintain a positive quantity of long bonds and benefit from the movements in bond prices.

Financing Spending Shocks. For this interpretation to be correct, it must be that positive spending shocks are financed with long-term debt. Though this already seems consistent with Figure 20, we now turn our focus to the response of the portfolio to positive spending shocks, to make sure that indeed long-term financing is optimal in our model. Recall that when the government increases the supply of short-term debt, consumption is crowded in and the economy is led to a stronger expansion of output. Higher consumption implies a negative co-movement of bond prices with spending and so the fiscal insurance argument is not applicable. In contrast, if consumption is crowded out, then we get the fiscal hedging channel of debt management.

Figure 21 shows the responses of the government portfolio, inflation and consumption to a sequence of positive spending shocks. We assume that one standard deviation shocks occur between periods 1 and 5, thereafter spending reverts back to the mean value at rate ρ_g. As can be seen from the Figure short-term debt (expressed as a percentage deviation from short debt when no shock hits the economy) falls after the spending shock.[23] In contrast, long-term debt increases to finance the shock.

The bottom panels show the reactions of inflation (left) and consumption (right). As expected, inflation increases persistently in response to the shock, to reduce the real payout of long-term government debt outstanding. Importantly, following the increase in spending, private sector consumption

[21] For example, in Lustig et al. (2008) the motivation behind 'No Lending' is that governments are not willing to bear the idiosyncratic default risk inherent in investments in private assets.

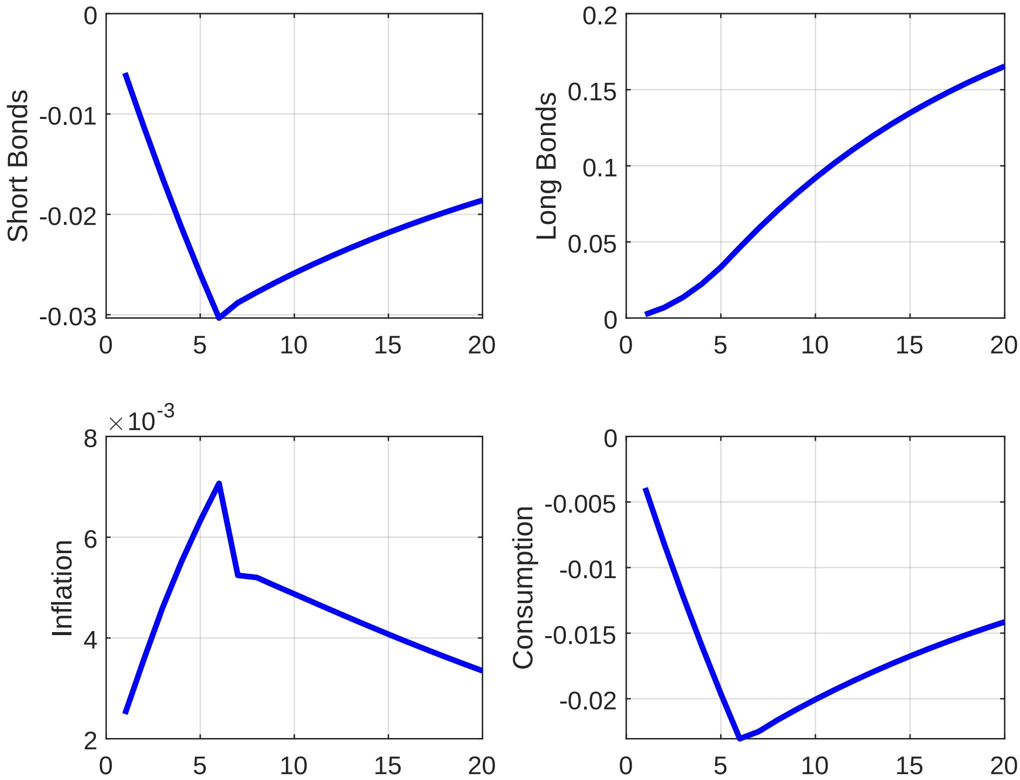
[22] Visually, it might appear that there is a downward trend in the quantity of long bonds in Figure 20 which would lead the model economy to hit the lower bound eventually. This is not the case.

