Online appendix for *The long and short of financing government spending*

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Appendices

This appendix contains six sections. Section A provides supplementary material on the model. It solves the household's problem and derives log-linear approximations to the optimal policy first-order conditions, both in the baseline model and in the version with quasi-linear preferences.

Section B complements the optimal tax results presented in the main text. In addition to deriving the analytical expressions reported in the paper, we obtain further results on optimal taxation under binding financial frictions. Specifically, we extend the analysis to consider time inconsistency in optimal policy and revisit classical Ramsey results on the role of debt maturity in ensuring time-consistent taxation. This section also contains analytical results on optimal portfolios (including ABN-type examples referenced in Section 4 of the paper) and presents numerical simulations of optimal portfolios under effectively complete markets and binding financial frictions.

Section C contains proofs of the propositions stated in the main text. Section D provides details on the computational methods.

Section E presents analytical results for optimal policy in the canonical model. We show that optimal inflation in this setting is determined by the government's budget constraint and derive the interest rate rules that implement the optimal policy.

Finally, Section F supplements the analysis of optimal inflation target. We provide results on the properties of inflation and taxation under both binding financial frictions and in the canonical model.

A Model Supplements

In this section, we solve the household's optimization program and derive the optimality conditions used in the competitive equilibrium (Section A.1). We also provide expressions for the log-linearized equations of our baseline model (Section A.2) and the quasi-linear utility model (Section A.3) , which we use in various analytical results in the main text and the appendix.

A.1 Household Optimality in the Baseline Model

We first present the household's dynamic program and constraints, and then derive the first-order conditions and Euler equations.

The dynamic program of household i is

(A.1)
$$V_t(B^i_{L,t-1}, B^i_{S,t-1,2}, \mathcal{S}_t) = \max_{B^i_{L,t}, B^i_{S,t}, C^i_t, c^i_t, h^i_t} \left\{ u(C^i_t) + E_\theta \theta v(c^i_t) - \chi \frac{{h_t}^{i,1+\gamma}}{1+\gamma} + \beta E_t \left[V_{t+1}(B^i_{L,t}, B^i_{S,t,2}, \mathcal{S}_{t+1}) \right] \right\}$$

subject to

$$P_t C^i_t + q_{L,t} B^i_{L,t} + q_{S,t} B^i_{S,t} = P_t(1-\tau_t) w_t h^i_t + (1 + q_{L,t}\delta) B^i_{L,t-1} + B^i_{S,t-1,2} + D_t P_t - P_t \bar{C}^i_t, \tag{A.2}$$

$$B^i_{S,t,2} = E_\theta (B^i_{S,t} - P_t c^i_t(\theta)) + P_t \bar{C}^i_{t,S}, \tag{A.3}$$

$$P_t c^i_t(\theta) \leq B^i_{S,t}. \tag{A.4}$$

We use the state variable $\mathcal{S}_t$ to denote the vector of aggregate shocks and $E/_\theta$ denotes expectation over idiosyncratic shocks θ.

Let λ_t, $\omega_{S,t}$, and $\psi_t(\theta)$ denote the multipliers associated with constraints (A.2), (A.3), and (A.4), respectively. The first-order conditions for the portfolio choice variables are

$$B^i_{S,t}: \qquad \lambda_t q_{S,t} - \omega_{S,t} + \int \psi_t(\theta) dF_\theta = 0$$

$$B^i_{L,t}: \qquad \lambda_t q_{L,t} - \beta E_t \frac{\partial V_{t+1}}{\partial B^i_{L,t}} = 0 \quad \Rightarrow \quad \lambda_t q_{L,t} = \beta E_t \lambda_{t+1}$$

$$B^i_{S,t,2}: \qquad \omega_{S,t} = -\beta E_t \frac{\partial V_{t+1}}{\partial B^i_{S,t,2}} = \beta E_t \lambda_{t+1}$$

$$c^i_t(\theta): \qquad \theta v'(c^i_t) f_\theta + \omega_{S,t} P_t f_\theta - \psi_t(\theta) P_t f_\theta = 0.$$

In deriving these conditions we used the envelope conditions

$$\frac{\partial V_t}{\partial B^i_{S,t-1,2}} = -\lambda_t, \qquad \frac{\partial V_t}{\partial B^i_{L,t-1}} = -\lambda_t(1 + \delta q_{L,t}).$$

Complementary slackness implies

$$\psi_t(\theta) \geq 0, \qquad \psi_t(\theta)\big(B^i_{S,t} - P_t c^i_t(\theta)\big) = 0.$$

The optimal allocation is characterized by a threshold $\widetilde{\theta}_t$ such that the constraint (A.4) binds whenever $\theta \geq \widetilde{\theta}_t$.

Using the FOC for consumption, we have $\lambda_t = -\frac{u'(C^i_t)}{P_t}$.

Substituting into the FOC for $B^i_{S,t}$ yields

$$\lambda_t q_{S,t} - \omega_{S,t} + \int_{\widetilde{\theta}_t} \psi_t(\theta) dF_\theta = 0$$

$$\lambda_t q_{S,t} - \omega_{S,t} + \int_{\widetilde{\theta}_t} \frac{\theta v'(c^i_t(\theta))}{P_t} dF_\theta + \omega_{S,t}(1 - F_{\widetilde{\theta}_t}) = 0.$$

Rearranging, we obtain the Euler equation for short-term debt

$$q_{S,t} \frac{u'(C^i_t)}{P_t} = \int_{\widetilde{\theta}_t} \frac{\theta v'(c^i_t(\theta))}{P_t} dF_\theta + \beta F_{\widetilde{\theta}_t} E_t \frac{u'(C^i_{t+1})}{P_{t+1}}. \tag{A.5}$$

Similarly substituting $\lambda_t = -\frac{u'(C^i_t)}{P_t}$ into the FOC for $B^i_{L,t}$ yields the Euler equation for long-term bonds.

Finally, the labor supply condition follows directly from the FOC for h^i_t and is therefore omitted.

A.2 Log-Linear baseline model

We derive the log-linear approximation of the competitive equilibrium equations in the baseline model.

For convenience, we repeat the system of equilibrium conditions here, including the expressions

for the terms $\frac{d\widetilde{U}_t}{db_{S,t}}$, $\frac{d\widetilde{U}_t}{d\widetilde{\theta}_t}$ etc shown in text.

$$\widetilde{\theta}_t: \qquad \widetilde{\theta}_t f(\widetilde{\theta}_t)\log C_t + \widetilde{\theta}_t f(\widetilde{\theta}_t)\log\widetilde{\theta}_t - \widetilde{\theta}_t f(\widetilde{\theta}_t)\log b_{S,t}$$
$$+\psi_{RC,t}\left(\widetilde{\theta}_t f_{\widetilde{\theta}_t} C_t - b_{S,t} f_{\widetilde{\theta}_t}\right) + \psi_{\widetilde{\theta},t} C_t + \psi_{gov,t} b_{S,t}\left(\beta f_{\widetilde{\theta}_t} E_t \frac{u'(C_{t+1})}{\pi_{t+1}} - \widetilde{\theta}_t f_{\widetilde{\theta}_t}\frac{1}{b_{S,t}}\right) = 0$$
$$Y_t: \qquad -\chi Y_t^{\gamma} - \psi_{RC,t} + \psi_{PC,t}\left(\frac{1+\gamma}{1-\tau_t}\frac{\eta}{\omega}\chi Y_t^{\gamma} - \frac{1+\eta}{\omega}u'(C_t)\right) + \psi_{gov,t}\frac{\tau_t}{1-\tau_t}(1+\gamma)\chi Y_t^{\gamma} = 0$$
$$\tau_t: \qquad \psi_{PC,t}\frac{\eta}{\omega} + \psi_{gov,t} = 0$$

$$C_t: \qquad \left(1 + \int_0^{\widetilde{\theta}_t}\theta dF_\theta\right)\frac{1}{C_t} + \psi_{RC,t}\left(1 + \int_0^{\widetilde{\theta}_t}\theta dF_\theta\right)$$
$$+u''(C_t)\pi_t(\pi_t - 1)\Delta\psi_{PC,t} - \psi_{PC,t}\frac{1+\eta}{\omega}u''(C_t)Y_t + \psi_{\widetilde{\theta},t}\widetilde{\theta}_t$$
$$-\psi_{gov,t}u''(C_t)G_t - \frac{b_{t-1,S}}{\pi_t}u''(C_t)\left(\psi_{gov,t} - F_{\widetilde{\theta}_{t-1}}\psi_{gov,t-1}\right) - u''(C_t)\sum_{j\geq 0}\delta^j(\psi_{gov,t-j} - \psi_{gov,t-j-1})\frac{b_{L,t-j-1}}{\Pi_{k=0}^{j}\pi_{t-k}} = 0$$

$$\pi_t: \qquad \omega\psi_{RC,t}(\pi_t - 1) + u'(C_t)(2\pi_t - 1)\Delta\psi_{PC,t} + \frac{b_{S,t-1}}{\pi_t^2}u'(C_t)(\psi_{gov,t} - \psi_{gov,t-1}F_{\widetilde{\theta}_{t-1}})$$
$$+u'(C_t)(1+\delta q_{L,t})\sum_{j\geq 0}\delta^j\frac{b_{L,t-j-1}}{\pi_{t-j}...\pi_t^2}\Delta\psi_{gov,t-j} = 0$$

$$b_{S,t}: \qquad \int_{\widetilde{\theta}_t}^{\infty}\theta dF_\theta\frac{1}{b_{S,t}} + \psi_{RC,t}\left(1 - F_{\widetilde{\theta}_t}\right) - \psi_{\widetilde{\theta},t} + \psi_{gov,t}\beta F_{\widetilde{\theta}_t}E_t\frac{u'(C_{t+1})}{\pi_{t+1}} - \beta E_t\frac{u'(C_{t+1})}{\pi_{t+1}}\psi_{gov,t+1} = 0$$

$$b_{L,t}: \qquad \psi_{gov,t}\sum_{j\geq 1}\beta^j\delta^{j-1}E_t\frac{u'(C_{t+j})}{\Pi_{k=1}^{j}\pi_{t+k}} = E_t\psi_{gov,t+1}\sum_{j\geq 1}\beta^j\delta^{j-1}\frac{u'(C_{t+j})}{\Pi_{k=1}^{j}\pi_{t+k}}$$

We log-linearize the system of first-order conditions around the steady state. We use bars to denote the steady-state values of the variables except for the steady-state value of $\widetilde{\theta}_t$ which is $\widetilde{\theta}$. We use hats to denote log-deviations from the steady state.

The first equation can be simplified. The terms in $\frac{dV_t}{d\widetilde{\theta}_t}$ cancel out because $C_t\widetilde{\theta} = b_{S,t}$. The terms multiplied by $\psi_{RC,t}$ also drop for the same reason. We thus get:

$$\overline{\psi}_{\widetilde{\theta}}\overline{C}(\hat{C}_t + \hat{\psi}_{\widetilde{\theta},t}) + \overline{\psi}_{gov}\overline{b}_S\left(\beta f(\widetilde{\theta})\frac{u'(\overline{C})}{\overline{\pi}} - \widetilde{\theta}f(\widetilde{\theta})\frac{1}{\overline{b}_S}\right)\left(\hat{\psi}_{gov,t} + \hat{b}_{S,t}\right)$$
$$+\overline{\psi}_{gov}\overline{b}_S\Bigg(\beta f'(\widetilde{\theta})\widetilde{\theta}\frac{u'(\overline{C})}{\overline{\pi}}\hat{\widetilde{\theta}}_t + f(\widetilde{\theta})\frac{u''(\overline{C})\overline{C}}{\overline{\pi}}E_t\hat{C}_{t+1} - f(\widetilde{\theta})\frac{u'(\overline{C})}{\overline{\pi}}E_t\hat{\pi}_{t+1}$$
$$-\widetilde{\theta}f(\widetilde{\theta})(1+\epsilon_{f,\widetilde{\theta}})\frac{1}{\overline{b}_S}\hat{\theta}_t + \widetilde{\theta}f(\widetilde{\theta})\frac{1}{\overline{b}_S}\hat{b}_{S,t}\Bigg) = 0$$

as the log-linear approximation of the FONC with respect to $\widetilde{\theta}_t$.

We then have:

$$0 = -\chi\overline{Y}^{\gamma}\gamma\hat{Y}_t - \overline{\psi}_{RC}\hat{\psi}_{RC,t} + \overline{\psi}_{PC}\hat{\psi}_{PC,t}\left(\frac{1+\gamma}{1-\overline{\tau}}\frac{\eta}{\omega}\chi\overline{Y}^{\gamma} - \frac{1+\eta}{\omega}u'(\overline{C})\right) +$$
$$\overline{\psi}_{PC}\left(\frac{1+\gamma}{1-\overline{\tau}}\frac{\eta}{\omega}\chi\overline{Y}^{\gamma}(\gamma\hat{Y}_t - \frac{\overline{\tau}}{1-\overline{\tau}}\hat{\tau}_t) - \frac{1+\eta}{\omega}u''(\overline{C})\overline{C}\hat{C}_t\right) + \overline{\psi}_{gov}\frac{\overline{\tau}}{1-\overline{\tau}}(1+\gamma)\chi\overline{Y}^{\gamma}\left(\gamma\hat{Y}_t + \hat{\psi}_{gov,t} + \frac{\hat{\tau}_t}{1-\overline{\tau}}\right)$$

$$\overline{\psi}_{PC}\hat{\psi}_{PC,t}\frac{\eta}{\omega} + \overline{\psi}_{gov}\hat{\psi}_{gov,t} = 0$$

$$\omega\overline{\psi}_{RC}(\overline{\pi}-1)\hat{\psi}_{RC,t} + \omega\overline{\psi}_{RC}\overline{\pi}\hat{\pi}_t + u'(\overline{C})(2\overline{\pi}-1)\overline{\psi}_{PC}\Delta\hat{\psi}_{PC,t} + \frac{u'(\overline{C})}{\overline{\pi}-\beta\delta}\overline{b}_L\overline{\psi}_{gov}\sum_{j\geq 0}(\frac{\delta}{\overline{\pi}})^j\Delta\hat{\psi}_{gov,t-j}$$
$$\frac{\overline{b}_S}{\overline{\pi}^2}u'(\overline{C})\overline{\psi}_{gov}(1-F_{\widetilde{\theta}})\left(\hat{b}_{S,t-1} - 2\hat{\pi}_t - \hat{C}_t\right) + \frac{\overline{b}_S}{\overline{\pi}^2}u'(\overline{C})\overline{\psi}_{gov}(\hat{\psi}_{gov,t} - F_{\widetilde{\theta}}\left(\hat{\psi}_{gov,t-1} + \frac{f_{\widetilde{\theta}}}{F_{\widetilde{\theta}}}\widetilde{\hat{\theta}}_{t-1}\right)) = 0$$

$$-\int_{\widetilde{\theta}}^{\infty}\theta dF_\theta\frac{1}{\overline{b}_S}\hat{b}_{S,t} - \widetilde{\theta}f(\widetilde{\theta})\frac{1}{\overline{b}_S}\hat{\widetilde{\theta}}_t + \overline{\psi}_{RC}\left(1-F_{\widetilde{\theta}}\right)\hat{\psi}_{RC,t} - \overline{\psi}_{RC}f_{\widetilde{\theta}}\widetilde{\theta}\hat{\widetilde{\theta}}_t - \overline{\psi}_{\widetilde{\theta}}\hat{\psi}_{\widetilde{\theta},t}$$
$$\beta\frac{u'(\overline{C})}{\overline{\pi}}\overline{\psi}_{gov}\left(\hat{\psi}_{gov,t}F_{\widetilde{\theta}} + f_{\widetilde{\theta}}\widetilde{\theta}\hat{\widetilde{\theta}}_t - E_t\hat{\psi}_{gov,t+1}\right) - \beta E_t\frac{u'(\overline{C})}{\overline{\pi}}\overline{\psi}_{gov}(F_{\widetilde{\theta}}-1)E_t\left(\hat{C}_{t+1} + \hat{\pi}_{t+1}\right) = 0$$

$$\widetilde{\theta}f(\widetilde{\theta})(\frac{1}{\overline{C}} + \overline{\psi}_{RC})\hat{\widetilde{\theta}}_t + \left(1 + \int_0^{\widetilde{\theta}}\theta dF_\theta\right)\left(-\frac{1}{\overline{C}}\hat{C}_t + \overline{\psi}_{RC}\hat{\psi}_{RC,t}\right)$$
$$+u''(\overline{C})\overline{\pi}(\overline{\pi}-1)\overline{\psi}_{PC}\Delta\hat{\psi}_{PC,t} - \overline{\psi}_{PC}\frac{1+\eta}{\omega}u''(\overline{C})\overline{Y}\left(\hat{Y}_t + \hat{\psi}_{PC,t} - 2\hat{C}_t\right) + \overline{\psi}_{\widetilde{\theta}}\widetilde{\theta}(\hat{\psi}_{\widetilde{\theta},t} + \hat{\widetilde{\theta}}_t)$$
$$-\overline{\psi}_{gov}u''(\overline{C})\overline{G}\left(\hat{\psi}_{gov,t} - 2\hat{C}_t + \hat{G}_t\right) - u''(\overline{C})\overline{\psi}_{gov}\overline{b}_L\sum_{j\geq 0}(\frac{\delta}{\overline{\pi}})^j\Delta\hat{\psi}_{gov,t-j}$$
$$-\frac{\overline{b}_S}{\overline{\pi}}u''(\overline{C})\overline{\psi}_{gov}(1-F_{\widetilde{\theta}})\left(\hat{b}_{t-1,S} - \hat{\pi}_t - 2\hat{C}_t\right) - \frac{\overline{b}_S}{\overline{\pi}}u''(\overline{C})\overline{\psi}_{gov}\left(\hat{\psi}_{gov,t} - F_{\widetilde{\theta}}(\hat{\psi}_{gov,t-1} + \frac{f_{\widetilde{\theta}}}{F_{\widetilde{\theta}}}\widetilde{\hat{\theta}}_{t-1})\right) = 0$$

where we make use of the assumption that $u'' = -\frac{1}{C^2}$ to derive the $\hat{C}_t$ terms. Finally, from the log-linear approximation of the long bond FONC we get:

$$\hat{\psi}_{gov,t} = E_t\hat{\psi}_{gov,t+1}$$

The above equations, together with the linearized constraints (omitted for brevity) form the system of equations that needs to be resolved to compute the equilibrium.

A.3 Quasilinear Utility

We derive the log-linear equations in the quasi-linear utility model (with sticky prices). For simplicity, we assume that f has unbounded support and expand around the optimum with binding financial frictions. The expressions below can be easily modified to have a finite bound $\overline{\theta}$, as assumed for various propositions.

$$\widetilde{U}(C_t, b_{S,t}, Y_t, \widetilde{\theta}_t) = C_t + \int_0^{\widetilde{\theta}_t} \theta \log(\theta) dF_\theta + \int_{\widetilde{\theta}_t}^{\infty} \theta \log(b_{S,t}) dF_\theta - \chi \frac{Y_t^{1+\gamma}}{1+\gamma}$$

and the condition $\widetilde{\theta}_t = b_{S,t}$ determines the optimal threshold $\widetilde{\theta}_t$.[1]

The resource constraint, the Phillips curve, and the government budget constraint are given by:

$$C_t + \int_0^{\widetilde{\theta}_t} \theta dF_\theta + \int_{\widetilde{\theta}_t}^{\infty} b_{S,t} dF_\theta + G_t + \frac{\omega}{2}(\pi_t - 1)^2 - Y_t = 0$$

$$\pi_t(\pi_t - 1) - \frac{\eta}{\omega}\left(\frac{1+\eta}{\eta} - \frac{1}{1-\tau_t}\chi Y_t^{\gamma}\right) Y_t - \beta E_t \pi_{t+1}(\pi_{t+1} - 1) = 0$$

$$b_{L,t} E_t \sum_{j \geq 1} \frac{\beta^j \delta^{j-1}}{\Pi_{k=1}^j \pi_{t+k}} + b_{S,t}\left[\int_{\widetilde{\theta}_t}^{\infty} \theta v'(b_{S,t}) dF_\theta + F(\widetilde{\theta}_t) E_t \frac{1}{\pi_{t+1}} \beta\right] = G_t - \frac{\tau_t}{1-\tau_t}\chi Y_t^{1+\gamma}$$
$$+ \frac{b_{t-1,S}}{\pi_t} + \frac{b_{t-1,L}}{\pi_t}\left(1 + \delta E_t \sum_{j \geq 1} \frac{\beta^j \delta^{j-1}}{\Pi_{k=1}^j \pi_{t+k}}\right)$$

Using the same approach to solve for the optimal policy as in the baseline model, it is straightforward to show that the first-order optimality conditions are given by:

$$1 - \chi Y_t^{\gamma} + \psi_{PC,t}\left(\frac{1+\gamma}{1-\tau_t}\frac{\eta}{\omega}\chi Y_t^{\gamma} - \frac{1+\eta}{\omega}\right) + \psi_{gov,t}\frac{\tau_t}{1-\tau_t}(1+\gamma)\chi Y_t^{\gamma} = 0$$

$$\psi_{PC,t}\frac{\eta}{\omega} + \psi_{gov,t} = 0$$

$$-\omega(\pi_t - 1) + (2\pi_t - 1)\Delta\psi_{PC,t} + \frac{b_{S,t-1}}{\pi_t^2}(\psi_{gov,t} - \psi_{gov,t-1}F_{\widetilde{\theta}_{t-1}}) + (1 + \delta q_{L,t}) \sum_{j \geq 0} \delta^j \frac{b_{L,t-j-1}}{\pi_{t-j}...\pi_t^2} \Delta\psi_{gov,t-j} = 0$$

where $q_{L,t} \equiv E_t \sum_{j \geq 1} \frac{\beta^j \delta^{j-1}}{\Pi_{k=1}^j \pi_{t+k}}$ in this model.

$$\int_{\widetilde{\theta}_t}^{\infty} \theta dF_\theta \frac{1}{b_{S,t}} - \left(1 - F_{\widetilde{\theta}_t}\right) + \psi_{gov,t} b_{S,t} f_{\widetilde{\theta}_t}\left(\beta E_t \frac{1}{\pi_{t+1}} - 1\right) + \psi_{gov,t}\beta F_{\widetilde{\theta}_t} E_t \frac{1}{\pi_{t+1}} - \beta E_t \frac{1}{\pi_{t+1}} \psi_{gov,t+1} = 0$$

[1] As in the main text, we drop the individual index i under quasi-linear utility.

$$\psi_{gov,t} E_t \sum_{j\geq 1} \frac{\beta^j \delta^{j-1}}{\Pi_{k=1}^{j} \pi_{t+k}} = E_t \psi_{gov,t+1} \sum_{j\geq 1} \frac{\beta^j \delta^{j-1}}{\Pi_{k=1}^{j} \pi_{t+k}}$$

The resulting linear system is:

$$\text{(A.6)} \qquad -\omega \overline{\pi} \hat{\pi}_t + (2\overline{\pi} - 1)\overline{\psi}_{PC} \Delta \hat{\psi}_{PC,t} + \frac{\overline{b}_L \overline{\psi}_{gov}}{\overline{\pi} - \delta\beta} \sum_{j\geq 0} (\frac{\delta}{\overline{\pi}})^j \Delta \hat{\psi}_{gov,t-j}$$
$$+ \frac{\overline{b}_S}{\overline{\pi}^2} \overline{\psi}_{gov} (1 - F_{\overline{\tilde{\theta}}}) \left(\hat{b}_{S,t-1} - 2\hat{\pi}_t \right) + \frac{\overline{b}_S}{\overline{\pi}^2} \overline{\psi}_{gov} (\hat{\psi}_{gov,t} - F_{\overline{\tilde{\theta}}} \left(\hat{\psi}_{gov,t-1} + \frac{f_{\overline{\tilde{\theta}}}}{F_{\overline{\tilde{\theta}}}} \overline{\tilde{\theta}} \hat{\tilde{\theta}}_{t-1} \right)) = 0$$

$$\text{(A.7)} \qquad -\int_{\overline{\tilde{\theta}}}^{\infty} \theta dF_\theta \frac{1}{\overline{b}_S} \hat{b}_{S,t} - \overline{\tilde{\theta}}^2 f(\overline{\tilde{\theta}}) \frac{1}{\overline{b}_S} \hat{\tilde{\theta}}_t + f_{\overline{\tilde{\theta}}} \overline{\tilde{\theta}} \hat{\tilde{\theta}}_t$$
$$\beta \frac{1}{\overline{\pi}} \overline{\psi}_{gov} \left(\hat{\psi}_{gov,t} F_{\overline{\tilde{\theta}}} + f_{\overline{\tilde{\theta}}} \overline{\tilde{\theta}} \hat{\tilde{\theta}}_t - E_t \hat{\psi}_{gov,t+1} \right) - \beta E_t \frac{1}{\overline{\pi}} \overline{\psi}_{gov} (F_{\overline{\tilde{\theta}}} - 1) E_t \hat{\pi}_{t+1}$$
$$+ \overline{\psi}_{gov} \overline{b}_S f_{\overline{\tilde{\theta}}} \left(\beta \frac{1}{\overline{\pi}} - 1 \right) (\hat{\psi}_{gov,t} + \hat{b}_{S,t} + \frac{f'_{\overline{\tilde{\theta}}}}{f_{\overline{\tilde{\theta}}}} \overline{\tilde{\theta}} \hat{\tilde{\theta}}_t) - \overline{\psi}_{gov} \overline{b}_S f_{\overline{\tilde{\theta}}} \beta \frac{1}{\overline{\pi}} E_t \hat{\pi}_{t+1} = 0$$

where $-\overline{\tilde{\theta}}^2 f(\overline{\tilde{\theta}}) \frac{1}{\overline{b}_S} \hat{\tilde{\theta}}_t + f_{\overline{\tilde{\theta}}} \overline{\tilde{\theta}} \hat{\tilde{\theta}}_t = 0$. Finally, we again obtain $\hat{\psi}_{gov,t} = \hat{\psi}_{PC,t}$ and $\hat{\psi}_{gov,t} = E_t \hat{\psi}_{gov,t+1}$.