[23] We let the model run without any shock for 100 periods so that bond quantities have converged to the stochastic steady state. The debt to GDP ratio is 70 percent of annual GDP.

drops and reverts to steady state from period 5 onwards. Due to the fall in consumption and the expected recovery, long bond prices drop.[24]

In Figure 22 we verify these responses in the case where total government debt is initially low (15 percent of annual GDP). We consider this case to ensure that financing debt long-term remains optimal when the outstanding stock of long-term debt is low and the government cannot benefit as much from the negative comovement long bond prices and deficits.[25] As Figure 22 shows this is indeed the case. Following the increase in spending levels, the government reduces the quantity of short-term debt outstanding and focuses on financing spending shocks long-term. Consumption is crowded out. The reaction of inflation is weaker than in Figure 21 because, as discussed in subsection C.2, at lower initial debt outstanding using inflation to reduce the real value of debt is less effective.

Figure 21: Impulse responses to spending shocks: No Lending

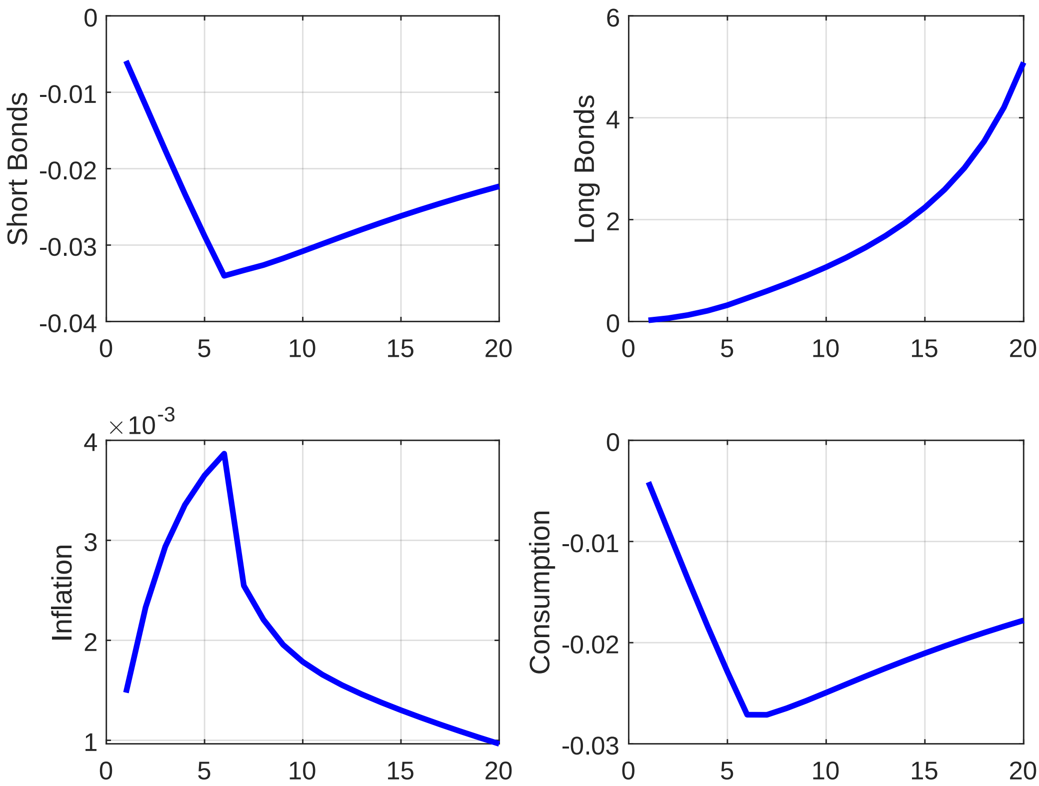


Notes: The Figure the impulse responses of short bonds (top left) long bonds (top right), inflation (bottom left) and consumption (bottom right) to an increase in government spending. We assume one standard deviation spending shocks for the initial 5 periods and subsequently spending reverts back to the mean value. Bond quantities and consumption are expressed in percentage deviations from the analogous objects when spending shocks are 0. Inflation the deviation in levels, scaled by a factor of 4.

[24] Since the positive shocks are unexpected in every period $t = 1, 2..., 5$ bond prices drop continuously. That is, agents expect in period 1 that consumption will revert to stochastic steady state from next period, then they are surprised by a new spending shock in period 2 and again do not expect a positive shock in period 3. Thus the fact that consumption drops for 5 periods, does not mean that short-term rates drop from $t = 1$ to $t = 5$.

[25] To get low initial debt, we simulate the model for 100 periods assuming that spending is 20 percent below the steady state value. We then shock the economy with 5 one standard deviation shocks as we did in Figure 21. Note that low total debt is almost coincident with low long-term debt since as we saw previously short bonds are quite stable in our simulations.

Figure 22: Impulse responses to spending shocks: No Lending, Low Debt Scenario



Notes: The Figure the impulse responses of short bonds (top left) long bonds (top right), inflation (bottom left) and consumption (bottom right) to an increase in government spending. We assume one standard deviation spending shocks for the initial 5 periods and subsequently spending reverts back to the mean value. Bond quantities and consumption are expressed in percentage deviations from the analogous objects when spending shocks are 0. Inflation the deviation in levels, scaled by a factor of 4. The debt to GDP ratio is initially 15% at annual horizon.

Comparing the model with US data. Our 'No Lending' model gives rise to a behavior of debt aggregates which is not far away from US data. In both the model and in the data we find that the government issues positive amounts of short and long-term debt, the short to long ratio we utilized in our empirical analysis is strictly positive and strictly below 1.

Given that the predictions of this model are comparable to the data and given that 'No Lending' constraints have been motivated by a number of papers in the related literature (Lustig et al., 2008; Nosbusch, 2008; Faraglia et al., 2019) as a simple modelling devise to capture that governments in practice are reluctant to invest in private assets, we now use this model to contrast its recommendations for optimal policy with the policy that is actually followed by the US Treasury. This exercise is in the same spirit to analogous experiments considered by Faraglia et al. (2019).

A striking difference between our optimal policy model and the US Treasury policy is that financing debt short-term in the model is suboptimal. Though STF leads to a larger fiscal multiplier, leading to a stronger increase in output and more revenues given the tax rates, it will also result in a positive comovement between long bond prices and spending (due to consumption crowding in) and to increases in the market value of long-term debt in times of high spending needs. Moreover, financing spending short-term reduces the rents extracted from liquidity provision, which further tightens the budget constraint.

Another important result of our model is that the short bond issuance is quite stable over time, the government does not want to strongly deviate from the target level when spending shocks occur. Hence, long-term debt displays considerably more volatility than short-term debt, it follows that the ratio of short over long ought to display different statistical properties (persistence and standard deviation) and different comovement with total debt to GDP than in the US data.

We now turn to the evaluation of the differences between the model and the data in terms of the behavior of debt aggregates. Figure 23 plots the ratio of short to long along with the debt to GDP ratio (annualized) and using the same simulation as in Figure 20. The US data was shown in Figure 1. In Table C2 we report data and model moments: the average of the share of short over long-term debt, the first order autocorrelation of the share, the standard deviation and the correlation with the debt to GDP ratio.[26]