For brevity, we omit the log-linear budget constraint. An expression is provided in one of the analytical results below.

B Optimal Taxes and Debt Portfolios

This subsection provides analytical results that characterize optimal taxation and debt management in the environment studied in the paper. We first derive explicit expressions for the optimal tax schedule and decompose it into key components, highlighting the role of financial frictions and the liquidity services provided by short-term government debt (Section B.1). We then study the optimal policy from the perspective of a date-0 program, emphasizing issues of time inconsistency and the role of timeless penalties (Section B.2). Finally, we analyze optimal debt portfolios under effectively complete markets, contrasting the canonical Ramsey benchmark with the case of binding financial frictions (Section B.3). Throughout, we focus on isolating the mechanisms that differentiate our framework from standard models without financial frictions.

B.1 Intertemporal tax distortions/ Analytical Tax Schedule

This subsection outlines the key components of optimal tax policy in our model. It complements section 3.2. of the main text. As discussed in the main text, we restrict attention to the limiting case $\omega \to \infty$, thereby abstracting from inflation.

Analytical Tax Schedule. Consider first the case where there is only short-term debt in the economy. Under these assumptions and by combining the first-order conditions of the Ramsey program, we can show that

$$A(\widetilde{\theta}_t)\left(1-\frac{\chi Y_t^{\gamma}}{u'(C_t)}+\psi_{gov,t}\frac{1+\eta}{\eta}-\psi_{gov,t}(1+\gamma)\frac{\chi Y_t^{\gamma}}{u'(C_t)}\right)$$

(B.8)

$$-\psi_{gov,t}f_{\widetilde{\theta}_t}\left(\beta E_t\frac{u'(C_{t+1})}{u'(C_t)}-1\right)+\psi_{gov,t}u'(C_t)(G_t-\frac{1+\eta}{\eta}Y_t)+b_{t-1,S}u'(C_t)\left(\psi_{gov,t}-F_{\widetilde{\theta}_{t-1}}\psi_{gov,t-1}\right)=0$$

where we define $A(\widetilde{\theta}_t)\equiv\left(1+\int_0^{\widetilde{\theta}_t}\theta dF_\theta\right)$.

Using $\frac{\chi Y_t^{\gamma}}{u'(C_t)}=\frac{1+\eta}{\eta}(1-\tau_t)$, we can rearrange equation (B.8) to derive an explicit expression for the tax rate in t.

$$-\tau_t=\frac{1}{\frac{1+\eta}{\eta}(1+\psi_{gov,t}(1+\gamma))A(\widetilde{\theta}_t)}\Bigg[A(\widetilde{\theta}_t)\left(1-\frac{1+\eta}{\eta}-\frac{1+\eta}{\eta}\psi_{gov,t}(\gamma+1)\right)-\psi_{gov,t}\underbrace{f_{\widetilde{\theta}_t}\left(\beta E_t\frac{u'(C_{t+1})}{u'(C_t)}-1\right)}_{D_{1,t}}$$

(B.9)

$$+\psi_{gov,t}\underbrace{\left(A(\widetilde{\theta}_t)\frac{1+\eta}{\eta}+u'(C_t)(G_t-\frac{1+\eta}{\eta}Y_t)\right)}_{MM}+\underbrace{b_{t-1,S}u'(C_t)\left(\psi_{gov,t}-F_{\widetilde{\theta}_{t-1}}\psi_{gov,t-1}\right)}_{D_{2,t}}\Bigg]$$

For notational convenience, we express the formula in terms of $-\tau_t$. We use this expression to inspect the mechanisms that affect the optimal tax schedule focusing specifically on the terms D_1 and D_2. We contrast (B.9) with the analogous closed form tax solution in the case where short bonds do not provide liquidity/safety services to the private sector (so that $F_{\widetilde{\theta}}=1$ $f_{\widetilde{\theta}}=0$ and $A(\widetilde{\theta})=\overline{A}$ is constant). We have

$$-\tau_t^{nf} = \frac{1}{\frac{1+\eta}{\eta}(1+\psi_{gov,t}(1+\gamma))\overline{A}}\Bigg[\overline{A}\left(1-\frac{1+\eta}{\eta}-\frac{1+\eta}{\eta}\psi_{gov,t}(\gamma+1)\right)$$

$$+\psi_{gov,t}\underbrace{\left(\overline{A}\frac{1+\eta}{\eta}+u'(C_t)(G_t-\frac{1+\eta}{\eta}Y_t)\right)}_{MM^{nf}}+\underbrace{b_{t-1,S}u'(C_t)\left(\psi_{gov,t}-\psi_{gov,t-1}\right)}_{D_{2,t}^{nf}}\Bigg] \tag{B.10}$$

where the superscript nf stands for "no frictions," in which, $D_1^{nf}=0$ holds.

Random Walk. In equation (B.9), an unanticipated spending shock that increases $\psi_{gov,t}$ implies a permanent rise in the optimal tax rate. Consequently, one component of the optimal tax schedule follows a random walk (Aiyagari, Marcet, Sargent, and Seppälä, 2002).

Markup Management. Consider terms MM and MM^{nf} in (B.9) and (B.10). These terms capture tax wedges arising from monopolistic competition. If we eliminate distortions from the economy by setting $\frac{1+\eta}{\eta}=1$, then, using the resource constraint, we obtain $G_t-Y_t=-A(\widetilde{\theta}_t)C_t$ and so $A(\widetilde{\theta}_t)-u'(C_t)C_tA(\widetilde{\theta}_t)=0$ (under log utility). Hence we have $MM=MM^{nf}=0$ independently of the degree of financial frictions (summarized here by $A(\widetilde{\theta}_t)$).

Recall that in our benchmark parametrization $\eta<-1$, implying $\frac{1+\eta}{\eta}<1$. Relative to this case, MM and MM^{nf} imply a reduction in the average tax rate in the steady state, reflecting the planner's incentive to mitigate distortions from monopolistic competition, as well as lower taxes in high-spending states. Importantly, again, this effect is independent of the magnitude of $A(\widetilde{\theta}_t)$.[2]

Real Discount Rate and Liquidity Cutoff terms. The terms D_1 D_2 D_2^{nf} were discussed extensively in the text. Here, we complement that discussion where necessary and derive the formulas presented there.

The per-period budget constraint in the canonical model is

$$E_t\beta\frac{u'(C_{t+1})}{u'(C_t)}b_{S,t}+\underbrace{\left(-G_t+\tau_tY_t\frac{1+\eta}{\eta}\right)}_{=\text{Surplus}_t}=b_{S,t-1}$$

Multiplying both sides of the constraint by the marginal utility $u'(C_t)$ and iterating forward, imposing the transversality condition $\lim_{j\to\infty}E_t\beta^ju'(C_{t+j})b_{S,t+j}=0$ yields the intertemporal budget constraint reported in the main text:

$$E_t\sum_{j\geq 0}\beta^j\frac{u'(C_{t+j})}{u'(C_t)}\underbrace{\left(-G_{t+j}+\tau_{t+j}Y_{t+j}\frac{1+\eta}{\eta}\right)}_{=\text{Surplus}_{t+j}}=b_{S,t-1}$$

Consider now the case where the debt supply is not high enough to eliminate the financial friction. The per period budget constraint when optimal short-term debt is positive in equilibrium but long term bonds are zero is zero for all t can be written as

$$\left[\frac{1}{u'(C_t)}\int_{\widetilde{\theta}_t}^{\infty}\theta dF_\theta+\beta b_{S,t}E_t\frac{u'(C_{t+1})}{u'(C_t)}F(\widetilde{\theta}_t)+\text{Surplus}_t\right]=b_{t-1,S}$$

[2]Specifically, $\frac{1+\eta}{\eta}A(\widetilde{\theta}_t)+u'(C_t)(G_t-\frac{1+\eta}{\eta}Y_t)=\frac{1+\eta}{\eta}A(\widetilde{\theta}_t)-u'(C_t)(\frac{1}{\eta}G_t+\frac{1+\eta}{\eta}A(\widetilde{\theta}_t)C_t)=-u'(C_t)\frac{1}{\eta}G_t$.

Multiplying the LHS and RHS of this equation by $u'(C_t)$ and iterating forward, imposing the limit condition $lim_{j\to\infty} E_t \beta^j u'(C_{t+j}) \prod_{k=0}^{j-1} F(\widetilde{\theta}_{t+k}) b_{S,t+j} = 0$ and assuming that the discounted surplus series converges, we obtain

$$E_t \sum_{j\geq 0} \beta^j \frac{\Pi_{k=0}^{j-1} F(\widetilde{\theta}_{t+k})}{u'(C_t)} \left[\int_{\widetilde{\theta}_{t+j}}^{\infty} \theta dF_\theta + u'(C_{t+j}) \text{Surplus}_{t+j} \right] = b_{t-1,S}$$

The term Surplus_{t+j} is as defined above.

Finally, consider the term D_1. As we explained in the main text this term introduces another intertemporal distortion into the optimal tax schedule. It emerges from the derivative of the value of newly issued debt with respect to $\widetilde{\theta}_t$ and thus captures indirectly the impact of the tax schedule on debt issuance and on the financial friction. Precisely, in this real model we have

$$K_t = u'(C_t) p_{t,S} b_{S,t} = \int_{\widetilde{\theta}_t}^{\infty} \theta dF(\theta) + b_{S,t} \beta F(\widetilde{\theta}_t) E_t u'(C_{t+1}) \tag{B.11}$$

expressing the issuance market value as a savings marginal utility cost for the agent and using $v = log$. Then:

$$K_t' = -f(\widetilde{\theta}_t)\widetilde{\theta}_t + b_{S,t}\beta f(\widetilde{\theta}_t) E_t u'(C_{t+1}) = -f(\widetilde{\theta}_t)\widetilde{\theta}_t + \beta \widetilde{\theta}_t f(\widetilde{\theta}_t) E_t \frac{u'(C_{t+1})}{u'(C_t)}$$

again under log utility and using the definition $\widetilde{\theta}_t = \frac{b_{S,t}}{C_t}$.

In the main text we gave the following economic interpretation of D_1: Since $\widetilde{\theta}_t$ is the ratio of public debt to consumption, and f is the change in the fraction of agents for which the financial friction binds in equilibrium, D_1 can be seen as a marginal net benefit (or loss) from alleviating the financial friction. Precisely, a marginal increase in $\widetilde{\theta}_t$ (increase in the face value of savings of households expressed in terms of the marginal utility cost) will imply that for an additional f agents the financial friction will not be relevant in period t and the marginal benefit from holding government debt is $\beta E_t \frac{u'(C_{t+1})}{u'(C_t)}$. When setting the optimal tax schedule the planner internalizes this margin, through its effect on the government budget constraint.

To discuss further how this term acts when spending shocks hit the economy, it is useful to use a first-order approximation around the steady state. We have

$$\hat{D}_{1,t} = f'(\overline{\widetilde{\theta}})\overline{\widetilde{\theta}}(1-\beta)\hat{\widetilde{\theta}}_t - f(\overline{\widetilde{\theta}})\beta(\hat{C}_t - E_t\hat{C}_{t+1})$$

Focus on the second component. Generically, $\frac{d(\hat{C}_t - E_t\hat{C}_{t+1})}{d\hat{G}_t} < 0$ since after a positive government spending shock consumption typically falls on impact and then gradually rises toward its new steady state. Along this margin, D_1 therefore calls for a reduction in taxes on impact, followed by a gradual increase as the economy transitions to the new steady state. Importantly, as we showed in text, this mechanism is not confined to periods t and $t+1$. It applies to the entire transition path toward the new long-run equilibrium. As a result, the planner optimally distorts the intertemporal tax profile throughout the adjustment path.

Consider now the component $f'(\overline{\widetilde{\theta}})\overline{\widetilde{\theta}}(1-\beta)\hat{\widetilde{\theta}}_t$, noting that this term can influence the optimal tax schedule both in the transition and after the economy has settled to a new steady state, insofar as $\hat{\widetilde{\theta}}_t$ will permanently be non-zero. The sign of this term hinges on the derivative $f'(\overline{\widetilde{\theta}})$ capturing the change in the mass of households for which the financial friction is relaxed when/if $\hat{\widetilde{\theta}}_t$ increases, or tightens if $\hat{\widetilde{\theta}}_t$ drops, after the shock in spending.

The numerical examples presented in the main text assumed that f is a log-normal distribution and for the calibration used, we had $f'(\widetilde{\overline{\theta}}) < 0$. Applying these as assumptions here, the term $f'(\widetilde{\overline{\theta}})\widetilde{\overline{\theta}}(1-\beta)\hat{\widetilde{\theta}}_t$ will lead to permanently lower taxes after the shock if $\hat{\widetilde{\theta}}_t$ drops in the new steady-state equilibrium.

Long-term debt. The analogue of (B.9) in the full model where the government can issue both long and short term bonds is

$$-\tau_t = \frac{1}{\frac{1+\eta}{\eta}(1+\psi_{gov,t}(1+\gamma))A(\widetilde{\theta}_t)}\Bigg[A(\widetilde{\theta}_t)\left(1-\frac{1+\eta}{\eta}-\frac{1+\eta}{\eta}\psi_{gov,t}(\gamma+1)\right)-\psi_{gov,t}\underbrace{f_{\widetilde{\theta}_t}\left(\beta E_t\frac{u'(C_{t+1})}{u'(C_t)}-1\right)}_{D_{1,t}}$$
$$+\psi_{gov,t}\underbrace{\left(A(\widetilde{\theta}_t)\frac{1+\eta}{\eta}+u'(C_t)(G_t-\frac{1+\eta}{\eta}Y_t)\right)}_{MM}+\underbrace{b_{t-1,S}u'(C_t)\left(\psi_{gov,t}-F_{\widetilde{\theta}_{t-1}}\psi_{gov,t-1}\right)}_{D_{2,t}}+$$
$$\underbrace{+u'(C_t)\sum_{j\geq 0}\delta^j\left(\psi_{gov,t-j}-\psi_{gov,t-j-1}\right)b_{L,t-j-1}}_{D_{3,t}}\Bigg] \tag{B.12}$$

This expression follows from combining the FOCs for C_t, Y_t, as in the previous paragraph, but without imposing that long bonds equal zero.

In the presence of long debt, the per-period budget constraint of the government can be written as:

$$E_t\sum_{j\geq 1}\beta^j\delta^{j-1}\frac{u'(C_{t+j})}{u'(C_t)}b_{L,t}+\frac{1}{u'(C_t)}\int_{\widetilde{\theta}_t}^{\infty}\theta dF_\theta+F(\widetilde{\theta}_t)E_t\beta\frac{u'(C_{t+1})}{u'(C_t)}b_{S,t}+\text{Surplus}_t$$
$$=b_{t-1,S}+b_{t-1,L}(1+\delta E_t\sum_{j\geq 1}\delta^{j-1}\beta^j\frac{u'(C_{t+j})}{u'(C_t)})$$

Multiplying by $u'(C_t)$ and rearranging, we can write this equation as

$$\underbrace{\beta E_t\sum_{j\geq 1}\beta^{j-1}\delta^{j-1}u'(C_{t+j})b_{L,t}+\beta E_t u'(C_{t+1})b_{S,t}}_{\beta E_t Z_{t+1}}+\int_{\widetilde{\theta}_t}^{\infty}\theta dF_\theta+(F(\widetilde{\theta}_t)-1)E_t\beta u'(C_{t+1})b_{S,t}$$
$$+u'(C_t)\text{Surplus}_t=\underbrace{b_{t-1,S}u'(C_t)+b_{t-1,L}E_t\sum_{j\geq 0}\delta^j\beta^j u'(C_{t+j})}_{Z_t}$$

Thus, Z_t denotes the utility-valued continuation payoff associated with outstanding government liabilities. Iterating forward on this constraint and imposing the transversality condition we get the intertemporal debt constraint:

$$E_t\sum_{j\geq 0}\beta^j\frac{1}{u'(C_t)}\Bigg[\underbrace{\int_{\widetilde{\theta}_{t+j}}^{\infty}\theta dF_\theta+(F(\widetilde{\theta}_{t+j})-1)u'(C_{t+j+1})b_{S,t+j}\beta}_{\text{Liquidity/Safety rents}}+u'(C_{t+j})\text{Surplus}_{t+j}\Bigg]$$
$$=b_{t-1,S}+b_{t-1,L}E_t\sum_{j\geq 0}\delta^j\beta^j\frac{u'(C_{t+j})}{u'(C_t)} \tag{B.13}$$

B.2 Date 0 program/Timeless Penalties/ Revisiting Lucas and Stokey (1983)

In this subsection we study optimal taxation from the perspective of a date 0 program. We first characterize the optimal tax schedule for a specific initial maturity structure of government debt (when long bonds are consols) contrasting the canonical model outcome with the outcome under binding financial frictions. We then show that the optimal fiscal policy derived from solving the date 0 program, is in general time inconsistent under financial frictions, so that if the planner is allowed after date 0 to re-optimize, they will implement policies that differ from those implied by the continuation Ramsey program. We derive the augmented Ramsey program (adding timeless penalties) that restores time consistency. Lastly, we revisit a key result of Lucas and Stokey (1983) showing that if the government can issue a sufficiently rich maturity structure, then the optimal date 0 policy can be time consistent.

Introducing into the model with binding frictions an arbitrary maturity structure of zero coupon bonds that do not provide safety/liquidity services (while maintaining that only short bonds have this attribute) we show that time consistency of the optimal policy cannot be ensured by maturity alone. The results of this paragraph are complementary to the results shown in text, and may be of separate interest.

Consider an initial allocation problem whereby the optimal policy starts in period 0, given the outstanding debt levels $b_{S,-1}$ and $b_{L,-1}$ and given Lagrange multipliers $\psi_{gov,-1} = \psi_{gov,-2} = \psi_{gov,-3} = ... = 0$.

Lemma B.1 derives the optimal tax schedule of the date 0 program in the canonical Ramsey model. We assume for tractability that long bonds are consols.

Lemma B.1. *Consider the initial allocation problem; the planner maximizes household utility subject to the sequence of budget constraints (B.13) and the resource constraints. Assume* $G_t = \overline{G}$ *and* $f(\widetilde{\theta}_t) = 0$ *for all* t*. Then if long bonds are consols* $\delta = 1$ *the optimal plan is* $\tau_0 < \tau_1 = \tau_2 = \tau_3 = ...\overline{\tau}$.

Proof. For brevity, we provide only a partial proof of Lemma B.1, together with a sketch of the complete argument. Notice that in the absence of spending shocks, we have $\psi_{gov,t} = \overline{\psi}_{gov}$ for all t. Since we also assume that $A(\widetilde{\theta}_t) = \overline{A}$ it is immediate that the leading term and the denominator in (B.12) are constant. Moreover, $D_{1,t} = 0$ for all t, $MM = -\frac{1}{\eta}\overline{G}u'(C_t)$, and $D_{2,0} = b_{S,-1}u'(C_0)\overline{\psi}_{gov}$ and $D_{2,t} = 0$ for $t > 0$. Furthermore,

$$D_{3,t} = u'(C_t)b_{L,-1}\overline{\psi}_{gov}$$

Hence, for taxes to be constant after period 1 we need to show that $C_{t\geq 1} = \overline{C}$ and $Y_{t\geq 1} = \overline{Y}$. This is simple. Note that for the maximization program in Lemma 1 only the date-0 constraint (B.13) is binding. For $t > 0$ we can always construct a portfolio of short- and long-term bonds that satisfies the constraints given the optimal allocation. It is then straightforward to show that consumption and output will be constant for period 1 onwards. ■

Lemma B.1 is not a novel (or surprising) result. It provides a useful benchmark that allows us to compare with the case in which the economy is subject to the financial friction. Our next result shows that generically, optimal taxes are not constant in this case when long bonds are consols.

Proposition B.2. *Consider the initial allocation problem; the planner maximizes household welfare subject to the sequence of budget constraints (B.13) and the resource constraints. Assume $G_t = \overline{G}$ for all t. Suppose that the equilibrium policy satisfies $F(\widetilde{\theta}_t) < 1$. Then, when long bonds are consols, optimal taxes are generically not constant after period 0.*

Proof. We prove this result by explicitly deriving the optimal plan. To that end, we simplify the planner's program by substituting the bond sequence $b_{S,t}$ using $\widetilde{\theta}_t C_t$, and using the resource constraint to substitute output from the objective and the surplus functions. After making these substitutions and dropping the corresponding equality constraints, we write the planner's program as the following Lagrangian function:

$$\mathcal{L} = \sum_{t\geq 0}\beta^t \widetilde{V}(C_t,\widetilde{\theta}_t) + \widetilde{\psi}_{gov}\Bigg(\sum_{t\geq 0}\beta^t\left[\int_{\widetilde{\theta}_t}^{\infty}\theta dF_\theta + u'(C_t)\text{Surplus}_t(C_t,\widetilde{\theta}_t)\right] + \sum_{t\geq 0}\beta^t(F(\widetilde{\theta}_t)-1)\beta u'(C_{t+1})\widetilde{\theta}_t C_t\Bigg]$$
$$-b_{-1,S}u'(C_0) - b_{-1,L}\sum_{t\geq 0}\beta^t u'(C_t)\Bigg)$$

The only constraint is the date-0 version of (B.13). For brevity, we omit the algebra required to arrive to the expression in $\mathcal{L}$.

The first-order conditions for the optimum are

$$\widetilde{V}_C(C_t,\widetilde{\theta}_t) + \widetilde{\psi}_{gov}u''(C_t)\text{Surplus}_t(C_t,\widetilde{\theta}_t) + \widetilde{\psi}_{gov}u'(C_t)\text{Surplus}_C(C_t,\widetilde{\theta}_t) + \widetilde{\psi}_{gov}(F(\widetilde{\theta}_t)-1)\widetilde{\theta}_t\beta u'(C_{t+1})$$
$$+\widetilde{\psi}_{gov}(F(\widetilde{\theta}_{t-1})-1)\widetilde{\theta}_{t-1}C_{t-1}u''(C_t) - \widetilde{\psi}_{gov}b_{-1,L}u''(C_t) = 0 \quad t>0 \tag{B.14}$$

(B.15)
$$\widetilde{V}_{\widetilde{\theta}_t}(C_t,\widetilde{\theta}_t) + \widetilde{\psi}_{gov}\frac{d}{d\widetilde{\theta}_t}\left[\int_{\widetilde{\theta}_t}^{\infty}\theta dF_\theta + u'(C_t)\text{Surplus}_t(C_t,\widetilde{\theta}_t)\right] + \widetilde{\psi}_{gov}u'(C_{t+1})\beta\Big((f(\widetilde{\theta}_t)-1)\widetilde{\theta}_t + (F(\widetilde{\theta}_t)-1)\Big) = 0$$

and finally,

$$\widetilde{V}_C(C_0,\widetilde{\theta}_0) + \widetilde{\psi}_{gov}u''(C_0)\text{Surplus}_0(C_0,\widetilde{\theta}_0) + \widetilde{\psi}_{gov}u'(C_0)\text{Surplus}_C(C_0,\widetilde{\theta}_0) + \widetilde{\psi}_{gov}(F(\widetilde{\theta}_0)-1)\widetilde{\theta}_0\beta u'(C_1)$$
$$-\widetilde{\psi}_{gov}(b_{-1,S}u''(C_0) + b_{-1,L}u''(C_0)) = 0 \tag{B.16}$$

Key to establishing the result is the observation that, after period-0, the optimality conditions for consumption feature the lagged values $\widetilde{\theta}_{t-1}, C_{t-1}$. Thus, the optimal solution to (B.14) and (B.15) (given initial conditions from (B.16) and the date 0 constraint (B.13) that will pin down $\widetilde{\psi}_{gov}$) is not (in general) a history-independent allocation whereby $C_t, \widetilde{\theta}_t$ are constant through time.

Moreover, given the time-varying solution for $C_t, \widetilde{\theta}_{t-1}$, it follows that the optimal tax rates

$$-\tau_t = \frac{1}{\frac{1+\eta}{\eta}(1+\widetilde{\psi}_{gov}(1+\gamma))A(\widetilde{\theta}_t)}\Bigg[A(\widetilde{\theta}_t)\Big(1-\frac{1+\eta}{\eta}-\frac{1+\eta}{\eta}\widetilde{\psi}_{gov}(\gamma+1)\Big) - \widetilde{\psi}_{gov}f_{\widetilde{\theta}_t}\Big(\beta\frac{u'(C_{t+1})}{u'(C_t)}-1\Big)$$
$$-\widetilde{\psi}_{gov}u'(C_t)\frac{\overline{G}}{\eta} + \widetilde{\psi}_{gov}C_{t-1}\widetilde{\theta}_{t-1}u'(C_t)\Big(1-F_{\widetilde{\theta}_{t-1}}\Big) + u'(C_t)\widetilde{\psi}_{gov}b_{L,-1}\Bigg]^{3} \tag{B.17}$$

are not constant through time. ■

[3] We used $b_{t-1,S} = C_{t-1}\widetilde{\theta}_{t-1}$ to substitute out the lag of short term debt from (B.12)

Unfortunately, deriving an explicit solution based on the nonlinear system (B.14) and (B.15) is not an easy task. Our global numerical solutions reported in the paper show that $C_t, \widetilde{\theta}_t$ display transitional dynamics and these numerical results confirm that taxes are time-varying.[4]

Time inconsistency/Timeless penalties.

We now explain that the optimal Ramsey policy in the model with financial frictions is not time-consistent when long-term bonds take the form of consols. Time inconsistency arises from two sources. First, along the transition path, the optimal portfolio changes — $b_{S,t}, b_{L,t}$ are generically different than the initial conditions $b_{S,-1}, b_{L,-1}$. Second, in the presence of financial frictions, lags values of C and $\widetilde{\theta}$ influence the solution. When setting the optimal consumption path, the planner commits to a path of frictions to determine equilibrium bond prices.

We now formalize these two sources of time inconsistency and explain how to introduce timeless penalties (Marcet and Marimon, 2019; Woodford, 2001; Dávila and Schaab, 2023) to the policy objective function to eliminate time inconsistency.

Proposition B.3. *Assume that the economy has been following the Ramsey equilibrium until period t and that unexpectedly the planner can choose alternative policies to maximize*

$$\sum_{\bar{t}\geq 0} \beta^{\bar{t}} \widetilde{V}(C_{t+\bar{t}}, \widetilde{\theta}_{t+\bar{t}}) \tag{B.18}$$

subject to the date t implementability condition

$$\sum_{\bar{t}\geq 0} \beta^{\bar{t}} \left[\int_{\widetilde{\theta}_{t+\bar{t}}}^{\infty} \theta dF_\theta + u'(C_{t+\bar{t}}) Surplus_{t+\bar{t}} \right] + \sum_{\bar{t}\geq 0} \beta^{\bar{t}} (F(\widetilde{\theta}_{t+\bar{t}}) - 1) u'(C_{t+\bar{t}+1}) \widetilde{\theta}_{t+\bar{t}} C_{t+\bar{t}} \beta \Big]$$
$$- b_{t-1,S} u'(C_t) - b_{t-1,L} \sum_{\bar{t}\geq 0} \beta^{\bar{t}} u'(C_{t+\bar{t}}) = 0 \tag{B.19}$$

i) The solution to this program differs from the continuation Ramsey policy (equations (B.14) and (B.15)).

ii) Let

$$P_t \equiv \widetilde{\psi}_{gov} F(\widetilde{\theta}_{t-1}) b_{S,t-1} u'(C_t) + b_{L,t-1} \widetilde{\psi}_{gov} \sum_{\bar{t}\geq 0} \beta^{\bar{t}} u'(C_{t+\bar{t}}) - \widetilde{\psi}_{gov} b_{L,-1} \sum_{\bar{t}\geq 0} \beta^{\bar{t}} u'(C_{t+\bar{t}}) \tag{B.20}$$

Then solving the augmented Ramsey program

$$\max \sum_{\bar{t}\geq 0} \beta^{\bar{t}} \widetilde{V}(C_{t+\bar{t}}, \widetilde{\theta}_{t+\bar{t}}) + P_t \tag{B.21}$$

subject to (B.19) delivers the same solution as in the continuation Ramsey program.

The two statements of Proposition B.3 are straightforward to prove. Consider part i). The first-order condition for period t is

$$\widetilde{V}_C(C_t, \widetilde{\theta}_t) + \widetilde{\widetilde{\psi}}_{gov} u''(C_t) \text{Surplus}_t(C_t, \widetilde{\theta}_t) + \widetilde{\widetilde{\psi}}_{gov} u'(C_t) \text{Surplus}_C(C_t, \widetilde{\theta}_t) + \widetilde{\psi}_{gov} (F(\widetilde{\theta}_t) - 1) \widetilde{\theta}_t \beta u'(C_{t+1})$$
$$- \widetilde{\widetilde{\psi}}_{gov} (b_{t-1,S} u''(C_t) + b_{t-1,L} u''(C_t)) = 0 \tag{B.22}$$

[4]See, for example, subsection B.3 (Figure 1) for a numerical illustration of the optimal allocation in a complete market with aggregate shocks, highlighting the transitional dynamics in this class of models. For brevity, we do not present the corresponding examples without aggregate shocks; however, the solutions confirm that taxes are time-varying.

This is different from (B.14) in three main respects. First, the debt composition in period t will in general differ from that at date 0. Therefore, generically $b_{L,t-1} \neq b_{L,-1}$. Second, even if the portfolio remains constant through time (i.e. $b_{S,t-1} = b_{S,-1}$ $b_{L,t-1} = b_{L,-1}$) the planner will still distort date t consumption relative to the solution implied by (B.14) according to the term $b_{t-1,S}u''(C_t)$. Third, re-optimization eliminates the past commitment regarding the financial friction, and therefore the lagged ratio $\widetilde{\theta}_{t-1}$ does not appear in (B.22).

Consider ii). It is straightforward to show that the first-order conditions with respect to C_t and $C_{t+\bar{t}}$ in the augmented Ramsey program become

$$\begin{aligned}&\widetilde{V}_C(C_t,\widetilde{\theta}_t)+\widetilde{\widetilde{\psi}}_{gov}u''(C_t)\text{Surplus}_t(C_t,\widetilde{\theta}_t)+\widetilde{\widetilde{\psi}}_{gov}u'(C_t)\text{Surplus}_C(C_t,\widetilde{\theta}_t)+\widetilde{\psi}_{gov}(F(\widetilde{\theta}_t)-1)\widetilde{\theta}_t\beta u'(C_{t+1})\\ &+(\widetilde{\psi}_{gov}F(\widetilde{\theta}_{t-1})-\widetilde{\widetilde{\psi}}_{gov})b_{S,t-1}u''(C_t)+(\widetilde{\psi}_{gov}-\widetilde{\widetilde{\psi}}_{gov})b_{t-1,L}u''(C_t)-\widetilde{\psi}_{gov}b_{L,-1}u''(C_t))=0\end{aligned} \tag{B.23}$$

$$\begin{aligned}&\widetilde{V}_C(C_{t+\bar{t}},\widetilde{\theta}_{t+\bar{t}})+\widetilde{\widetilde{\psi}}_{gov}u''(C_{t+\bar{t}})\text{Surplus}_{t+\bar{t}}(C_{t+\bar{t}},\widetilde{\theta}_{t+\bar{t}})+\widetilde{\widetilde{\psi}}_{gov}u'(C_{t+\bar{t}})\text{Surplus}_C(C_{t+\bar{t}},\widetilde{\theta}_{t+\bar{t}})\\ &+\widetilde{\widetilde{\psi}}_{gov}(F(\widetilde{\theta}_{t+\bar{t}})-1)\widetilde{\theta}_{t+\bar{t}}\beta u'(C_{t+\bar{t}+1})+(\widetilde{\psi}_{gov}-\widetilde{\widetilde{\psi}}_{gov})b_{t-1,L}u''(C_{t+\bar{t}})-\widetilde{\psi}_{gov}b_{L,-1}u''(C_{t+\bar{t}})=0 \quad \bar{t}>0\end{aligned} \tag{B.24}$$

which are equivalent to (B.14) if $\widetilde{\psi}_{gov} = \widetilde{\widetilde{\psi}}_{gov}$. When this holds, it is straightforward to show that the FOCs with respect to $\widetilde{\theta}$ are exactly (B.15).

To show that indeed $\widetilde{\psi}_{gov} = \widetilde{\widetilde{\psi}}_{gov}$ in the augmented Ramsey program, note that the continuation Ramsey policy that solves (B.14) and (B.15), satisfies the intertemporal constraint in period t given the portfolio in $b_{S,t-1}, b_{L,t-1}$. Therefore, the value of the multiplier that satisfies the first-order condition of the Lagrangean in the augmented Ramsey program will indeed be $\widetilde{\psi}_{gov}$. ■

Time Consistency with an Arbitrary Maturity structure. Having explained the time inconsistency problem in the context of the model with binding financial frictions, we now revisit the well-known result of Lucas and Stokey (1983) showing that if the government can issue an arbitrary maturity structure, the optimal policy can be made time consistent. If given the opportunity to re-optimize in period t, the planner will follow the continuation Ramsey policy determined at date 0.

Assume that from period 0 onward the planner can (sequentially) issue debt in a short liquid asset and an infinite number of zero coupon bonds $\left\{b^{\bar{t}}_{t-1,L}\right\}$, where $\bar{t}$ denotes the maturity of a debt instrument, so that a bond issued in $t-1$ with $\bar{t}=0$ will pay one unit of income in period t.

Under this assumption, the implementability condition (B.19) is replaced by

$$\begin{aligned}&\sum_{\bar{t}\geq 0}\beta^{\bar{t}}\left[\int_{\widetilde{\theta}_{t+\bar{t}}}^{\infty}\theta dF_\theta+u'(C_{t+\bar{t}})\text{Surplus}_{t+\bar{t}}\right]+\sum_{\bar{t}\geq 0}\beta^{\bar{t}}(F(\widetilde{\theta}_{t+\bar{t}})-1)u'(C_{t+\bar{t}+1})\widetilde{\theta}_{t+\bar{t}}C_{t+\bar{t}}\beta\Bigg]\\ &-b_{t-1,S}u'(C_t)-\sum_{\bar{t}\geq 0}b^{\bar{t}}_{t-1,L}\beta^{\bar{t}}u'(C_{t+\bar{t}})=0\end{aligned} \tag{B.25}$$

The planner reoptimizes in period t maximizing household welfare, subject to (B.25).

We focus first on the case where the financial friction is not binding (equivalently, in the canonical model). To keep the notation consistent with the previous paragraph, we represent the non-binding friction by imposing $\widetilde{\theta}_t = \overline{\theta}$ such that $F(\overline{\theta}) = 1$.[5]

[5]The exact value of $\widetilde{\theta}_t$ is then not determined in the model and so the FONC for $\widetilde{\theta}_t$ do not have to be derived.

The first-order condition of the new Ramsey program solved at date t with respect to $C_{t+\bar{t}}$ is:

$$\widetilde{V}_C(C_{t+\bar{t}},\overline{\theta})+\widetilde{\widetilde{\psi}}_{gov}u''(C_{t+\bar{t}})\text{Surplus}_{t+\bar{t}}(C_{t+\bar{t}},\overline{\theta})+\widetilde{\widetilde{\psi}}_{gov}u'(C_{t+\bar{t}})\text{Surplus}_C(C_{t+\bar{t}},\overline{\theta})$$
$$-\widetilde{\widetilde{\psi}}_{gov}u''(C_{t+\bar{t}})(b_{t-1,S}\mathbf{1}_{\{\bar{t}=0\}}+b^{\bar{t}}_{t-1,L})=0$$

whereas from the original date 0 program we have:

$$\widetilde{V}_C(C_{t+\bar{t}},\overline{\theta})+\widetilde{\psi}_{gov}u''(C_{t+\bar{t}})\text{Surplus}_{t+\bar{t}}(C_{t+\bar{t}},\overline{\theta})+\widetilde{\psi}_{gov}u'(C_{t+\bar{t}})\text{Surplus}_C(C_{t+\bar{t}},\overline{\theta})$$
$$-\widetilde{\psi}_{gov}u''(C_{t+\bar{t}})b_{-1,L}=0$$

It is simple to show that, for a given consumption sequence, satisfaction of these conditions requires to set

(B.26)
$$b^{\bar{t}}_{t-1,L}=(1-\frac{\widetilde{\psi}_{gov}}{\widetilde{\widetilde{\psi}}_{gov}})\left(\text{Surplus}_{t+\bar{t}}(C_{t+\bar{t}},\overline{\theta})+\frac{u'(C_{t+\bar{t}})}{u''(C_{t+\bar{t}})}\text{Surplus}_C(C_{t+\bar{t}},\overline{\theta})\right)+\frac{\widetilde{\psi}_{gov}}{\widetilde{\widetilde{\psi}}_{gov}}b_{-1,L}-b_{t-1,S}\mathbf{1}_{\{\bar{t}=0\}}$$

The result of Lucas and Stokey is that given the date 0 multiplier $\widetilde{\psi}_{gov}$ and the optimal allocation $C_{t+\bar{t}}$ one can construct a sequence $b^{\bar{t}}_{t-1,L}$ and a multiplier $\widetilde{\widetilde{\psi}}_{gov}$ that satisfy the above conditions and the date t implementability condition (B.25).

In terms of the initial allocation program we solved in Lemma 1, featuring constant consumption for $t\geq 1$ we will have $\widetilde{\psi}_{gov}=\widetilde{\widetilde{\psi}}_{gov}$ and $b^{\bar{t}}_{t-1,L}=b_{-1,L}-b_{t-1,S}\mathbf{1}_{\{\bar{t}=0\}}$.[6]

We now show that this logic fails under binding financial frictions. In this case, we get the following expressions for the consumption FONC under reoptimization:

$$\widetilde{V}_C(C_t,\widetilde{\theta}_t)+\widetilde{\widetilde{\psi}}_{gov}u''(C_t)\text{Surplus}_t(C_t,\widetilde{\theta}_t)+\widetilde{\widetilde{\psi}}_{gov}u'(C_t)\text{Surplus}_C(C_t,\widetilde{\theta}_t)$$
(B.28)
$$+\widetilde{\widetilde{\psi}}_{gov}(F(\widetilde{\theta}_t)-1)\widetilde{\theta}_t\beta u'(C_{t+1})-\widetilde{\widetilde{\psi}}_{gov}u''(C_t)(b_{t-1,S}+b^{\bar{t}}_{t-1,L})=0$$

$$\widetilde{V}_C(C_{t+\bar{t}},\widetilde{\theta}_{t+\bar{t}})+\widetilde{\widetilde{\psi}}_{gov}u''(C_{t+\bar{t}})\text{Surplus}_{t+\bar{t}}(C_{t+\bar{t}},\widetilde{\theta}_{t+\bar{t}})+\widetilde{\widetilde{\psi}}_{gov}u'(C_{t+\bar{t}})\text{Surplus}_C(C_{t+\bar{t}},\widetilde{\theta}_{t+\bar{t}})$$
(B.29)
$$+\widetilde{\widetilde{\psi}}_{gov}(F(\widetilde{\theta}_{t+\bar{t}})-1)\widetilde{\theta}_{t+\bar{t}}\beta u'(C_{t+\bar{t}+1})+\widetilde{\widetilde{\psi}}_{gov}(F(\widetilde{\theta}_{t+\bar{t}-1})-1)\widetilde{\theta}_{t+\bar{t}-1}u''(C_{t+\bar{t}})-\widetilde{\widetilde{\psi}}_{gov}b^{\bar{t}}_{t-1,L}u''(C_{t+\bar{t}})=0\quad\bar{t}>0$$

which together with the first-order condition with respect to $\widetilde{\theta}_{t+\bar{t}}$ complete the system of equations that we need to solve.

[6] To see this, use (B.26) in the date t constraint under constant consumption. We will get

(B.27)
$$\frac{1}{1-\beta}\text{Surplus}=b_{t-1,S}+\frac{1}{1-\beta}\left[(1-\frac{\widetilde{\psi}_{gov}}{\widetilde{\widetilde{\psi}}_{gov}})\left(\text{Surplus}+\frac{u'(C)}{u''(C)}\text{Surplus}_C\right)+\frac{\widetilde{\psi}_{gov}}{\widetilde{\widetilde{\psi}}_{gov}}b_{-1,L}\right]-b_{t-1,S}$$

which is clearly satisfied at the date 0 allocation if we set $\widetilde{\psi}_{gov}=\widetilde{\widetilde{\psi}}_{gov}$.

Using the same approach as above, it is straightforward to show that the portfolio issued at $t-1$ must satisfy

$$b^{\bar{t}}_{t-1,L} = (1 - \frac{\widetilde{\psi}_{gov}}{\widetilde{\widetilde{\psi}}_{gov}})\left(\text{Surplus}_{t+\bar{t}}(C_{t+\bar{t}}, \widetilde{\theta}_{t+\bar{t}}) + \frac{u'(C_{t+\bar{t}})}{u''(C_{t+\bar{t}})}\text{Surplus}_C(C_{t+\bar{t}}, \widetilde{\theta}_{t+\bar{t}})\right)$$

(B.30)

$$+(1 - \frac{\widetilde{\psi}_{gov}}{\widetilde{\widetilde{\psi}}_{gov}})(F(\widetilde{\theta}_{t+\bar{t}}) - 1)\widetilde{\theta}_{t+\bar{t}}\beta\frac{u'(C_{t+\bar{t}+1})}{u''(C_{t+\bar{t}})} - \left[b_{t+\bar{t}-1,S} + \frac{\widetilde{\psi}_{gov}}{\widetilde{\widetilde{\psi}}_{gov}}(F(\widetilde{\theta}_{t+\bar{t}-1}) - 1)\widetilde{\theta}_{t+\bar{t}-1}C_{t+\bar{t}-1}\right]\mathbf{1}_{\bar{t}=0}$$

$$+\left[(1 - \frac{\widetilde{\psi}_{gov}}{\widetilde{\widetilde{\psi}}_{gov}})(F(\widetilde{\theta}_{t+\bar{t}-1}) - 1)\widetilde{\theta}_{t+\bar{t}-1}C_{t+\bar{t}-1}\right]\mathbf{1}_{\bar{t}>0} + \frac{\widetilde{\psi}_{gov}}{\widetilde{\widetilde{\psi}}_{gov}}b_{-1,L}$$

However, now, the Lagrange multiplier is pinned down by optimality with respect to $\widetilde{\theta}_{t+\bar{t}}$((B.15) and its analogue in the date t program). For the same date 0 allocation we clearly need to have $\frac{\widetilde{\psi}_{gov}}{\widetilde{\widetilde{\psi}}_{gov}} = 1$, and consequently,

$$b^{\bar{t}}_{t-1,L} = b_{-1,L} - \left[(F(\widetilde{\theta}_{t+\bar{t}-1}) - 1)\widetilde{\theta}_{t+\bar{t}-1}C_{t+\bar{t}-1} + b_{t-1,S}\right]\mathbf{1}_{\bar{t}=0} \tag{B.31}$$

The portfolio defined in (B.31) ensures that the first-order conditions of the date 0 program will continue to be satisfied under reoptmization. However, it is not clear that, at the implied debt level, the period-t intertemporal budget will hold (given also that $b_{t-1,S}$ is pinned down by $\widetilde{\theta}_{t-1}, C_{t-1}$).

To give a concrete example, consider an allocation which in period $t-1$ (or before) has converged to a steady state. Then, (B.31) becomes

$$b^{\bar{t}}_{t-1,L} = b_{-1,L} - \left[(F(\widetilde{\theta}) - 1)\widetilde{\theta}C + \widetilde{\theta}C\right]\mathbf{1}_{\bar{t}=0}$$

and the intertemporal budget constraint can be written as

$$\frac{1}{1-\beta}\left[\int_{\widetilde{\theta}}^{\infty}\theta dF_\theta + u'(C)\text{Surplus} + (F(\widetilde{\theta}) - 1)\widetilde{\theta}\beta\right] = \widetilde{\theta} + b_{-1,L}\frac{u'(C)}{1-\beta} - \left[(F(\widetilde{\theta}) - 1)\widetilde{\theta} + \widetilde{\theta}C\right]u'(C),$$

or

$$\left[C\int_{\widetilde{\theta}}^{\infty}\theta dF_\theta + \text{Surplus} + (F(\widetilde{\theta}) - 1)C\widetilde{\theta}\right] = b_{-1,L}$$

But this condition is unlikely to hold, since initial debt $b_{-1,L}$ is arbitrary.

We showed that the LS time-consistency result generically fails in the presence of financial frictions. We demonstrated this by focusing on equilibria in which the frictions bind in the date-0 optimal policy program. To conclude this paragraph, we note an additional reason why the LS result may fail: the inherent global non-concavity of the optimal policy program, which we highlighted in previous paragraphs and in the main text.

For example, there may be cases in which initial debt is negative and the date-0 program commits to $\widetilde{\theta} < \overline{\theta}$. However, when the planner re-evaluates the program at date t, she eliminates the financial friction by setting $\widetilde{\theta} \geq \overline{\theta}$. For such cases, it may be also difficult to find portfolios that implement the date-0 allocation. We choose not to analyze these cases here.

B.3 Debt management under effectively complete markets

In this paragraph, we provide the results we referred to in Section 4.1 of the paper. We first present analytical examples illustrating the properties of optimal portfolios in the canonical Ramsey model developed by Angeletos (2002) and Buera and Nicolini (2004) (henceforth ABN). We then provide a numerical example showing that, under binding financial frictions, completing the market with debt portfolios is not possible.

Examples affirming ABN. We consider the canonical Ramsey model with two bonds and two states (as described in the main text). Let $\widetilde{V}$ denote the household utility after using the resource constraint to substitute aggregate output with $C_t + G_t$.

For convenience, we assume that long-term government debt is issued as consols. As in the analogous example in the previous subsection, this assumption limits transitional dynamics (the optimal policy allocation converges after one period). We solve for the Ramsey optimum by maximizing the household's utility subject to the date-0 implementability constraint $\Upsilon_0 = z_0$. The Lagrangian is

$$\mathcal{L} = E_0 \sum_{t\geq 0} \beta^t \widetilde{V}(C_t) + \widetilde{\psi}_{gov} \underbrace{\left(E_0 \sum_{t\geq 0} \beta^t u'(C_t) \text{Surplus}_t(C_t) - b_{-1,S} u'(C_0) - b_{-1,L} \sum_{t\geq 0} \beta^t E_0 u'(C_t) \right)}_{\Upsilon_0 = z_0}$$

The first-order condition for C_t is

$$\widetilde{V}_C(C_t) + \widetilde{\psi}_{gov} \Big(u''(C_t)\text{Surplus}_t(C_t) + u'(C_t)\text{Surplus}_C(C_t) \Big) - \widetilde{\psi}_{gov}(b_{-1,S}\mathbf{1}_{t=0} + b_{-1,L}) u''(C_t) = 0$$

Since $\widetilde{V}$ and the surplus function are implicitly a function of spending, the solution satisfies $C_t = C(g_t), t \geq 1$ (i.e., consumption is a function of the current spending realization). Thus, after one period, the optimal allocation converges.

Let C^-, C^+ denote the two consumption levels. Under standard assumptions for the functional forms in the model, we have $C^- > C^+$.
Analogously, defining $z_t(\mathbf{g}^t) \equiv E_t \sum_{j\geq 0} \beta^j u'(C_{t+j}) \text{Surplus}_{t+j}(C_{t+j})$ and $q_{t,L}(\mathbf{g}^t) \equiv E_t \sum_{j\geq 1} \beta^j \frac{u'(C_{t+j})}{u'(C_t)}$, we have $z_t(\mathbf{g}^t) \in \{z^-, z^+\}$ and $q_{t,L}(\mathbf{g}^t) \in \{q_L^-, q_L^+\}$, respectively.