Table C2: Data and model outcomes

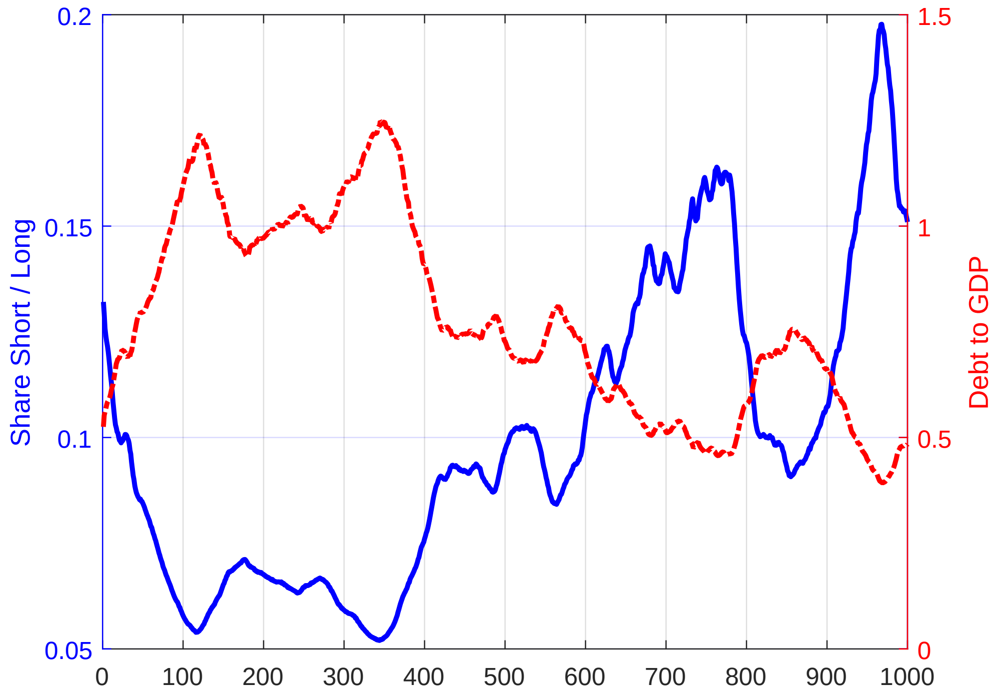
	Data	Model
Mean share	0.124	0.099
Auto-correlation	0.89	0.99
Standard deviation	0.024	0.020
Correlation with debt-GDP	-0.43	-0.94

Notes: The first column reports the mean share of short over long, the first order serial autocorrelation coefficient and the standard deviation of the share and the correlation of the share with the debt to GDP ratio in the data. The second column reports the analogous moments in the optimal policy model under 'No Lending'.

Figures 1 and 23 show that over the samples considered the ranges of values for the debt to GDP ratio and the share of short and long-term debt are analogous in data and model. In particular, the model share fluctuates between 5 percent and 20 percent over the simulated sample, the analogous

[26]To compute the standard deviation we split the sample used to construct Figure 20 into 4 subsamples of 250 quarters. We report the average of these samples. Note also that whether we use 4 samples or 1000 makes little difference for the estimates of the standard deviation, the model behavior does not change considerably across samples.

Figure 23: Short / Long Share and Debt to GDP ratio



Notes: The Figure plots the share of short-term over long-term debt in the model under 'No Lending' along with the debt to GDP ratio. The sample is the same as the one used to construct Figure 20.

range in the data is 6 and 19 percent. The debt to GDP ratio reaches a maximum of 140 percent and a minimum of 70 percent. Analogously, in the model, this range is between 50 and 130 percent.

The first row of Table C2 reports the means of the short to long ratio in the model and in the data. The model statistic is 9.9 percent whereas in the data the sample average is 12.4 percent. Thus, according to the model, the supply of short-term debt in the US economy is suboptimally high. The second row reports the first order autocorrelation coefficients of the model and data shares. The data moment is 0.89 and the analogous object in the model is 0.999. Clearly, the share is considerably more stable in the model than in the data, a property which is clearly evident in Figures 1 and 23 and in the third row of Table C2 which reports the sample standard deviations.

Finally, the 4th row of the table reports the correlation coefficients of the share with the debt to GDP ratio. The data moment is -0.43 whereas the model counterpart is -0.96. In other words, in the data, fluctuations of the share of short over long may occur even when the debt to GDP ratio does not change, whereas in the model the key factor driving fluctuations in the share is the overall debt burden of the government.

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