It is straightforward to show that the optimal portfolios that solve the system $\Upsilon_t = z_t$ for $t \geq 1$ are

$$\underbrace{b_S = \frac{q_L^+ z^- - q_L^- z^+}{q_L^+ - q_L^-}}_{\text{Short Bonds}} \quad \text{and} \quad \underbrace{b_L = \frac{z^+ - z^-}{q_L^+ - q_L^-}}_{\text{Long Bonds}}$$

which makes clear that when the intertemporal surplus and the bond prices satisfy $z^- > z^+$ and $q_L^- > q_L^+$ (both hold under standard specifications of utility) then $b_L > 0$.

We show now, using analytical examples, that $b_S < 0$, which confirms the ABN result.

Example 1. Consider $b_{-1,L} = b_{-1,S} = 0$ and $g_0 = g^+$. Then the allocation converges in period 0, and it follows from the date-0 implementability constraint that $z^+ = 0$. Therefore, $b_S = \frac{q_L^+ z^-}{q_L^+ - q_L^-} < 0$.

Example 2. Consider $b_{-1,L}, b_{-1,S} \neq 0$ $g_0 = g^+$ and an optimal policy that sets $s_0 = \text{Surplus}_0 > 0$ so that

$$b_{-1,S} + b_{-1,L} \sum_{t\geq 1} \beta^{t-1} E \frac{u'(C_t)}{u'(C_0)} = b_{-1,S} + b_{-1,L} \frac{C_0}{1-\beta} \underbrace{\frac{u'(C^-) + u'(C^+)}{2}}_{\equiv E u'(C)} = 0$$

(assuming that spending follows an i.i.d. process.)

From the implementability constraint, it follows that

$$z_0 = s_0 + \frac{\beta}{1-\beta} C_0 E u'(C) s = 0 \to s^- = -2s_0 \frac{1-\beta}{\beta C_0} - \frac{s^+ C^-}{C^+}$$

where $s^- > s^+$ denote the surplus in the low and high spending states, respectively.

If $b_S < 0$, then it must hold that

$$q_L^+ z^- - q_L^- z^+ > 0 \to C^+ \frac{Eu'(C)}{1-\beta}(s^- + \frac{\beta}{1-\beta} C^- E u'(C) s) - C^- \frac{Eu'(C)}{1-\beta}(s^+ + \frac{\beta}{1-\beta} C^+ E u'(C) s) =$$
$$C^+ s^- - C^- s^+ > 0$$

which is equivalent to

$$s^- > \frac{C^-}{C^+} s^+.$$

A geometric argument shows that this inequality holds when $s_0 > 0$. First, note that the feasible pairs (s^+, s^-) lie on the line $s^- = -2s_0 \frac{1-\beta}{\beta C_0} - \frac{C^-}{C^+} s^+$, which has slope $-\frac{C^-}{C^+} < -1$ and a negative intercept when $s_0 > 0$. Since $C^- > C^+$, the line $s^- = \frac{C^-}{C^+} s^+$ has slope greater than one and therefore lies strictly below the line $s^- = s^+$ for $s^+ < 0$. Hence, any point satisfying $s^- > s^+$ must also satisfy

$$s^- > \frac{C^-}{C^+} s^+,$$

which implies $b_S < 0$.

Financial Frictions. We now provide a numerical example of the model under curved utility and rigid prices. We demonstrate that, with two bonds and two states, completing the market when financial frictions bind is impossible. As in the previous subsection, assume that government spending takes two possible values, (g^-, g^+). Substituting output from the resource constraint and using $b_{S,t} = \widetilde{\theta}_t C_t$, we write the optimal policy Lagrangian as:

$$\mathcal{L} = E_0 \sum_{t \geq 0} \beta^t \widetilde{V}(C_t, \widetilde{\theta}_t) + \widetilde{\psi}_{gov} \Bigg(E_0 \sum_{t \geq 0} \beta^t \Bigg[\int_{\widetilde{\theta}_t}^{\infty} \theta dF_\theta + u'(C_t) \text{Surplus}_t(C_t, \widetilde{\theta}_t) \Bigg] +$$
$$E_0 \sum_{t \geq 0} \beta^t (F(\widetilde{\theta}_t) - 1) u'(C_{t+1}) C_t \widetilde{\theta}_t \beta - b_{-1,S} u'(C_0) - b_{-1,L} \sum_{t \geq 0} \beta^t E_0 u'(C_t) \Bigg)$$

The first-order conditions are

$$\widetilde{V}_C(C_t, \widetilde{\theta}_t) + \widetilde{\psi}_{gov} \Bigg(u''(C_t) \text{Surplus}_t(C_t, \widetilde{\theta}_t) + u'(C_t) \text{Surplus}_C(C_t, \widetilde{\theta}_t) \Bigg)$$

(B.32)
$$+\widetilde{\psi}_{gov}(F(\widetilde{\theta}_{t-1}) - 1)\widetilde{\theta}_{t-1} C_{t-1} u''(C_t) + \widetilde{\psi}_{gov} \widetilde{\theta}_t (F(\widetilde{\theta}_t) - 1) E_t u'(C_{t+1}) - \widetilde{\psi}_{gov}(b_{-1,S} \mathbf{1}_{t=0} + b_{-1,L}) u''(C_t) = 0$$

(B.33)
$$\widetilde{V}_{\widetilde{\theta}_t}(C_t, \widetilde{\theta}_t) + \widetilde{\psi}_{gov} \frac{d}{d\widetilde{\theta}_t} \Bigg[\int_{\widetilde{\theta}_t}^{\infty} \theta dF_\theta + u'(C_t) \text{Surplus}_t(C_t, \widetilde{\theta}_t) \Bigg] + \widetilde{\psi}_{gov} E_t \frac{u'(C_{t+1})}{u'(C_t)} \beta \Bigg(f(\widetilde{\theta}_t) \widetilde{\theta}_t + (F(\widetilde{\theta}_t) - 1) \Bigg) = 0$$

Model solution. Equation (B.32) shows that, in contrast to standard Ramsey problems with complete markets, the optimal allocation under binding financial frictions is not history-independent.

In particular, the presence of lagged $\widetilde{\theta}_{t-1}$ and lagged consumption implies that the model cannot be solved by treating each period t and each realization of the spending shock independently through the FONCs. Instead, solving the model requires a fully dynamic approach, which we implement by using simulations.

We use a PEA algorithm, as in the baseline model with incomplete markets.[7] Briefly, the algorithm proceeds as follows: i) Approximate the conditional expectation $E_t u'(C_{t+1})$ as a polynomial in the state variables $(G_t, \widetilde{\theta}_{t-1})$ and guess an initial value for the multiplier $\widetilde{\psi}_{gov}$. ii) Solve the model over long simulated samples. iii) Update the coefficients of the approximating polynomial to obtain a fixed point and determine $\widetilde{\psi}_{gov}$ such that the date-0 implementability constraint is satisfied.

Based on the model solution, we retrieve two additional approximations: the price of long-term debt $q_{L,t} = E_t \sum_{j=0}^{\infty} \beta^j \frac{u'(C_{t+j})}{u'(C_t)}$, and the present value of surpluses (including liquidity rents), z_t.

We then obtain the optimal portfolios by solving the following system:

$$\text{(B.34)} \qquad \begin{bmatrix} 1 & q_{t,L}(\mathbf{g}^{t-1}, g^-) \\ 1 & q_{t,L}(\mathbf{g}^{t-1}, g^+) \end{bmatrix} \begin{bmatrix} b_{S,t-1} \\ b_{L,t-1} \end{bmatrix} = \begin{bmatrix} z_t(\mathbf{g}^{t-1}, g^-) \\ z_t(\mathbf{g}^{t-1}, g^+) \end{bmatrix}$$

where $(\mathbf{g}^{t-1}, g^-)$ denotes the history up to period t with current state g^- (analogously for g^+).

Figure 1 plots the solution.[8] The blue and red lines denote the quantities of short- and long-term bonds that solve system (B.34). As in ABN, the government saves in private short-term assets, that is, $b_{S,t-1} < 0$. The optimal bond positions vary over time. This is consistent with the lack of history independence. However, these fluctuations are quantitatively negligible.

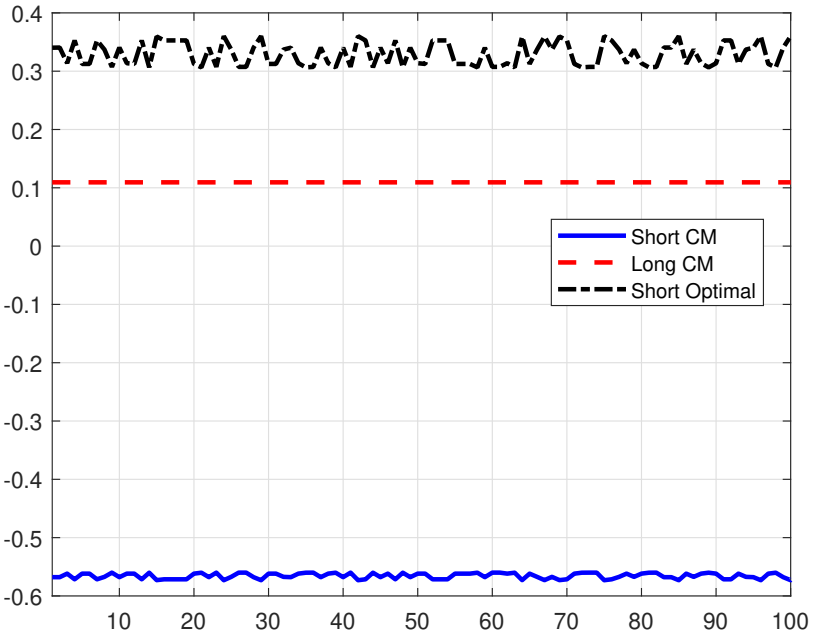
The black dotted line shows the optimal quantity of short-term debt implied by the FOCs. Unsurprisingly, the optimal policy prescribes $b_{S,t} > 0$. Since the solution to the linear system (B.34) is unique and does not coincide with the optimal short-term debt issuance implied by the model, we conclude that markets cannot be completed in this environment.

[7] In our simulations, we assume that the economy always remains in an equilibrium with binding financial frictions. In the calibrated example we consider, the variance of f is assumed to be ten times larger than in the benchmark case. As a result, the solved value of $b_{S,t}$ exceeds 0.3 (the liquidity-abundance cutoff in the baseline model). Eliminating the friction approximately would therefore require an extremely large issuance of short bonds. Hence, the support of f can be viewed as effectively unbounded, implying that an equilibrium without financial frictions is likely not attainable in the environment considered in this paragraph.

Notice, however, that allowing for the possibility that frictions do not always bind would require a welfare-function approximation, as in the baseline model. In contrast to the incomplete-markets baseline, under effectively complete markets public debt is a stationary variable, so total debt does not drift to very negative values. Applying our previous results, we are confident that focusing on the equilibrium with binding frictions is justified.

[8] The parameter values are the same as in the baseline model, except for the variance of f (as noted previously), the value of δ (set equal to 1, corresponding to a consol bond), and $\omega = \infty$, reflecting the assumption of rigid prices.

Figure 1: Optimal short and long term bonds



Notes: We plot the optimal quantities of short- and long-term debt. The blue and red lines correspond to the solution of system (B.34), where q_L and z are determined by the optimal policy under effectively complete markets. The black dotted line represents the optimal level of short bonds in the allocation implied by the optimal policy.

C Main Text Proofs

This subsection provides the formal proofs of the propositions, lemmas, and corollaries stated in the main text. We begin by establishing the characterization of competitive equilibrium and proving Proposition 1. We then present proofs of the remaining results in sequence, including the characterization of inflation dynamics (Proposition 2), the analysis of optimal allocations in the quasi-linear case (Lemma 1, Proposition 3, and Corollary 1), and the implications for inflation and debt dynamics (Propositions 4 and 5). To streamline the exposition, we focus on the key steps of each argument and omit standard derivations where they follow directly from well-known results.

C.1 Proof of Proposition 1

We begin with the definition of the competitive equilibrium. In our notation, we define the sequences of aggregate variables $\left\{x_t\right\}_t$ as stochastic processes whose date t elements are measurable with respect to $\left\{\mathbf{g}^t, \mathbf{B}_{-1}\right\}$ where $\mathbf{g}^t = \left(G_0, G_1, ..., G_t\right)$ and $\mathbf{B}_{-1}$ denotes the initial bond quantities (assumed to be equally distributed across individuals). Analogously, we define individual variables as processes measurable with respect to $\left\{\mathbf{g}^t, \mathbf{B}_{-1}, \theta_i^t\right\}$ where θ_i^t denotes the history of preference shocks of agent i.

Definition 1. The competitive equilibrium consists of sequences of government fiscal policies $\left\{\tau_t, b_{L,t}, b_{S,t}\right\}_t$, prices $\left\{\pi_t, w_t, q_{S,t}, q_{L,t}\right\}_t$, aggregates $\left\{Y_t, C_t, \widetilde{c}_t, D_t\right\}_t$, individual decision rules $\left\{C_t^i, c_t^i, B_{S,t}^i, B_{s,t,2}^i, B_{L,t}^i, h_t^i\right\}_t$, transfers $\left\{\overline{C}_t^i\right\}_t$ and the threshold $\left\{\widetilde{\theta}_t^i\right\}_t$ such that households optimize, firms optimize, and goods and bonds markets clear.

Proof. According to Proposition 1 the resource constraint, (see below for a derivation), the government budget constraint, the Phillips curve and condition $C_t\widetilde{\theta}_t = b_{S,t}$ are sufficient for a competitive equilibrium. For brevity, we present formal statements for selected elements of the proof of the proposition, while the remaining components are established through intuitive arguments.

First, note that we can dispense with the individual household budget constraints. Given that in our model the family pools the resources of all agents and uses transfers to redistribute at the end of every period, agent-specific budget constraints will be slack. That is, given an allocation, we can always find $\overline{C}_t^i$ satisfying the individual constraints (A.2) and (A.3). These can therefore be dropped from the Ramsey program.

Second, given that in equilibrium agents will make the same choices $C_t^i, B_{S,t}^i, B_{L,t}^i$ and transfers will ensure that $E_\theta B_{S,t,2}^i = E_\theta B_{S,t}^i$ we can consolidate all individual budgets (integrating over θ and dropping superscripts i) into the following object:

$$P_tC_t + P_tE_\theta c_t(\theta)) + q_{L,t}B_{L,t} + q_{S,t}B_{S,t} = P_t(1-\tau_t)w_th_t + (1+q_{L,t}\delta)B_{L,t-1} + B_{S,t-1} + D_tP_t \tag{C.35}$$

where $E_\theta c_t(\theta)) = \int c_t(\theta)dF_\theta$.[9]

[9]By the properties of this allocation we can be sure that it is sufficient to express i variables as functions of the current idiosyncratic state.

Moreover, making use of the optimality condition $c_t = \theta C_t$ when $\theta < \widetilde{\theta}_t$ and $c_t = b_{S,t}$ when $\theta \geq \widetilde{\theta}_t$ (again dropping i for simplicity) we can express aggregate consumption in subperiod 2 as:

$$\widetilde{c}_t \equiv \int c_t(\theta) dF_\theta = C_t \int_0^{\widetilde{\theta}_t} \theta dF_\theta + \int_{\widetilde{\theta}_t}^{\infty} b_{S,t} dF_\theta$$

Given this, it is straightforward to derive the economy's resource constraint (C.36) by equating total expenditure to output.

$$C_t \left(1 + \mathbf{1}_{\widetilde{\theta}_t < \overline{\theta}} \int_0^{\widetilde{\theta}_t} \theta dF_\theta + \mathbf{1}_{\widetilde{\theta}_t > \overline{\theta}} \int_0^{\overline{\theta}} \theta dF_\theta \right) + \mathbf{1}_{\widetilde{\theta}_t < \overline{\theta}} \int_{\widetilde{\theta}_t}^{\infty} b_{S,t} dF_\theta + G_t + \frac{\omega}{2}(\pi_t - 1)^2 = Y_t \tag{C.36}$$

With these objects, we can show that (C.35) is obtained by adding the resource constraint and the government budget constraint, and using the definition of real firm profits (dividends). Hence, any allocation that satisfies these constraints will also satisfy (C.35).

Finally, to derive (C.37), we proceed as follows: First, we use the Euler equations to substitute prices out of the government budget constraint. The two bond pricing conditions for short and long bonds are:

$$q_{t,S} u'(C_t) = \int_{\widetilde{\theta}_t}^{\infty} \theta v'(b^i_{S,t}) dF_\theta + \beta F(\widetilde{\theta}_t) E_t \frac{u'(C_{t+1})}{\pi_{t+1}}$$

$$q_{t,L} u'(C_t) = \sum_{j \geq 1} \beta^j \delta^{j-1} E_t \frac{u'(C_{t+j})}{\Pi_{k=1}^j \pi_{t+k}}$$

Given the optimal policy allocation we can back out prices to satisfy these conditions.

Second, we use the labour supply condition to express government revenue $w_t \tau_t Y_t$ as $\frac{\tau_t}{1-\tau_t} \chi \frac{Y_t^{1+\gamma}}{u'(C_t)}$. Third, we multiply both sides of the government budget constraint by the marginal utility of consumption, $u'(C_t)$.

$$\begin{aligned} &\mathbf{1}_{\widetilde{\theta}_t < \overline{\theta}} \left[\int_{\widetilde{\theta}_t}^{\infty} \theta dF_\theta + b_{S,t} F(\widetilde{\theta}_t) E_t \frac{u'(C_{t+1})}{\pi_{t+1}} \beta \right] + \mathbf{1}_{\widetilde{\theta}_t \geq \overline{\theta}} b_{t,S} E_t \frac{u'(C_{t+1})}{\pi_{t+1}} + b_{t,L} \sum_{j \geq 1} \beta^j \delta^{j-1} E_t \frac{u'(C_{t+1})}{\Pi_{k=1}^j \pi_{t+k}} \\ &= \frac{b_{t-1,S}}{\pi_t} u'(C_t) + \frac{b_{t-1,L}}{\pi_t} (u'(C_t) + \delta \sum_{j \geq 1} \beta^j \delta^{j-1} E_t \frac{u'(C_{t+j})}{\Pi_{k=1}^j \pi_{t+k}}) + G_t u'(C_t) - \frac{\tau_t}{1-\tau_t} \chi Y_t^{1+\gamma} \end{aligned} \tag{C.37}$$

Sufficiency.

Household Utility. To derive the expression for the household utility $\widetilde{U}_t$ in Proposition 1, we use the following:

$$\int_0^{\infty} \theta \log c_t dF_\theta = \int_0^{\widetilde{\theta}_t} \theta \log C_t dF_\theta + \int_{\widetilde{\theta}_t}^{\infty} \theta \log(b_{S,t}) dF_\theta$$

C.2 Proof of Proposition 2

As discussed in the main text, the proof follows immediately from the first-order condition for inflation evaluated at the steady state.

C.3 Proofs of Lemma 1, Proposition 3 and Corollary 1

We repeat Lemma 1 for convenience.

Lemma 1. Consider the assumptions $u(C_t) = C_t$, $\frac{1+\eta}{\eta} = 1$ and $G_t = \overline{G}$ for all t. Moreover, suppose that f is uniformly distributed on $[0, \overline{\theta}]$. The optimal date-0 policy can be found as a solution to the following static optimization problem.

$$\max_{\overline{Y}, \widetilde{\theta} \leq \overline{\theta}} \overline{Y} - \frac{1}{\overline{\theta}} \left(\frac{1}{2} \widetilde{\theta}^2 + \widetilde{\theta}(\overline{\theta} - \widetilde{\theta}) \right) + \frac{\overline{\theta}}{2} \log(\widetilde{\theta}_t) - \frac{\widetilde{\theta}_t^2}{4\overline{\theta}} - \chi \frac{\overline{Y}^{1+\gamma}}{1+\gamma}$$

subject to

$$\Upsilon = \beta(\frac{\widetilde{\theta}}{\overline{\theta}} - 1)\widetilde{\theta} + \frac{1}{2\overline{\theta}} \left(\overline{\theta}^2 - \widetilde{\theta}^2 \right) + \overline{Y}(1 - \chi \overline{Y}^{\gamma})$$

where $\Upsilon = (1-\beta)B_{-1} + \overline{G}$ *and* B_{-1} denotes total initial debt (both long and short).

Proof. Consider the full date 0 program.

$$\max_{Y_t, C_t, \widetilde{\theta}_t, b_{S,t}, b_{L,t}} \sum_{t \geq 0} \beta^t \left[C_t + \mathcal{I}_{\widetilde{\theta}_t < \overline{\theta}} \left(\int_0^{\widetilde{\theta}_t} \theta \log(\theta) \frac{1}{\overline{\theta}} d\theta + \int_{\widetilde{\theta}_t}^{\overline{\theta}} \theta \log(\widetilde{\theta}_t) \frac{1}{\overline{\theta}} d\theta \right) + \mathcal{I}_{\widetilde{\theta}_t \geq \overline{\theta}} \int_0^{\overline{\theta}} \log(\theta) \frac{1}{\overline{\theta}} d\theta - \chi \frac{Y_t^{1+\gamma}}{1+\gamma} \right]$$

subject to

$$\sum_{j \geq 0} \beta^j \left[\left(\int_{\widetilde{\theta}_{t+j}}^{\overline{\theta}} \theta dF_\theta + (\frac{\widetilde{\theta}_{t+j}}{\overline{\theta}} - 1) b_{S,t+j} \right) \mathcal{I}_{\widetilde{\theta}_{t+j} < \overline{\theta}} \beta + Y_{t+j}(1 - \chi Y_{t+j}^{1+\gamma}) - \overline{G} \right]$$

$$= b_{t-1,S} + b_{t-1,L} E_t \sum_{j \geq 0} \delta^j \beta^j \qquad t = 0, 1, 2, ... \tag{C.38}$$

$$b_{S,t} = \widetilde{\theta}_t \tag{C.39}$$

$$C_t + \overline{G} + \mathcal{I}_{\widetilde{\theta}_t < \overline{\theta}} \left(\int_0^{\widetilde{\theta}_t} \theta \frac{1}{\overline{\theta}} d\theta + \widetilde{\theta}_t \int_{\widetilde{\theta}_t}^{\overline{\theta}} \frac{1}{\overline{\theta}} d\theta \right) + \mathcal{I}_{\widetilde{\theta}_t \geq \overline{\theta}} \left(\int_0^{\overline{\theta}} \theta \frac{1}{\overline{\theta}} d\theta \right) = Y_t \tag{C.40}$$

First, note that the sequence of constraints (C.38) can be dropped from the maximization problem except at $t = 0$. For a given optimal allocation $\widetilde{\theta}_t, C_t, Y_t, b_{S,t}$ the planner can adjust $b_{L,t}$ to ensure satisfaction of the date $t > 0$ constraints. Moreover, using (C.39) together with the resource constraint (C.40), it is straightforward to see that we can eliminate the sequences $b_{S,t}, C_t$ and instead solve a reduced program in which the planner chooses only $\widetilde{\theta}_t, Y_t$.

Note further that whenever $\widetilde{\theta}_t \geq \overline{\theta}$ the optimal quantity $\widetilde{\theta}_t$ is not defined in the resulting program. Hence, we can let the planner choose $\widetilde{\theta}_t$ subject to the inequality constraint $\widetilde{\theta}_t \leq \overline{\theta}$. At this corner, no safety or liquidity services are provided by short-term debt, and the relevant integrals remain well-defined. We have:

$$\left(\int_0^{\widetilde{\theta}_t} \theta \log(\theta) \frac{1}{\overline{\theta}} d\theta + \int_{\widetilde{\theta}_t}^{\overline{\theta}} \theta \log(\widetilde{\theta}_t) \frac{1}{\overline{\theta}} d\theta \right) = \frac{\overline{\theta}}{2} \log(\widetilde{\theta}_t) - \frac{\widetilde{\theta}_t^2}{4\overline{\theta}}$$

$$\left(\int_0^{\widetilde{\theta}_t} \theta \frac{1}{\overline{\theta}} d\theta + \widetilde{\theta}_t \int_{\widetilde{\theta}_t}^{\overline{\theta}} \frac{1}{\overline{\theta}} d\theta \right) = \frac{1}{\overline{\theta}} \left(\frac{1}{2} \widetilde{\theta}_t^2 + \widetilde{\theta}_t(\overline{\theta} - \widetilde{\theta}_t) \right)$$

Using the above, it is clear that the optimization program can be written as:

$$\max_{Y_t,\widetilde{\theta}_t\leq\overline{\theta}}\sum_t\beta^t\left(Y_t-\frac{1}{\overline{\theta}}\left(\frac{1}{2}\widetilde{\theta}_t^2+\widetilde{\theta}_t(\overline{\theta}-\widetilde{\theta}_t)\right)+\frac{\overline{\theta}}{2}\log(\widetilde{\theta}_t)-\frac{\widetilde{\theta}_t^2}{4\overline{\theta}}-\chi\frac{Y_t^{1+\gamma}}{1+\gamma}\right)$$

subject to

$$\frac{\Upsilon}{1-\beta}\equiv b_{S,-1}+\frac{b_{L,-1}}{1-\beta\delta}+\frac{\overline{G}}{1-\beta}=\sum_t\beta^t\left(\beta(\frac{\widetilde{\theta}_t}{\overline{\theta}}-1)\widetilde{\theta}_t+\frac{1}{2\overline{\theta}}\left(\overline{\theta}^2-\widetilde{\theta}_t^2\right)+Y_t(1-\chi Y_t^{\gamma})\right)$$

It is quite evident that the solution to this optimization problem will define a constant $\widetilde{\theta}_t$ at the optimum. Hence, we can solve the static optimization program as stated in the Lemma. ■

Proof of Proposition 3. Let m denote the multiplier attached to the inequality constraint $\widetilde{\theta}\leq\overline{\theta}$ The first-order conditions for the optimum are the following:

$$1-\chi\overline{Y}^{\gamma}+\widetilde{\psi}_{gov}(1-(1+\gamma)\chi\overline{Y}^{\gamma})=0 \tag{C.41}$$

$$-1+\frac{\widetilde{\theta}}{2\overline{\theta}}+\frac{\overline{\theta}}{2\widetilde{\theta}}+\widetilde{\psi}_{gov}\left(\beta(2\frac{\widetilde{\theta}}{\overline{\theta}}-1)-\frac{\widetilde{\theta}}{\overline{\theta}}\right)-m=0 \tag{C.42}$$

$$m\geq 0\qquad m(\overline{\theta}-\widetilde{\theta})=0\qquad\textbf{Complementary Slackness} \tag{C.43}$$

It is straightforward to show that a solution with $\widetilde{\theta}=\overline{\theta}$ is not optimal unless the economy is at the first-best equilibrium. For an optimal allocation, it must be that $(1-(1+\gamma)\chi\overline{Y}^{\gamma})<0$, otherwise the economy operates on the wrong side of the Laffer curve. Therefore, for $\widetilde{\psi}_{gov}=\frac{1-\chi\overline{Y}^{\gamma}}{(1+\gamma)\chi\overline{Y}^{\gamma}-1}$ (>0 if the tax rate is positive). Evaluating (C.42) at $\widetilde{\theta}=\overline{\theta}$ we get that

$$\widetilde{\psi}_{gov}\Big(\beta-1\Big)=m<0$$

which is inconsistent with the first-order conditions.

Notice that this result generalizes to any f probability distribution function.[10] It is also not dependent on the assumption that $\frac{1+\eta}{\eta}=1$ since assuming monopoly distortions will not modify the first-order condition with respect to $\widetilde{\theta}$. ■

Monotone FONC. Maintaining $\widetilde{\psi}_{gov}>0$, the system of first-order conditions yields a unique solution. To see this, consider (C.42) (off corners). $-1+\frac{\widetilde{\theta}}{2\overline{\theta}}+\frac{\overline{\theta}}{2\widetilde{\theta}}$ defines a continuous, monotonically decreasing function; it is infinite for $\widetilde{\theta}=0$ and zero when $\widetilde{\theta}=\overline{\theta}$. $\widetilde{\psi}_{gov}\left(\beta(2\frac{\widetilde{\theta}}{\overline{\theta}}-1)-\frac{\widetilde{\theta}}{\overline{\theta}}\right)$ is increasing and negative in $(0,\overline{\theta})$ if $\beta>\frac{1}{2}$. Though the sum could hypothetically be zero at more than one point, the second derivative of the FONC is $\frac{\overline{\theta}}{\widetilde{\theta}^3}>0$. By the convexity of this function, it is evident that the optimum is (locally) unique (the function intersects zero once).

Global uniqueness requires that the following condition to hold

$$\mathcal{H}\equiv\frac{1}{2\overline{\theta}}-\frac{\overline{\theta}}{2\widetilde{\theta}^2}+\widetilde{\psi}_{gov}(2\beta-1)\frac{1}{\overline{\theta}}+\frac{d\widetilde{\psi}_{gov}}{d\widetilde{\theta}}\left(\beta(2\frac{\widetilde{\theta}}{\overline{\theta}}-1)-\frac{\widetilde{\theta}}{\overline{\theta}}\right)<0 \tag{C.44}$$

[10] This is trivial to show as for any f the condition $\widetilde{\psi}_{gov}\Big(\beta-1\Big)=m_t$ would continue to hold at the bound $\overline{\theta}$.

The third term captures the indirect effect of $\widetilde{\theta}$ through the constraint. Notice that $\frac{d\widetilde{\psi}_{gov}}{d\widetilde{\theta}} > 0$ since increasing $\widetilde{\theta}$ tightens the constraint. Thus, $\frac{d\widetilde{\psi}_{gov}}{d\widetilde{\theta}}\left(\beta(2\frac{\widetilde{\theta}}{\overline{\theta}}-1)-\frac{\widetilde{\theta}}{\overline{\theta}}\right) < 0$ so that (C.44) indeed holds for all $\widetilde{\theta} \in (0, \overline{\theta})$.

Our next result imposes $\mathcal{H} < 0$.

Corollary 1. Let $\widetilde{\theta}^*(\Upsilon)$ denote the interior optimum when (C.44) is satisfied. Then $\frac{d\widetilde{\theta}^*(\Upsilon)}{d\Upsilon} < 0$.

Proof. By the implicit function theorem, the FONC of output and the constraint we have:

$$\frac{d\widetilde{\psi}_{gov}}{d\overline{Y}} = -\frac{\chi\gamma^2\overline{Y}^{\gamma-1}}{(1-(1+\gamma)\chi\overline{Y}^{\gamma})^2} < 0$$

$$\underbrace{(1-(1+\gamma)\chi\gamma\overline{Y}^{\gamma})}_{<0}\,d\overline{Y} = d\Upsilon - \underbrace{\left[\beta(2\frac{\widetilde{\theta}}{\overline{\theta}}-1)-\frac{\widetilde{\theta}}{\overline{\theta}}\right]}_{<0}d\widetilde{\theta}$$

Therefore,

$$\left(\frac{1}{2\overline{\theta}}-\frac{\overline{\theta}}{2\widetilde{\theta}^2}+\widetilde{\psi}_{gov}(2\beta-1)\frac{1}{\overline{\theta}}\right)d\widetilde{\theta}+\left(\beta(2\frac{\widetilde{\theta}}{\overline{\theta}}-1)-\frac{\widetilde{\theta}}{\overline{\theta}}\right)\frac{d\widetilde{\psi}_{gov}}{dY}\frac{\overbrace{\left[d\Upsilon-\left[\beta(2\frac{\widetilde{\theta}}{\overline{\theta}}-1)-\frac{\widetilde{\theta}}{\overline{\theta}}\right]d\widetilde{\theta}\right]}^{d\overline{Y}}}{(1-(1+\gamma)\chi\gamma\overline{Y}^{\gamma})}=0$$

or

$$\overbrace{\left(\frac{1}{2\overline{\theta}}-\frac{\overline{\theta}}{2\widetilde{\theta}^2}+\widetilde{\psi}_{gov}(2\beta-1)\frac{1}{\overline{\theta}}-\left(\beta(2\frac{\widetilde{\theta}}{\overline{\theta}}-1)-\frac{\widetilde{\theta}}{\overline{\theta}}\right)^2\frac{d\widetilde{\psi}_{gov}}{dY}\frac{1}{(1-(1+\gamma)\chi\gamma\overline{Y}^{\gamma})}\right)}^{\mathcal{H}<0}d\widetilde{\theta}=$$

$$\underbrace{-\frac{d\widetilde{\psi}_{gov}}{d\overline{Y}}\frac{\left(\beta(2\frac{\widetilde{\theta}}{\overline{\theta}}-1)-\frac{\widetilde{\theta}}{\overline{\theta}}\right)}{(1-(1+\gamma)\chi\gamma\overline{Y}^{\gamma})}}_{>0}d\Upsilon$$

■

C.4 Proof of Proposition 4

The first-order condition for inflation with quasilinear preferences is given by:

$$-\omega\overline{\pi}\hat{\pi}_t-(2\overline{\pi}-1)\overline{\psi}_{gov}\frac{\omega}{\eta}\Delta\hat{\psi}_{gov,t}+\frac{\overline{b}_S}{\overline{\pi}^2}\overline{\psi}_{gov}(1-F_{\widetilde{\theta}})(\hat{b}_{S,t-1}-2\hat{\pi}_t)$$

$$+\frac{\overline{b}_S}{\overline{\pi}^2}\overline{\psi}_{gov}(\hat{\psi}_{gov,t}-F_{\widetilde{\theta}}(\hat{\psi}_{gov,t-1}+\frac{f_{\widetilde{\theta}}}{F_{\widetilde{\theta}}}\widehat{\widetilde{\theta}\theta}_{t-1}))+\frac{\overline{b}_L}{\overline{\pi}-\beta\delta}\sum_{j\geq 0}(\frac{\delta}{\overline{\pi}})^j\Delta\hat{\psi}_{gov,t-j}=0 \tag{C.45}$$

where $\overline{b}_S = \widetilde{\theta}$ and $\hat{b}_{S,t} = \hat{\widetilde{\theta}}_t$. Moreover, $\overline{\pi} > 1$ follows readily from the steady-state relation

$$\omega(\overline{\pi}-1) = \frac{\overline{b}_S}{\overline{\pi}^2}\overline{\psi}_{gov}(1-F_{\overline{b}_S})$$

when $\overline{\psi}_{gov} > 0$ and the financial friction is binding. Using the steady-state relation, we can substitute $\frac{\overline{b}_S}{\overline{\pi}^2}\overline{\psi}_{gov}$ with $\frac{\omega(\overline{\pi}-1)}{(1-F_{\widetilde{\theta}})}$. With this we can write (C.45) as:

$$\omega(3\overline{\pi}-2)\hat{\pi}_t = (2\overline{\pi}-1)\overline{\psi}_{gov}\frac{\omega}{\eta}\Delta\hat{\psi}_{gov,t} + \frac{\omega(\overline{\pi}-1)}{(1-F_{\overline{\widetilde{\theta}}})}\Delta\hat{\psi}_{gov,t}$$

$$+\omega(\overline{\pi}-1)[1-\frac{f_{\overline{\widetilde{\theta}}}\overline{\widetilde{\theta}}}{(1-F_{\overline{\widetilde{\theta}}})}]\hat{b}_{S,t-1} + \omega(\overline{\pi}-1)\hat{\psi}_{gov,t-1} + \frac{\overline{b}_L}{\overline{\pi}-\beta\delta}\sum_{j\geq 0}(\frac{\delta}{\overline{\pi}})^j\Delta\hat{\psi}_{gov,t-j}$$

or

$$\hat{\pi}_t = \frac{(2\overline{\pi}-1)}{(3\overline{\pi}-2)}\frac{\overline{\psi}_{gov}}{\eta}\Delta\hat{\psi}_{gov,t} + \frac{(\overline{\pi}-1)}{(3\overline{\pi}-2)}\frac{1}{(1-F_{\overline{\widetilde{\theta}}})}\Delta\hat{\psi}_{gov,t} \tag{C.46}$$

$$+\underbrace{\frac{(\overline{\pi}-1)}{(3\overline{\pi}-2)}\left[1-\frac{f_{\overline{\widetilde{\theta}}}\overline{\widetilde{\theta}}}{(1-F_{\overline{\widetilde{\theta}}})}\right]}_{\equiv\gamma_1}\hat{b}_{S,t-1} + \underbrace{\frac{(\overline{\pi}-1)}{(3\overline{\pi}-2)}}_{\equiv\gamma_2}\hat{\psi}_{gov,t-1} + \underbrace{\frac{\overline{b}_L}{\omega(\overline{\pi}-\beta\delta)(3\overline{\pi}-2)}}_{\equiv\gamma_3}\sum_{j\geq 0}(\frac{\delta}{\overline{\pi}})^j\Delta\hat{\psi}_{gov,t-j}$$

Using the random walk property of the Lagrange multiplier it is straightforward to show that:

$$E_t\hat{\pi}_{t+1} = \gamma_1\hat{b}_{S,t} + \gamma_2\hat{\psi}_{gov,t} + \gamma_3\delta\sum_{j\geq 0}(\frac{\delta}{\overline{\pi}})^j\Delta\hat{\psi}_{gov,t-j}$$

Next, we use the first-order condition for short term debt.

$$-\int_{\widetilde{\theta}}^{\infty}\theta dF_\theta\frac{1}{\widetilde{\theta}\overline{\psi}_{gov}}\hat{b}_{S,t} + \frac{\beta}{\overline{\pi}}\left(\hat{\psi}_{gov,t}F_{\overline{\widetilde{\theta}}} + f_{\widetilde{\theta}}\hat{\widetilde{\theta}}\widetilde{\theta}_t - E_t\hat{\psi}_{gov,t+1}\right)$$

$$+\widetilde{\theta}f_{\widetilde{\theta}}\left(\frac{\beta}{\overline{\pi}}-1\right)\left(\hat{\psi}_{gov,t}+\hat{b}_{S,t}+\frac{f'_{\widetilde{\theta}}}{f_{\widetilde{\theta}}}\hat{\widetilde{\theta}}\widetilde{\theta}_t\right) - \beta\frac{1}{\overline{\pi}}\left[\widetilde{\theta}f_{\widetilde{\theta}} - (1-F_{\widetilde{\theta}})\right]E_t\hat{\pi}_{t+1} = 0$$

Substituting expected inflation we get:

$$-\int_{\widetilde{\theta}}^{\infty}\theta dF_\theta\frac{1}{\widetilde{\theta}\overline{\psi}_{gov}}\hat{b}_{S,t} + \frac{\beta}{\overline{\pi}}\left(\hat{\psi}_{gov,t}F_{\overline{\widetilde{\theta}}} + f_{\widetilde{\theta}}\hat{\widetilde{\theta}}\widetilde{\theta}_t - E_t\hat{\psi}_{gov,t+1}\right)$$

$$+\widetilde{\theta}f_{\widetilde{\theta}}\left(\frac{\beta}{\overline{\pi}}-1\right)\left(\hat{\psi}_{gov,t}+\hat{b}_{S,t}+\frac{f'_{\widetilde{\theta}}}{f_{\widetilde{\theta}}}\hat{\widetilde{\theta}}\widetilde{\theta}_t\right) - \beta\frac{1}{\overline{\pi}}\left[\widetilde{\theta}f_{\widetilde{\theta}} - (1-F_{\widetilde{\theta}})\right]\left(\gamma_1\hat{b}_{S,t}+\gamma_2\hat{\psi}_{gov,t}+\gamma_3\delta\sum_{j\geq 0}(\frac{\delta}{\overline{\pi}})^j\Delta\hat{\psi}_{gov,t-j}\right) = 0$$

Finally, assuming a uniform distribution (wlog) using the random walk property, gathering terms and rearranging we get:

$$\left[-\int_{\widetilde{\theta}}^{\overline{\theta}}\frac{\theta}{\overline{\theta}}d\theta\frac{1}{\widetilde{\theta}\overline{\psi}_{gov}} + \frac{\widetilde{\theta}}{\overline{\theta}}\left(2\frac{\beta}{\overline{\pi}}-1\right) - \frac{\beta}{\overline{\pi}}\left(2\frac{\widetilde{\theta}}{\overline{\theta}}-1\right)\gamma_1\right]\hat{b}_{S,t} + \left[\frac{\beta}{\overline{\pi}}\left(2\frac{\widetilde{\theta}}{\overline{\theta}}-1\right)(1-\gamma_2) - \frac{\widetilde{\theta}}{\overline{\theta}}\right]\hat{\psi}_{gov,t}$$

$$-\frac{\beta}{\overline{\pi}}\left(2\frac{\widetilde{\theta}}{\overline{\theta}}-1\right)\gamma_3\delta\sum_{j\geq 0}(\frac{\delta}{\overline{\pi}})^j\Delta\hat{\psi}_{gov,t-j} = 0$$

We can write this relation as

$$\hat{b}_{S,t} = \gamma_4 \hat{\psi}_{gov,t} + \gamma_5 \sum_{j\geq 0} (\frac{\delta}{\overline{\pi}})^j \Delta\hat{\psi}_{gov,t-j})$$

for appropriately defined coefficients γ_4, γ_5. Making use of this expression in (C.46) we get:

$$\hat{\pi}_t = \gamma_6 \Delta\hat{\psi}_{gov,t} + \underbrace{(\gamma_1\gamma_4 + \gamma_2)\hat{\psi}_{gov,t-1}}_{\textbf{Time Varying Inflation Target}} + \underbrace{(\gamma_1\gamma_5 + \gamma_3)\sum_{j\geq 0}(\frac{\delta}{\overline{\pi}})^j \Delta\hat{\psi}_{gov,t-j}}_{\textbf{Real Value of Long Debt Channel}} \tag{C.47}$$

where γ_6 collects all $\Delta\hat{\psi}_{gov,t}$ terms in (C.47). ■

As discussed in the main text, the second term on the RHS of (C.47) represents a time-varying inflation target. Since $\psi_{gov,t}$ follows a random walk we have $\Delta\psi_{gov,t} = \nu\epsilon_{G,t}$ where ν is a constant pinned down by the intertemporal budget constraint. Hence $\psi_{gov,t-1} = \nu\sum_j \epsilon_{G,t-j-1}$.[11] Whenever a spending shock occurs, the optimal inflation target shifts permanently. This permanent change arises only when the financial friction is binding.

C.5 Proof of Proposition 5

We now use the quasi-linear version of the model to derive analytically the impulse response of taxes and the debt portfolio to a spending shock. This section contains the proof of Proposition 5 for which we utilize a local approximation focusing on an equilibrium with financial frictions.

In this model, the steady-state multiplier is given by

$$\overline{\psi}_{gov} = \frac{1 - \chi\overline{Y}^{\gamma}}{(1+\gamma)\chi\overline{Y}^{\gamma} - \frac{1+\eta}{\eta}} > 0$$

Approximating the first-order condition for output around the steady state yields

$$-\chi\overline{Y}^{\gamma}\gamma\hat{Y}_t + \overline{\psi}_{gov}\hat{\psi}_{gov,t}(\frac{1+\eta}{\eta} - (1+\gamma)\chi\overline{Y}^{\gamma}) - \overline{\psi}_{gov}(1+\gamma)\chi\overline{Y}^{\gamma}\gamma\hat{Y}_t = 0$$

which implies

$$-\chi\overline{Y}^{\gamma}\gamma(1 + \overline{\psi}_{gov}(1+\gamma))\hat{Y}_t - (1 - \chi\overline{Y}^{\gamma})\hat{\psi}_{gov,t} = 0$$

rearranging this equation we get

$$\hat{Y}_t = -\frac{(1 - \chi\overline{Y}^{\gamma})}{\chi\overline{Y}^{\gamma}\gamma(1 + \overline{\psi}_{gov}(1+\gamma))}\hat{\psi}_{gov,t} \equiv \widetilde{h}\hat{\psi}_{gov,t} \tag{C.48}$$

where $\widetilde{h} < 0$ so that a tighter debt constraint leads to a drop in output (higher taxes) in equilibrium.

The log linear first-order conditions for short and long term debt give us the following relations:

$$\left[-\frac{\int_{\widetilde{\theta}}^{\infty}\theta dF_\theta}{\overline{b}_S\beta\overline{\psi}_{gov}} + \overline{b}_S f_{\widetilde{\theta}}\left(1 - \frac{1}{\beta}\right)(1 + \frac{f'_{\widetilde{\theta}}}{f_{\widetilde{\theta}}}\widetilde{\theta}) + f_{\widetilde{\theta}}\widetilde{\theta}\right]\hat{b}_{S,t} + \left[F_{\widetilde{\theta}} + \overline{b}_S f_{\widetilde{\theta}}\left(1 - \frac{1}{\beta}\right)\right]\hat{\psi}_{gov,t} = E_t\hat{\psi}_{gov,t+1} \tag{C.49}$$

$$E_t\hat{\psi}_{gov,t+1} = \hat{\psi}_{gov,t} \tag{C.50}$$

[11] See the proof of Proposition 5 for an analytical solution of ν in a model similar to this one.

Combining these equations, imposing a uniform distribution and using $\overline{b}_S = \widetilde{\theta}$ we get

$$\upsilon_1 \equiv \frac{\left[-\frac{(\frac{\overline{\theta}}{2}-\frac{\widetilde{\theta}^2}{2\overline{\theta}})}{\widetilde{\theta}\beta\overline{\psi}_{gov}} + \frac{\widetilde{\theta}}{\overline{\theta}}\left(2-\frac{1}{\beta}\right)\right]}{1-\frac{\widetilde{\theta}}{\overline{\theta}}\left(2-\frac{1}{\beta}\right)}\hat{b}_{S,t} = \hat{\psi}_{gov,t} \tag{C.51}$$

We continue considering that the program is well behaved in the sense that the Lagrangian is concave in $\widetilde{\theta}, \overline{Y}$. We then have $\upsilon_1 < 0$.[12]

Focusing on one shock to government spending in date t, assuming that initially the public debt portfolio is at steady state we can write the intertemporal budget constraint of the government as:

$$E_t \sum_{j\geq 0} \beta^j \left(\widetilde{\theta}\left[(2\beta-1)\frac{\widetilde{\theta}}{\overline{\theta}} - 1\right]\hat{b}_{S,t+j} - \overline{G}\hat{G}_{t+j} + \underbrace{\overline{Y}(\frac{1+\eta}{\eta} - (1+\gamma)\chi\overline{Y}^{\gamma})}_{<0}\hat{Y}_{t+j} \right) = 0 \tag{C.54}$$

which using the above derivations can be written as:

$$E_t \sum_{j\geq 0} \beta^j \left(\frac{\widetilde{\theta}}{\upsilon_1}\left[(2\beta-1)\frac{\widetilde{\theta}}{\overline{\theta}} - 1\right]\hat{\psi}_{gov,t+j} - \overline{G}\hat{G}_{t+j} + \overline{Y}(\frac{1+\eta}{\eta} - (1+\gamma)\chi\overline{Y}^{\gamma})\widetilde{h}\hat{\psi}_{gov,t+j} \right) = 0 \tag{C.55}$$

This boils down to the following condition:

$$\hat{\psi}_{gov,t} = \frac{\overline{G}}{\upsilon_2}\frac{(1-\beta)}{1-\beta\rho_G}\hat{G}_t \tag{C.56}$$

where

$$\upsilon_2 = \left(\frac{\widetilde{\theta}}{\upsilon_1}\left[(2\beta-1)\frac{\widetilde{\theta}}{\overline{\theta}} - 1\right] + \overline{Y}(\frac{1+\eta}{\eta} - (1+\gamma)\chi\overline{Y}^{\gamma})\widetilde{h} \right) > 0$$

[12] The denominator of the fraction is clearly positive since $\beta < 1$. Then, for the numerator we have

$$\text{sign}\left[-\frac{(\frac{\overline{\theta}}{2}-\frac{\widetilde{\theta}^2}{2\overline{\theta}})}{\widetilde{\theta}\beta\overline{\psi}_{gov}} + \frac{\widetilde{\theta}}{\overline{\theta}}\left(2-\frac{1}{\beta}\right)\right] = \text{sign}\left[(\frac{1}{2\overline{\theta}} - \frac{\overline{\theta}}{2\widetilde{\theta}^2}) + \overline{\psi}_{gov}\frac{1}{\overline{\theta}}\left(2\beta-1\right)\right] = \text{sign}(\mathcal{L}_{\widetilde{\theta},\widetilde{\theta}}) < 0 \tag{C.52}$$

Notice that this condition is sufficient, not necessary, for a strict maximum. Specifically, the bordered Hessian :is given by

$$H_B = \begin{pmatrix} 0 & 1-(1+\gamma)\chi\overline{Y}^{\gamma} & (2\beta-1)\frac{\widetilde{\theta}}{\overline{\theta}} - \beta \\ 1-(1+\gamma)\chi\overline{Y}^{\gamma} & -\chi\gamma\overline{Y}^{\gamma-1} - \psi^{gov}(1+\gamma)\gamma\chi\overline{Y}^{\gamma-1} & 0 \\ (2\beta-1)\frac{\widetilde{\theta}}{\overline{\theta}} - \beta & 0 & \frac{1}{2\overline{\theta}} - \frac{\overline{\theta}}{2\widetilde{\theta}^2} + \overline{\psi}_{gov}\frac{2\beta-1}{\overline{\theta}} \end{pmatrix}.$$

and for a strict maximum we must have $|H_B| > 0$ or

$$-(1-(1+\gamma)\chi\overline{Y}^{\gamma})^2\underbrace{(\frac{1}{2\overline{\theta}} - \frac{\overline{\theta}}{2\widetilde{\theta}^2} + \overline{\psi}_{gov}\frac{2\beta-1}{\overline{\theta}})}_{\mathcal{L}_{\widetilde{\theta},\widetilde{\theta}}} + ((2\beta-1)\frac{\widetilde{\theta}}{\overline{\theta}} - \beta)^2\underbrace{\chi\gamma\overline{Y}^{\gamma-1}(1+\psi^{gov}(1+\gamma))}_{-\mathcal{L}_{\overline{Y},\overline{Y}}} > 0 \tag{C.53}$$

Since the second term is positive we may have a strict maximum even with $(\mathcal{L}_{\widetilde{\theta},\widetilde{\theta}}) > 0$. This does not occur in the numerical solution of the problem for plausible parameter values.

We can now construct the sequence of long term debt. Using the intertemporal budget it is simple to show that

(C.57)
$$\frac{\overline{b}_L}{1-\beta\delta}\hat{b}_{L,t+\bar{t}-1} = (\frac{\upsilon_2}{1-\beta} - \frac{\widetilde{\theta}}{\upsilon_1})\hat{\psi}_{gov,t} - \frac{\overline{G}}{1-\beta\rho_G}\rho_G^{\bar{t}}\hat{G}_t = \left(1 - \frac{\widetilde{\theta}}{\upsilon_1}\frac{1-\beta}{\upsilon_2} - \rho_G^{\bar{t}}\right)\frac{\overline{G}}{1-\beta\rho_G}\hat{G}_t \qquad \bar{t} \geq 1$$

and so $\hat{b}_{L,t+\bar{t}-1} > 0$ independently of the shock persistence ρ_G.

Finally, using $\tau_t = 1 - \frac{\eta}{1+\eta}\chi Y_t^{\gamma}$ we obtain $\hat{\tau}_t = -\frac{\eta}{1+\eta}\chi\gamma\overline{Y}^{\gamma}\hat{Y}_t = -\frac{\eta}{1+\eta}\chi\gamma\overline{Y}^{\gamma}\widetilde{h}\hat{\psi}_{gov,t}$. Therefore, taxes permanently increase following a positive spending shock. ■

D Computational Appendix

This computational appendix is organized into four parts. Section D.1 develops the saddle–point functional equation (SPFE) representation of the Ramsey problem and shows how the infinite–dimensional state vector induced by long–term debt can be reduced to a tractable recursive state vector. Section D.2 explains how forward–looking conditional expectations are approximated using polynomial functions within the Parameterized Expectations Algorithm (PEA), and the construction of the approximation of the welfare function. Section D.3 addresses the numerical challenges arising from portfolio indeterminacy, discussing alternative resolution methods briefly mentioned in text. Finally, Section D.4 reports numerical results from the lending version of the model.

D.1 Saddle point functional equation

We begin by applying the arguments of Marcet and Marimon (2019) the saddle-point functional equation (SPFE) of our model. As is well known, that paper provides a recursive formulation of dynamic optimization problems with forward-looking (implementability) constraints by augmenting the state space with appropriate co-state variables. Revisiting these arguments is useful in our context because our numerical algorithm partially relies on value function approximations to characterize globally optimal policy rules.

As we discussed in text, in our model, with long-term debt featuring coupon payments that extend over an infinite horizon, the state vector is in principle infinite dimensional. The planner's commitments to inflation and consumption extend indefinitely into the future, which implies that an infinite number of lags in the quantity of bonds, inflation and associated multipliers would need to be tracked as state variables. However, a much lower-dimensional representation is sufficient. The following recursive state variable fully summarizes the planner's policy promises relevant for the pricing of long-term debt:

$$\Psi_t = \frac{b_{L,t-1}\Delta\psi_{gov,t}}{\pi_t} + \frac{\delta}{\pi_t}\Psi_{t-1} \tag{D.58}$$

With this definition we derive the **SPFE** as:

$$\begin{aligned} W(b_{S,-1}, b_{L,-1}, \widetilde{\theta}_{-1}, \psi_{gov,-1}, \psi_{PC,-1}, \Psi_{-1}, G) = \inf_{\nu_{gov},\nu_{PC}} \sup_{C,Y,\pi,b_L,b_S,\widetilde{\theta},\tau} \widetilde{U}(b_S, \widetilde{\theta}, Y, C) \\ +\psi_{gov,-1}\left(\frac{b_{S,-1}}{\beta}\int_{\widetilde{\theta}_{-1}}^{\overline{\theta}} \theta v'(b_{S,-1})dF_\theta + F_{\widetilde{\theta}_{-1}}\frac{u'(C)}{\pi}\right) \\ -\nu_{gov}b_{S,-1}\frac{u'(C)}{\pi} - \psi_{PC,-1}(\pi(\pi-1)u'(C) - f(Y;\tau;C)) + \nu_{PC}\pi(\pi-1)u'(C) \\ -\Psi_{-1}\delta\frac{u'(C)}{\pi} + \frac{u'(C)}{\pi}\psi_{gov,-1}b_{L,-1} - \nu_{gov}b_{L,-1}\frac{u'(C)}{\pi} + \beta EW(b_S, b_L, \widetilde{\theta}, \psi_{gov}, \psi_{PC}, \Psi, G') \end{aligned}$$

subject to:

$$\psi_{gov} = \nu_{gov} \qquad \psi_{PC} = \nu_{PC} \qquad \widetilde{\theta} = \frac{C}{b_S}$$

$$\Psi = \frac{b_{L,-1}(\nu_{gov} - \psi_{gov,-1})}{\pi} + \frac{\delta}{\pi}\Psi_{-1}$$

$$C\left(1 + \mathbf{1}_{\widetilde{\theta}<\overline{\theta}}\int_0^{\widetilde{\theta}} \theta dF_\theta + \mathbf{1}_{\widetilde{\theta}\geq\overline{\theta}}\int_0^{\overline{\theta}} \theta dF_\theta\right) + \int_{\widetilde{\theta}}^{\overline{\theta}} b_S dF_\theta \mathbf{1}_{\widetilde{\theta}<\overline{\theta}} + G + \frac{\omega}{2}(\pi-1)^2 = Y$$

where $v' = \frac{1}{b_S}$, $u'(C) = \frac{1}{C}$, and we define

$$\widetilde{U}(b_S, \widetilde{\theta}, Y, C) = \left(1 + \mathbf{1}_{\widetilde{\theta}<\overline{\theta}} \int_0^{\widetilde{\theta}} \theta dF_\theta + \mathbf{1}_{\widetilde{\theta}\geq\overline{\theta}} \int_0^{\overline{\theta}} \theta dF_\theta\right) \log(C_t) + \mathbf{1}_{\widetilde{\theta}<\overline{\theta}} \left(\int_0^{\widetilde{\theta}} \theta \log\theta dF_\theta + \int_{\widetilde{\theta}}^{\overline{\theta}} \theta \log b_S dF_\theta\right)$$
$$+\mathbf{1}_{\widetilde{\theta}\geq\overline{\theta}} \left(\int_0^{\overline{\theta}} \theta \log\theta dF_\theta\right) - \chi \frac{Y^{1+\gamma}}{1+\gamma}$$
$$f(Y; \tau; C)) = \frac{\eta}{\omega}\left(\frac{1+\eta}{\eta} u'(C) - \frac{1}{1-\tau}\chi Y^\gamma\right) Y$$

Our formulation of the SPFE allows for the possibility that the distribution function F has finite support. However, solving for the Ramsey policy using first-order conditions requires the problem to be differentiable. For this reason, we assume in what follows that $\overline{\theta} = \infty$. Under differentiability of the value function W, it can be shown that the optimality conditions characterizing the SPFE coincide with the first-order conditions obtained from the associated Lagrangian. For brevity, we omit these derivations.

D.2 Parameterizing Expectations

In the main text we explained that the dimensionality of the state vector can be further reduced by excluding the lagged Phillips curve multiplier. Therefore,

$$X_t \equiv \left(b_{S,t-1}, b_{L,t-1}, \widetilde{\theta}_{t-1}, \psi_{gov,t-1}, \Psi_{t-1}, G_t\right),$$

Our approximations of the conditional expectations in the model are the following:

$$\overbrace{E_t\big(u'(C_{t+1})\pi_{t+1}(\pi_{t+1}-1)\big) \approx \Phi_{(PC)}(X_t, \kappa_{(PC)})}^{\text{Phillips curve expectation term}} \qquad \overbrace{E_t\left(\frac{u'(C_{t+1})}{\pi_{t+1}}\right) \approx \Phi_{(q,S)}(X_t, \kappa_{(q,S)})}^{\text{Short bond price expectation term}}$$

$$\overbrace{E_t\left(\frac{u'(C_{t+1})}{\pi_{t+1}}\psi_{gov,t+1}\right) \approx \Phi_{(\psi,S)}(X_t, \kappa_{(\psi,S)})}^{\text{Short bond FONC expectation term}} \qquad \overbrace{E_t\left(\sum_{j\geq 1}\beta^j\delta^{j-1}\frac{u'(C_{t+j})}{\prod_{k=1}^j \pi_{t+k}}\right) \approx \Phi_{(q,L)}(X_t, \kappa_{(q,L)})}^{\text{Long bond price expectation term}}$$

$$\overbrace{E_t\left(\psi_{gov,t+1}\sum_{j\geq 1}\beta^j\delta^{j-1}\frac{u'(C_{t+j})}{\prod_{k=1}^j \pi_{t+k}}\right) \approx \Phi_{(\psi,L)}(X_t, \kappa_{(\psi,L)})}^{\text{Long bond FONC expectation term}}$$

where $\Phi_{(.)}$ belong to a class of functions that can approximate the conditional expectations arbitrarily well. We will take $\Phi_{(.)}$ to be polynomials of a given order and so $\kappa_{(.)}$ are coefficients on the variables in X as well as their squares, cubes, cross-products depending on the order of the approximation being used.

Given this notation, we can write the full system of equations that we need to solve as:

$$\psi_{\widetilde{\theta},t}C_t + \psi_{gov,t}b_{S,t}\left(\beta f_{\widetilde{\theta}_t}\Phi_{(q,S)}(X_t,\kappa_{(q,S)}) - \widetilde{\theta}_t f_{\widetilde{\theta}_t}\frac{1}{b_{S,t}}\right) = 0$$

$$\int_{\widetilde{\theta}_t}^{\infty}\frac{1}{b_{S,t}}\theta dF_\theta + \psi_{RC,t}\left(1-F_{\widetilde{\theta}_t}\right) - \psi_{\widetilde{\theta},t} + \psi_{gov,t}\beta F_{\widetilde{\theta}_t}\Phi_{(q,S)}(X_t,\kappa_{(q,S)}) - \beta\Phi_{(\psi,S)}(X_t,\kappa_{(\psi,S)}) = 0$$

$$-\chi Y_t^\gamma - \psi_{RC,t} + \psi_{PC,t}\left(\frac{1+\gamma}{1-\tau_t}\frac{\eta}{\omega}\chi Y_t^\gamma - \frac{1+\eta}{\omega}u'(C_t)\right) + \psi_{gov,t}\frac{\tau_t}{1-\tau_t}(1+\gamma)\chi Y_t^\gamma = 0$$

$$\psi_{PC,t}\frac{\eta}{\omega} + \psi_{gov,t} = 0$$

$$\left(\frac{1}{C_t} + \psi_{RC,t}\right)\left(1+\int_0^{\widetilde{\theta}_t}\theta dF_\theta\right) + u''(C_t)\pi_t(\pi_t-1)\Delta\psi_{PC,t} - \psi_{PC,t}\frac{1+\eta}{\omega}u''(C_t)Y_t + \psi_{\widetilde{\theta},t}\widetilde{\theta}_t$$

$$-\psi_{gov,t}u''(C_t)G_t - \frac{b_{t-1,S}}{\pi_t}u''(C_t)\left(\psi_{gov,t} - F_{\widetilde{\theta}_{t-1}}\psi_{gov,t-1}\right) - u''(C_t)\Psi_t = 0$$

$$u'(C_t)(2\pi_t-1)\Delta\psi_{PC,t} + \psi_{gov,t}\frac{1}{\pi_t^2}u'(C_t) - \psi_{gov,t-1}u'(C_t)\frac{1}{\pi_t^2}F_{\widetilde{\theta}_{t-1}} + (u'(C_t)+\delta q_{L,t}u'(C_t))\frac{1}{\pi_t}\Psi_t = 0$$

$$\psi_{gov,t}\Phi_{(q,L)}(X_t,\kappa_{(q,L)}) = \Phi_{(\psi,L)}(X_t,\kappa_{(\psi,L)})$$

together with the Phillips curve and government budget constraints,

$$\pi_t(\pi_t-1) = \frac{\eta}{\omega}\left(\frac{1+\eta}{\eta} - \chi\frac{Y_t^\gamma}{U'(C_t)}\right)Y_t + \beta\Phi_{(PC)}(X_t,\kappa_{(PC)})$$

$$b_{S,t}\left[\int_{\widetilde{\theta}_t}^{\infty}\theta v'(b_{S,t}^i)dF_\theta + \beta F(\widetilde{\theta}_t)\Phi_{(q,S)}(X_t,\kappa_{(q,S)})\right] + b_{L,t}\Phi_{(q,L)}(X_t,\kappa_{(q,L)})$$

$$= \frac{b_{t-1,S}}{\pi_t}u'(C_t) + \frac{b_{L,t-1}}{\pi_t}(u'(C_t) + \delta\Phi_{(q,L)}(X_t,\kappa_{(q,L)})) + G_t u'(C_t) - \frac{\tau_t}{1-\tau_t}\chi Y_t^{1+\gamma}$$

$$b_{S,t} = C_t\widetilde{\theta}_t$$

and the resource constraint (C.36).

The PEA algorithm initiates functions $\Phi_{(.)}(X_t,\kappa_{(.)}^0)$ with a guess for the values of the coefficients $\kappa_{(.)}$. The model is solved for a large number of periods and the simulated output is utilized to update the coefficients obtaining new approximations $\Phi_{(.)}(X_t,\kappa_{(.)}^1)$. If $\Phi_{(.)}(X_t,\kappa_{(.)}^0) \approx \Phi_{(.)}(X_t,\kappa_{(.)}^1)$ then the algorithm stops. Otherwise, we iterate using the updated polynomials as the initial guess until a fixed point of the coefficients is found.

Debt limits. As we discussed in text, we add to the model ad hoc debt constraints assuming (as in Faraglia, Marcet, Oikonomou, and Scott, 2013, 2019, 2016; Maliar and Maliar, 2003; Judd, Maliar, and Maliar, 2011) that the issuance of short- and long-term bonds is subject to

$$b_{x,t} \in [\underline{M}_x, \overline{M}_x], \qquad x \in S, L, \tag{D.59}$$

We account for these debt limits using the standard Kuhn–Tucker first-order conditions. Let $\underline{\xi}_{t,L}$ and $\overline{\xi}_{t,L}$ denote the multipliers associated with the lower and upper bounds, respectively. Accounting for these constraints, the first-order condition for $b_{L,t}$ can be written as

$$\psi_{gov,t}E_t\sum_{j\geq 1}\beta^j\delta^{j-1}\frac{u'(C_{t+j})}{\prod_{k=1}^j\pi_{t+k}} - E_t\psi_{gov,t+1}\sum_{j\geq 1}\beta^j\delta^{j-1}\frac{u'(C_{t+j})}{\prod_{k=1}^j\pi_{t+k}} - \overline{\xi}_{t,L} + \underline{\xi}_{t,L} = 0 \tag{D.60}$$

where $\overline{\xi}_{t,L}, \underline{\xi}_{t,L} \geq 0$ and the complementary slackness conditions $\underline{\xi}_{t,L}(b_{L,t} - \underline{M}_L) = 0$, $\overline{\xi}_{t,L}(\overline{M}_L - b_{L,t}) = 0$ hold.

Moreover, since short-term debt issuance is not subject to a lower bound constraint (as it would never bind in our model), the optimality condition for short-term debt can be written as

(D.61)
$$\int_{\widetilde{\theta}_t}^{\infty} \frac{1}{b_{S,t}} \theta \, dF_\theta + \psi_{RC,t}\big(1 - F(\widetilde{\theta}_t)\big) - \psi_{\widetilde{\theta},t} + \psi_{gov,t}\beta F(\widetilde{\theta}_t) E_t \frac{u'(C_{t+1})}{\pi_{t+1}} - \beta E_t \psi_{gov,t+1} \frac{u'(C_{t+1})}{\pi_{t+1}} - \overline{\xi}_{t,S} = 0$$

where $\overline{\xi}_{t,S} \geq 0$ and $\overline{\xi}_{t,S}(\overline{M}_S - b_{S,t}) = 0$ hold.

We solve the model with (D.60) and (D.61) as follows:

1. **Interior optimum.** In period t, set $\overline{\xi}_{t,S} = \overline{\xi}_{t,L} = \underline{\xi}_{t,L} = 0$. Solve the system of equations to recover $b_{S,t}$ from (D.61) and determine $b_{L,t}$ from the per-period budget constraint. The multiplier $\psi_{gov,t}$ is determined from (D.60).

   Check whether the resulting value of $b_{L,t}$ satisfies the bounds on long-term debt. If a bound is violated, set $b_{L,t}$ equal to the relevant bound and solve the remaining first-order conditions, excluding (D.60), to determine the remaining model variables.

2. **Corner solution.** Fix $b_{S,t} = \overline{M}_S$. Use the first-order conditions, excluding (D.61), to solve for the remaining model variables.

3. **Selection.** Evaluate the welfare objective using the solution vectors obtained in steps 1 and 2, and select the global optimum.

SPFE approximation. We complement the PEA algorithm with an approximation of $W(X)$. As shown in text, our approximation of the welfare objective function is:

$$W(X_t) \equiv E\left(\sum_{j \geq 0} \beta^j \widetilde{U}_{t+j} | X_t\right) \approx \Phi_W(X_t, \kappa_W)$$

where Φ_W is a polynomial of the state variables, κ_W denote the coefficients attached to the terms in Φ_W.

Given this approximation, our approach for finding the optimal policy in a generic period t is the following: First, we find the critical points as described in the previous paragraph. Second, we evaluate the welfare function at each of the points. The welfare associated with a generic critical point (denoted using the asterisk here) is

$$\widetilde{U}_t^* + \beta E_t W(X_{t+1}^*) = \widetilde{U}_t^* + \beta E_t \Phi_W(X_{t+1}^*, \kappa_W)$$

where $X_{t+1}^* = \left(b_{S,t}^*, b_{L,t}^*, \widetilde{\theta}_t^*, \psi_{gov,t}^*, \Psi_t^*, G_{t+1}\right)$.[13] Evaluating the welfare objective function at each of the critical points, we verify the global optimum.

Finally, the coefficients κ_W are obtained applying the usual iteration procedure: Assuming an initial guess, solving the model, simulating the welfare objective function over a large number of periods to update the coefficients and iterating until they have converged.

Polynomial specification. We approximate the functions $\Phi_{(.)}$ with orthogonal (Chebyshev) polynomials.[14] We specify the polynomial order as follows. For the state variables G_t, $\psi_{gov,t-1}$,

[13] The conditional expectation $E_t \Phi_W(X_{t+1}^*, \kappa_W)$ is straightforward to compute since the only random variable is government spending in $t+1$ and this can be easily integrated.

[14] This choice is motivated by Judd et al. (2011), who show that orthogonal polynomials yield more stable algorithms, particularly in the presence of higher-order terms.

Ψ_{t-1}, and $\widetilde{\theta}_{t-1}$, we employ first-order polynomials. This closely follows Faraglia et al. (2019), who show that first-order polynomials are sufficient to obtain an accurate approximation in their Ramsey framework.[15] Relative to their setup, our model features additional non-linearities arising from the safety and liquidity properties of short-term debt. To capture these effects, we allow for higher-order terms in the state variable $b_{S,t-1}$.[16] Specifically, we use third-order polynomials, thereby allowing squared, cubic, and interaction terms to affect the solution. Finally, we treat symmetrically the state $b_{L,t-1}$ (including terms up to the third order) to accommodate the possibly binding debt limits for this variable.

D.3 Alternative approaches to dealing with the indeterminacy

We now describe two alternative approaches to address the indeterminacy of the government portfolio. We first explain how we implemented "soft upper bound" on short-term debt. We then discuss the approach of Faraglia et al. (2019).

Soft upper bound on short term debt. We considered adding a transaction cost function to the model. Precisely, we assumed that when the government issues short term debt above the level $\overline{M}_S$ it incurs a resource cost $\mathcal{T}_S = \nu(b_{S,t} - \overline{M}_S)^2$. $\overline{M}_S$ is defined as in the text

$$\overline{M}_S = \sup\left\{ b_S : F_{\widetilde{\theta}} < 1 - \epsilon \right\}$$

Moroever, the cost $\mathcal{T}_S$ enters directly into the government budget constraint and not in the resource constraint.[17]

For a small cost parameter ν, the optimal policy program changes only minimally, but the indeterminacy problem is avoided. In particular, the governments optimal choice of debt when $b_{S,t} \geq \overline{M}_S$ is given by

$$\overbrace{\int_{\widetilde{\theta}_t}^{\infty} \frac{1}{b_{S,t}} \theta \, dF_\theta + \psi_{RC,t}\big(1 - F(\widetilde{\theta}_t)\big) - \psi_{\widetilde{\theta},t}}^{\approx 0} + \psi_{gov,t} \beta \overbrace{F(\widetilde{\theta}_t)}^{\approx 1} E_t \frac{u'(C_{t+1})}{\pi_{t+1}}$$
$$- \beta E_t \psi_{gov,t+1} \frac{u'(C_{t+1})}{\pi_{t+1}} - 2\nu(b_{S,t} - \overline{M}_S) u'(C_t) = 0$$

which implies that this condition can be solved explicitly for the optimal quantity $b_{S,t}$ even when the first three terms in the top line are approximately zero and $F(\widetilde{\theta}_t) \approx 1$. Furthermore, to solve the model it is not necessary to approximate $W(X_t)$ numerically, since the optimal bond supply is determinate almost everywhere on the state space.

In solving the baseline model with this alternative specification, we essentially found no differences relative to the benchmark solution. Intuitively, in the canonical model without lending, the government always prefers to issue more long-term debt in order to benefit from fiscal insurance. The same logic applies in our setting in the region where liquidity (or safety) provided by short bonds is abundant. For brevity we will not show simulations from the model with transaction costs.

[15] Note that non-linearities are implicit in this approximation, since the state variable Ψ_{t-1} is defined as a weighted sum of lagged cross terms involving bonds and multipliers.

[16] This therefore also implicitly accounts for some of the non-linearities that may be present through $\widetilde{\theta}_{t-1}$. For this reason, we omit higher-order terms for this variable. A detailed discussion of why specifying superfluous polynomial terms in the PEA can lead to numerical instability can be found in Judd et al. (2011) and Faraglia et al. (2019).

[17] Implicitly, the households offer financial services to the government and so $\mathcal{T}_S$ will show up in the household budget constraint as a lump sum transfer. We assume that in choosing short bond holdings the households neglect this term. This follows Faraglia et al. (2019).

Solving the portfolio problem with Forward States. Before concluding this numerical section, we provide a brief overview of the approach proposed by Faraglia et al. (2019) to address the indeterminacy of optimal portfolios in Ramsey models.

Consider again the discussion on indeterminacy in text. The key issue was that given the PEA approximation the first-order conditions for short and long term bonds could not determine the optimal portfolio as the vector X_t contained only predetermined variables. The solution proposed by Faraglia et al. (2019) is to formulate approximations of the conditional expectations using the vector X_{t+1} (the date $t+1$ or 'forward' state variables) instead of X_t, since X_{t+1} contains the date t values of government bonds and multipliers.

Formally, consider an approximation of the term

$$\psi_{gov,t+1}, u'(C_{t+1})/\pi_{t+1} = \widetilde{\Phi}_{(\psi,S)}(X_{t+1}, \widetilde{\kappa}_{(\psi,S)}), \tag{D.62}$$

where tildes are used to distinguish this approximating polynomial from the one employed in our baseline formulation. Under this specification, the date- t conditional expectation can be obtained directly as

$$E_t\psi_{gov,t+1}u'(C_{t+1})/\pi_{t+1} = E_t\widetilde{\Phi}_{(\psi,S)}(X_{t+1}, \widetilde{\kappa}_{(\psi,S)}) \tag{D.63}$$

where the expectation is taken solely over the only random variable on the right-hand side, G_{t+1}. Applying analogous forward-state approximations to the remaining conditional expectations in the bond Euler equations, we obtain:

$$\psi_{gov,t} = \frac{E_t\psi_{gov,t+1}u'(C_{t+1})/\pi_{t+1}}{E_tu'(C_{t+1})/\pi_{t+1}} = \frac{E_t\widetilde{\Phi}_{(\psi,S)}(X_{t+1}, \widetilde{\kappa}_{(\psi,S)})}{E_t\widetilde{\Phi}_{(q,S)}(X_{t+1}, \widetilde{\kappa}_{(q,S)})} \tag{D.64}$$

$$\psi_{gov,t} = \frac{E_tE_{t+1}\psi_{gov,t+1}\sum_{j\geq 1}\beta^j\delta^{j-1}\frac{u'(C_{t+j})}{\Pi_{k=1}^{j}\pi_{t+k}}}{E_tE_{t+1}\sum_{j\geq 1}\beta^j\delta^{j-1}\frac{u'(C_{t+j})}{\Pi_{k=1}^{j}\pi_{t+k}}} = \frac{E_t\widetilde{\Phi}_{(\psi,L)}(X_{t+1}, \widetilde{\kappa}_{(\psi,L)})}{E_t\widetilde{\Phi}_{(q,L)}(X_{t+1}, \widetilde{\kappa}_{(q,L)})} \tag{D.65}$$

for $F(\widetilde{\theta}_t) = 1$.

Equations (D.64) and (D.65) along with the remaining equations of the model can be solved to obtain all of the date t endogenous variables, including the bond portfolio.

This approach is fully applicable to our model. However, unlike Faraglia et al. (2019), where indeterminacy spans the entire state space of the Ramsey problem, in our case it is confined to only a subset of the state space (for which our intuition suggests that the solution is likely to be trivial). We therefore find that applying 'forward states' to our model introduces complications in specifying the polynomial approximation, since the optimal policy problem differs across the two regions of the state space. This may in turn require different specifications, potentially introducing discontinuities in the solution.[18]

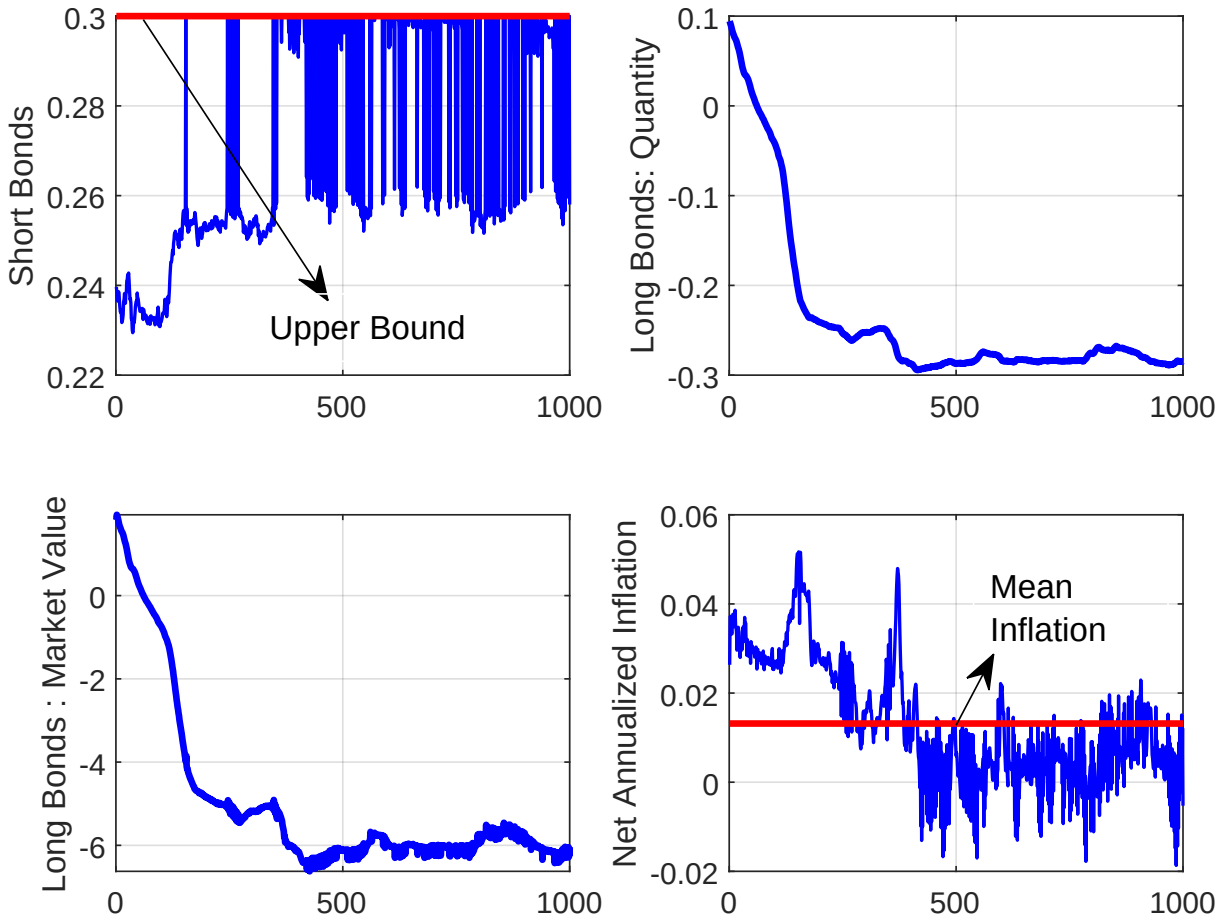
Finally, compared to our algorithm, the inclusion of 'forward states' generally complicates the computation of the equilibrium, as it requires conditional expectations to be taken over vectors of endogenous variables, thereby introducing additional nonlinearities into the problem.

D.4 The Lending Model Symmetric Bounds: Numerical Results

In this section, we present the output from the solution to the baseline model under symmetric bounds for long-term debt, $\underline{M}_L = -\overline{M}_L$.

[18] Obviously, in the region of the state space where the canonical outcome applies, $\widetilde{\theta}_{t-1}$ is not a state variable. It is thus likely that a different approximation should be applied in this region than in the region with binding frictions. This could result in several complications.

Figure 2: Model Simulation: Lending Symmetric Bounds



Notes: The Figure shows 1000 simulated periods from the optimal policy model under 'Lending'. The top left panel plots the quantity of short bonds issued. The flat red line is the upper bound on short-term debt which corresponds to the issuance that fulfills the liquidity demand for short debt. The top right panel plots the quantity of long bonds. The bottom left is the market value of long-term debt (quantity times price level). The bottom right panel shows annualized net inflation rates in the model. The flat line is the sample average inflation.

We summarize the model results in Figure 2, which shows a simulated path of short bonds (top-left panel), long bond quantities (top-right panel), the market value of long-term debt (quantity times price, bottom-left panel), and the (annualized) inflation rate (bottom-right panel). These objects are plotted over 1000 model periods, with spending shocks drawn randomly from the underlying distribution. Initially, the economy is at the deterministic Ramsey steady state.

Consider the bond quantities shown in the top panels. As evident from the graphs, the quantity of long-term bonds quickly turns negative and continues dropping until it fluctuates around (and occasionally hits) the lower bound. During the transition toward negative long-term debt, the issuance of short-term debt fluctuates within the region where short bonds provide safety and liquidity to the private sector. However, once long-term bonds become sufficiently negative, the supply of short-term debt increases, and we frequently observe in our simulations that short debt hits the upper bound.

Interpretation. It is clear that the optimal policies we recover from solving the model have a strong analogue in the theoretical results established in Section 3. There, using the date–0 optimal allocation problem, we showed that the optimal policy is to satiate the economy with short-term debt when the supply of long-term bonds is sufficiently negative. We recover the same property in the stochastic optimal policy equilibrium.

However, why does the planner choose to target a very negative supply of long-term bonds, given that this policy may entail potentially large intertemporal distortions through front-loading taxes to accumulate assets?

The intuition is straightforward. With a large stock of savings in the long-term asset, the cost of distortionary taxation is relatively low, and the government can use the returns on these savings to finance spending shocks and smooth taxes across time and states. We therefore find that the optimal policy in our model shares an analogous property with the canonical Ramsey model studied in Aiyagari et al. (2002) (and subsequently in Faraglia et al. (2016, 2019) and related work).

Accumulating savings in the long-term asset effectively means that the government gives up the fiscal hedging benefits typically associated with long-term debt.

Regime Fluctuations. As evident from Figure 2, when long-term debt turns negative, the government does not maintain a constant supply of short-term bonds at the upper bound. Instead, the equilibrium oscillates between periods of complete provision of liquidity and safety and periods where financial frictions are binding. We interpret this property of the model as follows: although accumulating long-term savings allows the government to smooth distortions over time, it still desires to use short-term debt as a shock absorber. Following an unanticipated increase in spending, it becomes optimal to contract the supply of short-term debt, financing this adjustment with a further reduction in the long-term asset stock.

Optimal Inflation. Finally, we discuss the properties of inflation in the optimal policy equilibrium with lending. As shown in the bottom-right panel of Figure 2, optimal inflation is positive throughout the transition period during which the government accumulates savings, and it is approximately zero on average in the subsample where financial frictions only occasionally bind. These patterns are consistent with our theoretical results: when frictions bind, the planner sets a strictly positive inflation target, whereas when frictions cease to bind, the target is zero, as in the canonical Ramsey model.

Interestingly, inflation displays considerably lower conditional volatility at the beginning of the sample, and much higher volatility once the transition to negative debt is completed. While our theory identifies mechanisms that can explain this pattern (i.e., inflation contains a random-walk component when frictions bind), the large and frequent fluctuations of inflation in the second part of the sample are likely partly driven by equilibrium regime oscillations. That is, the government switches from a temporarily positive inflation target to a zero target, resulting in excess inflation

volatility. The finding that inflation fluctuations can be large when government debt is negative is of independent interest.

D.5 The Lending Model Tight Bounds

In text we discussed that in more plausible calibrations of the Lending model the lower bound $\underline{M}_L$ should be significantly tighter than minus the upper bound. In particular, since in our model the consolidated budget is the relevant equilibrium object, a value of $\underline{M}_L$ corresponding to the level of asset purchases by the Fed during QE or the pandemic is more appropriate.[19]

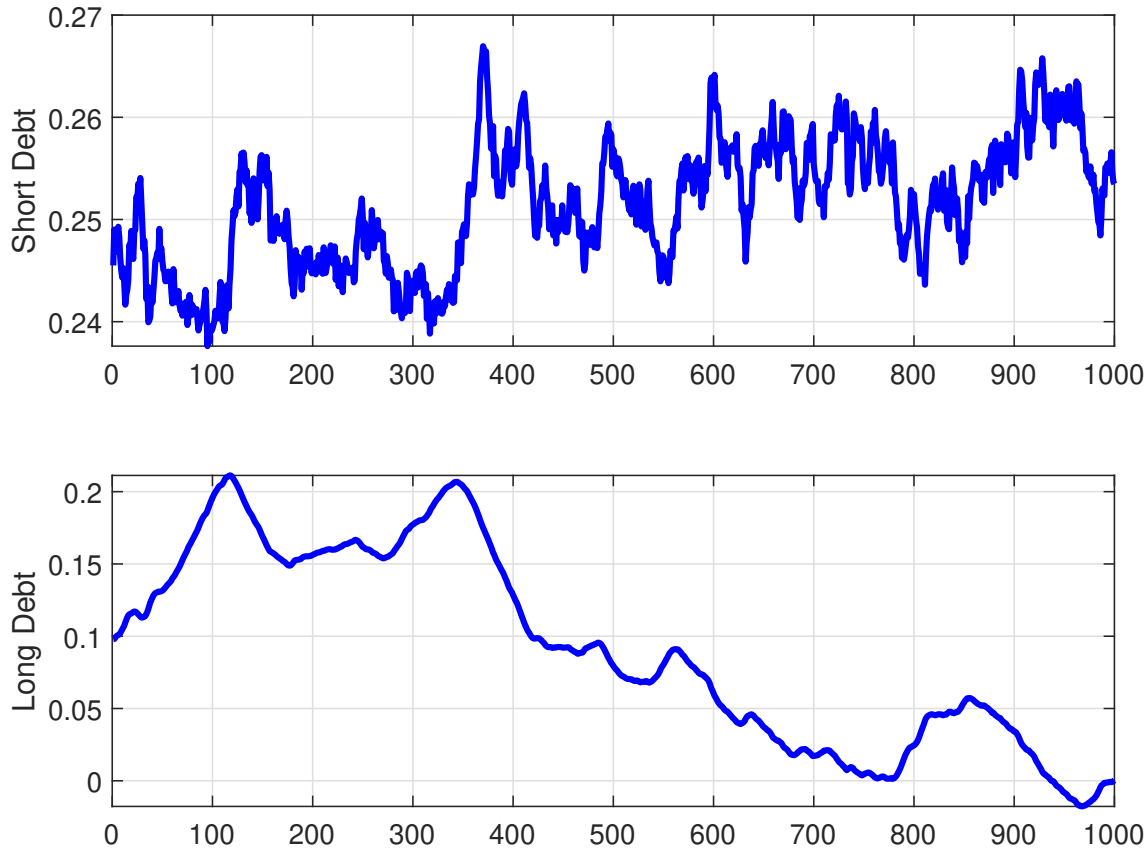
We therefore consider a calibration in which $\underline{M}_L$ is -.03, which is somewhat looser than the market value of the Fed's purchases of private assets (approximately 12 percent of GDP). The results obtained from solving the model are displayed in Figure 3. We focus on the issuance of short-term and long-term debt (top and bottom panels, respectively).

As can be seen from the figure, throughout the sample shown the optimal policy equilibrium remains within the region of binding frictions. We conclude that the optimal policy in a model with plausibly negative lower bounds is close to that in the baseline model studied in the text.

[19]While the private assets purchased by the Fed were clearly riskier than long-term government debt, we follow much of the literature in treating private and government debt as perfect substitutes.

Moreover, during QE the Fed also purchased long-term government bonds. However, these purchases are implicitly accounted for through the consolidated budget constraint. The appropriate lower bound for long-term debt is therefore zero, as it is implausible to assume that net long-term debt held by the public is ever negative.

Figure 3: Model Simulation: Lending Tight Bounds



Notes: The Figure shows 1000 simulated periods from the optimal policy model under 'Lending' with tight bounds. See text for details.

E Optimal Policy in the Canonical Model

In this section, we characterize optimal monetary and fiscal policies in the canonical Ramsey model in which short-term bonds do not provide liquidity or safety services to the private sector. In Section E.1, we show the optimality conditions. To simplify the derivations, we assume that the economy features a single debt instrument and abstract from idiosyncratic consumption risk in period t_2 (equivalently, we impose $f_\theta = 0$ a.e.). These assumptions are not restrictive. Short-term debt remains a special case of the framework and can be recovered by setting $\delta = 0$ so the derivations below encompass this case.

We use this section to establish several key results in Section E.2 for the efficient steady state and in Section E.3 for the distorted steady state. First, we show that optimal inflation in the canonical model does not feature a time-varying target, in contrast to environments with financial frictions. Second, inflation is pinned down by the government's intertemporal debt constraint and is determined by the maturity structure of government debt. Third, we demonstrate that the Ramsey-optimal policy admits an interest rate rule in which the nominal interest rate tracks the welfare-relevant real interest rate and responds contemporaneously to inflation. We characterize the optimal inflation coefficient analytically and show that optimal monetary policy is passive in the sense of Leeper (1991). This result further formalizes the conclusion that inflation in the model is fully determined by the intertemporal solvency condition, consistent with models of the fiscal theory of the price level. Finally, we characterize the optimal tax schedule in the canonical model in subsection E.4.

It should be noted that the optimal policy problem studied in this section is essentially the same as that considered by Benigno and Woodford (2006); Leeper and Zhou (2021) and related work. As in these studies, we derive analytical results using a log-linear approximation to the Ramsey policy equilibrium. Thus, as we discuss explicitly below, some of the results derived here are not entirely new to the literature. Nevertheless, it is useful to re-derive them within our framework. We contribute to the literature by providing a novel analytical characterization of the optimal interest rate rule.

E.1 Optimality Conditions

In our derivations below we will focus on the timelessly optimal policy. For brevity, we will not explicitly state the Lagrangian of the planner's program, since this can be inferred directly from the full model by setting $f(\theta) = 0$.

The first-order conditions characterizing the optimum are:

$$Y_t: \quad -\chi Y_t^\gamma - \psi_{RC,t} + \psi_{PC,t}\left(\frac{1+\gamma}{1-\tau_t}\frac{\eta}{\omega}\chi Y_t^\gamma - \frac{1+\eta}{\omega}u'(C_t)\right) + \psi_{gov,t}\frac{\tau_t}{1-\tau_t}(1+\gamma)\chi Y_t^\gamma = 0$$

$$\tau_t: \quad \psi_{PC,t}\frac{\eta}{\omega}\chi\frac{Y_t^{1+\gamma}}{(1-\tau_t)^2} + \psi_{gov,t}\chi\frac{Y_t^{1+\gamma}}{(1-\tau_t)^2} = 0$$

$$C_t: \quad u'(C_t) + \psi_{RC,t} + u''(C_t)\pi_t(\pi_t - 1)\Delta\psi_{PC,t} - \psi_{PC,t}\frac{1+\eta}{\omega}u''(C_t)Y_t$$

$$-\psi_{gov,t}u''(C_t)G_t - u''(C_t)\sum_{j\geq 0}\delta^j(\psi_{gov,t-j} - \psi_{gov,t-j-1})\frac{b_{L,t-j-1}}{\Pi_{k=0}^j\pi_{t-k}} = 0$$

$$\pi_t: \quad \omega\psi_{RC,t}(\pi_t - 1) + u'(C_t)(2\pi_t - 1)\Delta\psi_{PC,t} + u'(C_t)(1+\delta q_{L,t})\sum_{j\geq 0}\delta^j\frac{b_{L,t-j-1}}{\pi_{t-j}...\pi_t^2}\Delta\psi_{gov,t-j} = 0$$

$$b_{L,t}: \quad \psi_{gov,t}\sum_{j\geq 1}\beta^j\delta^{j-1}E_t\frac{u'(C_{t+j})}{\Pi_{k=1}^j\pi_{t+k}} = E_t\psi_{gov,t+1}\sum_{j\geq 1}\beta^j\delta^{j-1}\frac{u'(C_{t+j})}{\Pi_{k=1}^j\pi_{t+k}}$$

The interpretation of these objects is analogous to that of the FONCs in the baseline model. To allow for greater generality, we assume that $u'(C_t) = C_t^{-\sigma}$. This specification includes the two cases considered in the paper, $\sigma \in \{0, 1\}$.

By combining the optimality conditions for C_t and Y_t, we obtain the following consolidated expression:

$$\begin{aligned} & C_t^{-\sigma} - \chi Y_t^{\gamma} + \psi_{PC,t}\left(\frac{1+\gamma}{1-\tau_t}\frac{\eta}{\omega}\chi Y_t^{\gamma} - \frac{1+\eta}{\omega}C_t^{-\sigma}\right) + \psi_{gov,t}\frac{\tau_t}{1-\tau_t}(1+\gamma)\chi Y_t^{\gamma} \\ & -\sigma C_t^{-\sigma-1}\pi_t(\pi_t - 1)\Delta\psi_{PC,t} + \psi_{PC,t}\frac{1+\eta}{\omega}\sigma C_t^{-\sigma-1}Y_t \\ & +\psi_{gov,t}\sigma C_t^{-\sigma-1}G_t + \sigma C_t^{-\sigma-1}\sum_{j\geq 0}\delta^j(\psi_{gov,t-j} - \psi_{gov,t-j-1})\frac{b_{L,t-j-1}}{\Pi_{k=0}^{j}\pi_{t-k}} = 0 \end{aligned} \tag{E.66}$$

We first work with equation (E.66) and the optimal inflation condition. We derive a log-linear approximation of these equations around the (locally) unique zero-inflation Ramsey steady state.[20]

The log-linear system is :

$$\omega\overline{\psi}_{RC}\hat{\pi}_t + \overline{\psi}_{PC}\overline{C}^{-\sigma}\Delta\hat{\psi}_{PC,t} + \overline{\psi}_{gov}\overline{C}^{-\sigma}\frac{1}{1-\beta\delta}\sum_{j\geq 0}\delta^j\overline{b}_L\Delta\hat{\psi}_{gov,t-j} = 0 \tag{E.67}$$

and

$$\begin{aligned} & -\sigma\overline{C}^{-\sigma}\hat{C}_t - \chi\gamma\overline{Y}^{\gamma}\hat{Y}_t + \overline{\psi}_{PC}\frac{1+\gamma}{1-\overline{\tau}}\frac{\eta}{\omega}\chi\overline{Y}^{\gamma}(\gamma\hat{Y}_t + \hat{\psi}_{PC,t} + \frac{\overline{\tau}}{1-\overline{\tau}}\hat{\tau}_t) - \overline{\psi}_{PC}\frac{1+\eta}{\omega}\overline{C}^{-\sigma}(\hat{\psi}_{PC,t} - \sigma\hat{C}_t) \\ & +\overline{Y\psi}_{PC}\frac{1+\eta}{\omega}\sigma\overline{C}^{-\sigma-1}(\hat{\psi}_{PC,t} - (1+\sigma)\hat{C}_t + \hat{Y}_t) + \overline{\psi}_{gov}\frac{\overline{\tau}}{1-\overline{\tau}}(1+\gamma)\chi\overline{Y}^{\gamma}(\hat{\psi}_{gov,t} + \gamma\hat{Y}_t + \frac{1}{1-\overline{\tau}}\hat{\tau}_t) \\ & +\overline{\psi}_{gov}\sigma\overline{C}^{-\sigma-1}\overline{G}(\hat{\psi}_{gov,t} - (1+\sigma)\hat{C}_t + \hat{G}_t) + \overline{\psi}_{gov}\sigma\overline{b}_L\overline{C}^{-\sigma-1}\sum_{j\geq 0}\delta^j(\hat{\psi}_{gov,t-j} - \hat{\psi}_{gov,t-j-1}) = 0 \end{aligned}$$

which can be further simplified by using the envelope condition to eliminate tax terms.

$$\begin{aligned} & -\sigma\overline{C}^{-\sigma}\hat{C}_t - \chi\gamma\overline{Y}^{\gamma}\hat{Y}_t + \overline{\psi}_{PC}\frac{1+\gamma}{1-\overline{\tau}}\frac{\eta}{\omega}\chi\overline{Y}^{\gamma}(\gamma\hat{Y}_t + \hat{\psi}_{PC,t}) - \overline{\psi}_{PC}\frac{1+\eta}{\omega}\overline{C}^{-\sigma}(\hat{\psi}_{PC,t} - \sigma\hat{C}_t) \\ & +\overline{Y\psi}_{PC}\frac{1+\eta}{\omega}\sigma\overline{C}^{-\sigma-1}(\hat{\psi}_{PC,t} - (1+\sigma)\hat{C}_t + \hat{Y}_t) + \overline{\psi}_{gov}\frac{\overline{\tau}}{1-\overline{\tau}}(1+\gamma)\chi\overline{Y}^{\gamma}(\hat{\psi}_{gov,t} + \gamma\hat{Y}_t) \\ & +\overline{\psi}_{gov}\sigma\overline{C}^{-\sigma-1}\overline{G}(\hat{\psi}_{gov,t} - (1+\sigma)\hat{C}_t + \hat{G}_t) + \overline{\psi}_{gov}\sigma\overline{b}_L\overline{C}^{-\sigma-1}\sum_{j\geq 0}\delta^j(\hat{\psi}_{gov,t-j} - \hat{\psi}_{gov,t-j-1}) = 0 \end{aligned} \tag{E.68}$$

Steady state expressions. We solve for the multipliers in the Ramsey steady state. Define: $\frac{\chi\overline{Y}^{\gamma}}{\overline{C}^{-\sigma}} = \frac{1+\eta}{\eta}(1-\overline{\tau}) = \widetilde{\epsilon}$, where $\widetilde{\epsilon} \leq 1$ measures distortions arising from monopolistic competition ($\frac{1+\eta}{\eta} < 1$) and taxation ($1-\overline{\tau}$). In the efficient steady state $\widetilde{\epsilon} = 1$ can be accomplished by setting $\overline{\tau} = 1 - \frac{\eta}{1+\eta} < 0$. Notice that this outcome is not ruled out. In Ramsey policy equilibria, government debt can (in the longer term) take negative values (Aiyagari et al., 2002; Faraglia et al., 2016).[21]

[20] The property that the zero-inflation steady state is unique in this model is well known. See for example Benigno and Woodford (2005).

[21] Our formulae below can therefore be seen as successive local approximations to the Ramsey policy equilibrium, accommodating the possibility that, in the nonlinear model, government debt may become negative in the longer term.

At steady state, (E.66) can be written as:

$$1-\widetilde{\epsilon}+\overline{\psi}_{PC}\gamma\frac{1+\eta}{\omega}+\overline{\psi}_{gov}\frac{\overline{\tau}}{1-\overline{\tau}}(1+\gamma)\widetilde{\epsilon}+\overline{\psi}_{PC}\frac{1+\eta}{\omega}\sigma\frac{\overline{Y}}{\overline{C}}+\overline{\psi}_{gov}\sigma\frac{\overline{G}}{\overline{C}}=0$$

and using the optimality condition for the tax rate $\frac{\eta}{\omega}\overline{\psi}_{PC}+\overline{\psi}_{gov}=0$ we get

$$\overline{\psi}_{PC}=\frac{1-\widetilde{\epsilon}}{-\gamma\frac{1+\eta}{\omega}+\frac{\eta}{\omega}\frac{\overline{\tau}}{1-\overline{\tau}}(1+\gamma)\widetilde{\epsilon}-\frac{1+\eta}{\omega}\sigma\frac{\overline{Y}}{\overline{C}}+\frac{\eta}{\omega}\sigma\frac{\overline{G}}{\overline{C}}}$$

$$\overline{\psi}_{gov}=-\frac{1-\widetilde{\epsilon}}{-\gamma\frac{1+\eta}{\eta}+\frac{\overline{\tau}}{1-\overline{\tau}}(1+\gamma)\widetilde{\epsilon}-\frac{1+\eta}{\eta}\sigma\frac{\overline{Y}}{\overline{C}}+\sigma\frac{\overline{G}}{\overline{C}}}$$

Finally, from the first-order condition with respect to Y_t we find

$$\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}=-\widetilde{\epsilon}+\overline{\psi}_{PC}\gamma\frac{\eta}{\omega}\frac{\widetilde{\epsilon}}{1-\overline{\tau}}+\overline{\psi}_{gov}\frac{\overline{\tau}}{1-\overline{\tau}}(1+\gamma)\widetilde{\epsilon}$$

in steady state.

Using these steady-state relations we can rewrite equation (E.68) as:

(E.69)
$$\begin{aligned}&-\sigma\hat{C}_t-\gamma\widetilde{\epsilon}\hat{Y}_t+\overline{\psi}_{PC}\frac{1+\gamma}{1-\overline{\tau}}\frac{\eta}{\omega}\widetilde{\epsilon}\gamma\hat{Y}_t+\overline{\psi}_{PC}\frac{1+\eta}{\omega}\sigma\hat{C}_t\\&+\frac{\overline{Y}}{\overline{C}}\overline{\psi}_{PC}\frac{1+\eta}{\omega}\sigma(-(1+\sigma)\hat{C}_t+\hat{Y}_t)+\overline{\psi}_{gov}\frac{\overline{\tau}}{1-\overline{\tau}}(1+\gamma)\widetilde{\epsilon}\gamma\hat{Y}_t\\&+\overline{\psi}_{gov}\sigma\frac{\overline{G}}{\overline{C}}(-(1+\sigma)\hat{C}_t+\hat{G}_t)+(\widetilde{\epsilon}-1)\hat{\psi}_{gov,t}+\overline{\psi}_{gov}\sigma\bar{b}_L\overline{C}^{-1}\sum_{j\geq0}\delta^j(\hat{\psi}_{gov,t-j}-\hat{\psi}_{gov,t-j-1})=0\end{aligned}$$

or (gathering terms)

$$a_1\hat{Y}_t+a_2\hat{G}_t+(\widetilde{\epsilon}-1)\hat{\psi}_{gov,t}+\overline{\psi}_{gov}\sigma\bar{b}_L\overline{C}^{-1}\sum_{j\geq0}\delta^j(\hat{\psi}_{gov,t-j}-\hat{\psi}_{gov,t-j-1})=0$$

for

$$\begin{aligned}a_1=&-\sigma\frac{\overline{Y}}{\overline{C}}-\gamma\widetilde{\epsilon}+\overline{\psi}_{PC}\frac{1+\gamma}{1-\overline{\tau}}\frac{\eta}{\omega}\widetilde{\epsilon}\gamma+\overline{\psi}_{PC}\frac{1+\eta}{\omega}\sigma\frac{\overline{Y}}{\overline{C}}\\&+\frac{\overline{Y}}{\overline{C}}\overline{\psi}_{PC}\frac{1+\eta}{\omega}\sigma(-(1+\sigma)\frac{\overline{Y}}{\overline{C}}+1)+\overline{\psi}_{gov}\frac{\overline{\tau}}{1-\overline{\tau}}(1+\gamma)\widetilde{\epsilon}\gamma+\overline{\psi}_{gov}\sigma\frac{\overline{G}}{\overline{C}}(-(1+\sigma)\frac{\overline{Y}}{\overline{C}})\end{aligned}$$

and

$$a_2=\sigma\frac{\overline{G}}{\overline{C}}-\overline{\psi}_{PC}\frac{1+\eta}{\omega}\sigma\frac{\overline{G}}{\overline{C}}+\frac{\overline{Y}}{\overline{C}}\overline{\psi}_{PC}\frac{1+\eta}{\omega}\sigma(1+\sigma)\frac{\overline{G}}{\overline{C}}+\overline{\psi}_{gov}\sigma\frac{\overline{G}}{\overline{C}}((1+\sigma)\frac{\overline{G}}{\overline{C}}+1)$$

E.2 Policy at the efficient steady state.

We first characterize optimal policy in the neighborhood of the efficient allocation. This benchmark allows us to derive simple formulas, which we later generalize to encompass the case of distortions.

For the efficient allocation ($\widetilde{\epsilon}=1$) we get

$$\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}=-1 \quad \overline{\psi}_{PC}=\overline{\psi}_{gov}=0$$

The result that $\overline{\psi}_{PC}=\overline{\psi}_{gov}=0$ is intuitive. Since the economy operates at the efficient level of output, the planner has no incentive to distort inflation or taxes when the Ramsey program begins with government debt at its steady-state value (as an initial condition).[22] However, because these multipliers are zero, our earlier derivations must be slightly adjusted: we need a linear approximation of the multipliers around their steady-state values, and a log-linear approximation for the remaining model variables. Specifically, we now obtain:

$$a_1\hat{Y}_t+a_2\hat{G}_t+\left[\frac{(1+\gamma)}{\eta}+\sigma\frac{\overline{G}}{\overline{C}}+\frac{\omega}{\eta}\kappa\right]\hat{\psi}_{gov,t}+\sigma\overline{b}_L\overline{C}^{-1}\sum_{j\geq 0}\delta^j(\hat{\psi}_{gov,t-j}-\hat{\psi}_{gov,t-j-1})=0$$

where $\kappa\equiv-\frac{1+\eta}{\omega}(\gamma+\sigma\frac{\overline{Y}}{\overline{C}})$ is the slope of the Phillips curve. Moreover,

$$a_1=-\sigma\frac{\overline{Y}}{\overline{C}}-\gamma \qquad\qquad a_2=\sigma\frac{\overline{G}}{\overline{C}}$$

Defining natural output $\hat{Y}^n_t\equiv-\frac{a_2}{a_1}\hat{G}_t=\frac{\sigma\frac{\overline{G}}{\overline{C}}}{\sigma\frac{\overline{Y}}{\overline{C}}+\gamma}\hat{G}_t$ we can express the output gap as:

$$(\sigma\frac{\overline{Y}}{\overline{C}}+\gamma)\underbrace{(\hat{Y}_t-\hat{Y}^n_t)}_{=\widetilde{Y}_t}=\left[\frac{(1+\gamma)}{\eta}+\sigma\frac{\overline{G}}{\overline{C}}+\frac{\omega}{\eta}\kappa\right]\hat{\psi}_{gov,t}+\sigma\overline{b}_L\overline{C}^{-1}\sum_{j\geq 0}\delta^j(\hat{\psi}_{gov,t-j}-\hat{\psi}_{gov,t-j-1}) \tag{E.70}$$

The first-order condition for inflation is:

$$\omega\hat{\pi}_t=-\frac{\omega}{\eta}\Delta\hat{\psi}_{gov,t}+\frac{1}{1-\beta\delta}\sum_{j\geq 0}\delta^j\overline{b}_L\Delta\hat{\psi}_{gov,t-j} \tag{E.71}$$

Combining (E.70) and (E.71) we can derive the following condition linking inflation and the growth of the output gap.

$$\begin{aligned}\hat{\pi}_t+\frac{1}{\kappa\omega}\left(\sigma\frac{\overline{Y}}{\overline{C}}+\gamma\right)\Delta\widetilde{Y}_t=&\overbrace{\frac{1}{\omega\kappa}\left[\frac{(1+\gamma)}{\eta}+\sigma\frac{\overline{G}}{\overline{C}}\right]\Delta\hat{\psi}_{gov,t}}^{\text{Surplus Effect}}\\ &\underbrace{+\sigma\frac{1}{\kappa\omega}\overline{b}_L\overline{C}^{-1}\sum_{j\geq 0}\delta^j\left(\Delta\hat{\psi}_{gov,t-j}-\Delta\hat{\psi}_{gov,t-j-1}\right)}_{\text{Real bond price/interest rate effect}}+\underbrace{\frac{1}{(1-\beta\delta)\omega}\sum_{j\geq 0}\delta^j\overline{b}_L\Delta\hat{\psi}_{gov,t-j}}_{\text{Direct Inflation Effect}}=0\end{aligned} \tag{E.72}$$

It is useful to investigate equation (E.72) in detail. Consider the case $\Delta\hat{\psi}_{gov,t}=0$ for all t (i.e. no spending shocks). (E.72) reduces $\hat{\pi}_t+\frac{\lambda_Y}{\kappa}\Delta\widetilde{Y}_t=0$, where $\lambda_Y=\frac{1}{\kappa\omega}\left(\sigma\frac{\overline{Y}}{\overline{C}}+\gamma\right)$ is the weight on the

[22] The optimal plan is therefore time-consistent, and no timeless penalty is required when formulating the date 0 program (Woodford, 2003).

output gap in the usual microfounded loss function (Woodford, 2003). Therefore, inflation and the output gap are set according to the standard target criterion in the New Keynesian model (Giannoni and Woodford, 2003).[23]

According to (E.72), however, $\hat{\pi}_t + \frac{\lambda_Y}{\kappa}\Delta\widetilde{Y}_t$ is not equal to zero, but to a weighted sum of the current and lagged values of the multiplier attached to the consolidated budget constraint.

These terms capture how the consolidated budget constraint affects the optimal policy. To clarify, it is useful to derive the intertemporal debt solvency condition:

$$E_t \sum_{j\geq 0} \beta^j \overline{S}\hat{S}_{t+j} = \frac{\bar{b}_L}{1-\beta\delta}\hat{b}_{t-1,L} + \bar{b}_L \sum_{j\geq 0}(\beta\delta)^j \left[-\sigma\left(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}\right) - \sum_{k=0}^{j}\hat{\pi}_{t+k}\right] \tag{E.73}$$

The LHS is the present value of the government surplus stream (scaled by the marginal utility of consumption).[24] On the RHS we have the real market value of outstanding debt. Satisfaction of the sequence of constraints (E.73) is equivalent to satisfying the per period consolidated budget constraints in the planner's program.

Consider a shock that increases the level of spending in period t. For the planner's program, the constraint (E.73) tightens and the value of the multiplier $\hat{\psi}_{gov,t}$ increases. If the shock is not fully absorbed by taxes (so that the surpluses do not fully compensate for outstanding debt), the planner will need to engineer a drop in the real payout of debt either through increasing inflation and/or adjusting output when $\sigma > 0$. Moreover, since optimal policy features full commitment, it is feasible to adjust both the current and future values of these variables.

As discussed in text, the lagged multipliers in (E.72) admit a natural interpretation. They represent commitments made by the planner to adjust inflation and output in response to shocks that occurred in the past. Formally, in this model we can show that

$$\Delta\hat{\psi}_{gov,t-j} = \nu\epsilon_{G,t-j}$$

where ϵ_G denotes the government spending shock in $t-j$ and the constant ν is pinned down by the intertemporal budget constraint.[25] Substituting this relationship in (E.72) makes clear that the lagged multiplier terms reflect policy commitments arising from previously realized shocks.

Components of (E.72) We briefly explain the terms on the RHS of equation (E.72) and their respective impacts on the optimal policy. Consider first, the term labeled 'Direct Effect' (E.72). This term accounts for the immediate impact of inflation on the real value of government debt: An (unanticipated) 1 % increase in inflation in period t reduces the debt that matures in t by $\bar{b}_L 1\%$, the debt in $t+1$ by $\bar{b}_L\beta\delta 1\%$, in $t+2$ by $\bar{b}_L\beta^2\delta^2 1\%$ and so on. The overall impact is $\bar{b}_L\frac{1}{1-\beta\delta}1\%$ as the expression in (E.72) reveals.

However, the direct impact is not the only channel via which inflation can affect the real market value of government debt. The second term on the RHS of (E.72) ('Real bond price/interest rate effect') captures the impact that inflation has on output and through that on the real long bond prices and the real interest rates that discount the sequence of surpluses. Higher inflation in t increases

[23] Intuition: Changing inflation in period t by (say) 1 percent, leads to a change in output in t and $t-1$ by $\frac{1}{\kappa}$ and $-\frac{\beta}{\kappa}$ respectively. Since the planner discounts the future at rate β the relative weight attached to the output change in terms of this trade-off is λ_Y.

[24] That is $\overline{S}\hat{S}_{t+j}$ is an approximation of $S_{t+j} = u'(C_{t+j})(\tau_{t+j}Y_{t+j}\frac{1+\eta}{\eta} - G_{t+j})$.

[25] This follows from the property that $\hat{\psi}_{gov}$ follows a random walk. Note that it is possible to show that ν is uniquely determined. Although, $E_t\hat{\psi}_{gov,t+1} = \hat{\psi}_{gov,t}$ might appear consistent with multiple solutions, the government's intertemporal budget constraint pins down the value of the multiplier. This argument is essentially the FTPL equilibrium selection: as in FTPL models, inflation (or here, the multiplier) is determined by the intertemporal debt constraint. See Chafwehé, de Beauffort, and Oikonomou (2026) for a formal treatment in a setting similar to this one.

the current output lowering real interest rates (increasing long bond prices). This raises the market value of debt, on the one hand, and on the other, it raises the present value of government surpluses. For $\delta < 1$ the second effect dominates and this channel contributes to the intertemporal solvency of government debt.

Finally, the 'surplus effect' term measures the impact of inflation-through output-on the governments surplus.[26]

Mean reverting inflation. Using equations (E.71) and (E.72), it is straightforward to show that—unlike in the model with financial frictions—optimal inflation in this environment does not feature a time-varying inflation target. For simplicity, consider equation (E.71), which isolates the inflation process. It is simple to show that:

$$\hat{\pi}_t = (\frac{\overline{b}_L}{\omega(1-\beta\delta)} - \frac{1}{\eta})\Delta\hat{\psi}_{gov,t} + \frac{\delta}{\eta}\Delta\hat{\psi}_{gov,t-1} + \delta\hat{\pi}_{t-1} \tag{E.74}$$

and therefore for $\delta < 1$ (and $\Delta\hat{\psi}_{gov,t}$ an i.i.d random variable) inflation follows a stationary ARMA(1,2) process. A shock to spending occurring in period t does not generate a permanent increase in inflation. Instead, inflation jumps on impact and, after a short transition (governed by the terms $\Delta\hat{\psi}_{gov,t}, \Delta\hat{\psi}_{gov,t-1},$) it will revert to 0 at rate δ. (See, for example, Leeper and Zhou, 2021).

Moreover, based on our previous remarks suggesting that the terms $\Delta\hat{\psi}_{gov,t-j}$ are pinned down by the intertemporal debt constraints it is obvious that in this model inflation is fully pinned down by the sequence of constraints (E.73)

Finally, note that a special case in which inflation features a random walk component is $\delta = 1$ corresponding to long bonds taking the form of consols (Leeper and Zhou, 2021). This case is not relevant for our analysis, as we focus on $\delta < 1$.

The optimal interest rate rule around the efficient steady state. We derive the interest rate rule that implements the optimal policy allocation using equations (E.70) and (E.71).

First, from the Euler equation we get

$$\hat{i}_t = r_t^n + E_t\hat{\pi}_{t+1} + \sigma\frac{\overline{Y}}{\overline{C}}[E_t(\hat{Y}_{t+1} - \hat{Y}_{t+1}^n) - (\hat{Y}_t - \hat{Y}_t^n)]$$

where $r_t^n = -\frac{\gamma\overline{G}}{\overline{Y}+\frac{\gamma}{\sigma}\overline{G}}(E_t\hat{G}_{t+1} - \hat{G}_t)$ denotes the natural interest rate.

Then, from (E.70) we get

$$E_t\hat{\pi}_{t+1} = -\frac{1}{\eta}\underbrace{E_t\Delta\hat{\psi}_{gov,t+1}}_{=0} + E_t\frac{1}{(1-\beta\delta)\omega}\sum_{j\geq 0}\delta^j\overline{b}_L\Delta\hat{\psi}_{gov,t-j+1}. =$$
$$\frac{1}{(1-\beta\delta)\omega}\delta\sum_{j\geq 0}\delta^j\overline{b}_L\Delta\hat{\psi}_{gov,t-j} = \delta(\hat{\pi}_t + \frac{1}{\eta}\Delta\hat{\psi}_{gov,t})$$

Moreover,

$$(\sigma\frac{\overline{Y}}{\overline{C}} + \gamma)(E_t\widetilde{Y}_{t+1} - \widetilde{Y}_t) = \left[\frac{(1+\gamma)}{\eta} + \sigma\frac{\overline{G}}{\overline{C}} + \frac{\omega}{\eta}\kappa\right]E_t\Delta\hat{\psi}_{gov,t+1} + (\delta-1)\sigma\overline{b}_L\overline{C}^{-1}\sum_{j\geq 0}\delta^j\Delta\hat{\psi}_{gov,t-j} = \tag{E.75}$$

$$(\delta-1)(1-\beta\delta)\omega\sigma\overline{C}^{-1}(\hat{\pi}_t + \frac{1}{\eta}\Delta\hat{\psi}_{gov,t}) \tag{E.76}$$

[26] A lengthier discussion of these terms than we provide here can be found in Chafwehé, Priftis, Oikonomou, and Vogel (2024).

Combining the two expressions we get the following result:

Proposition E.1. *Assume the efficient steady state. Optimal monetary policy is characterized by an interest rate rule:*

$$\hat{i}_t = r_t^n + \overbrace{\left(\delta + (\delta-1)(1-\beta\delta)\frac{\sigma\omega\overline{C}^{-1}}{(\sigma\frac{\overline{Y}}{\overline{C}}+\gamma)}\right)}^{\textit{Inflation Coefficient}}\hat{\pi}_t + \underbrace{\frac{1}{\eta}(\delta + (\delta-1)(1-\beta\delta)\frac{\omega\sigma\overline{C}^{-1}}{(\sigma\frac{\overline{Y}}{\overline{C}}+\gamma)})}_{\textit{Stochastic Intercept}}\Delta\hat{\psi}_{gov,t}$$

where r_t^n denotes the natural interest rate.

Proposition E.1 (and its accompanying Proposition E.2, see below) constitutes a central result of this section. We show that optimal monetary policy in the model can be implemented as a simple rule with three components: (i) the natural interest rate, (ii) a systematic response to inflation, and (iii) a stochastic intercept term, which can be expressed as a function of the period-t government spending shock.

The first component is straightforward to interpret. Tracking the natural rate is a standard property of optimal policy in the New Keynesian model in the absence of a public debt constraint. In the presence of demand-driven fluctuations (here, spending shocks), it allows the planner to close the output gap—that is, to equate output to its natural level at all times.[27] According to Proposition E.1, this property carries over to optimal policy even in the presence of public debt and distortionary taxation.

The third term—the stochastic intercept—induces a temporary innovation to the interest rate rule whenever a shock hits the economy, leading to $\Delta\hat{\psi}_{\text{gov},t} \neq 0$. For example, suppose that government spending increases in period t, so that $\Delta\hat{\psi}_{\text{gov},t} > 0$. Then, according to Proposition E.1, optimal policy keeps the nominal interest rate slightly lower in period t than the value implied by the systematic component of the interest rate rule. This adjustment can be related to the MA terms in (E.74).

The most substantive result in Proposition E.1 is the analytical characterization of the optimal inflation coefficient. Specifically, the property

$$\delta + (\delta-1)(1-\beta\delta)\frac{\sigma\omega\overline{C}^{-1}}{\sigma\frac{\overline{Y}}{\overline{C}}+\gamma} < 1$$

establishes that optimal monetary policy is passive (Leeper, 1991). In light of this property, it is straightforward to argue that inflation in the model is determined by the intertemporal debt solvency condition.

Interpretation of the inflation coefficient. We will devote a few lines to briefly explain what the $\left(\delta + (\delta-1)(1-\beta\delta)\frac{\sigma\omega\overline{C}^{-1}}{(\sigma\frac{\overline{Y}}{\overline{C}}+\gamma)}\right)$ signifies.

To build intuition, consider first the case of quasilinear utility and abstract from the stochastic intercept, so that the optimal interest rate rule simplifies to $\hat{i}_t = \delta\hat{\pi}_t$. From the Euler equation (and given that the real interest rate is constant) we obtain $\delta\hat{\pi}_t = E_t\hat{\pi}_{t+1}$. Standard arguments then imply that inflation is a backward-looking process: following a one-off shock at time t inflation evolves according to $\hat{\pi}_{t+j} = \delta^j\hat{\pi}_t$ where the initial condition $\hat{\pi}_t$ is pinned down uniquely by the date

[27] Put differently, by tracking r_t^n, the planner can eliminate the shock from the Euler equation when $\widetilde{Y}_t = \widetilde{Y}_{t+1} = 0$.

t intertemporal debt constraint. To understand why a policy with these properties is optimal, note first that under quasilinear utility the output and spending terms drop out of the right-hand side of (E.73). As a result, because real long-bond prices are constant, the output path is (nearly) irrelevant for the satisfaction of (E.73).[28] Hence, satisfaction of (E.73) is achieved through inflation (the direct effect discussed above), and the planner optimally chooses policy along this margin.

An inflation coefficient equal to the coupon decay rate, δ, is therefore optimal, as it ensures that inflation reverts to steady state at the same rate as coupon payments decline. If the policy rule reacts more aggressively to inflation, inflation persistence under the interest rate rule becomes excessive: inflation remains far from zero even when coupon payments are nearly exhausted. Such elevated future inflation contributes little to debt sustainability. Conversely, if the reaction is weaker than δ, inflation becomes excessively frontloaded and insufficiently persistent. In this case, satisfying the intertemporal budget constraint requires higher inflation in period t. This path, however, fails to fully exploit the maturity structure of debt. A smoother and more persistent inflation path—achieved by relying more heavily on adjustments in the real value of future coupon payments—allows the fiscal inflation burden to be spread more efficiently over time.

Consider now the case $\sigma \geq 1$, in which output plays a role in fiscal solvency. When designing the optimal policy rule, the planner therefore internalizes the indirect effect of inflation on the output path (through the Phillips curve) and, consequently, on real interest rates and real long-bond prices. The term $\left((\delta-1)(1-\beta\delta)\frac{\sigma\omega\overline{C}^{-1}}{(\sigma\frac{\overline{Y}}{\overline{C}}+\gamma)}\right)$ captures this additional policy margin. Its magnitude declines as δ increases, because intertemporal distortions in the output path induce offsetting changes in both the real interest rates that discount government surpluses and the real prices of long bonds that determine the market value of outstanding debt. When long bonds take the form of consols, this margin becomes irrelevant: any change in long-bond prices is exactly offset by a corresponding change in the present value of surpluses, leaving (E.73) unchanged.

Finally, note that policy rules similar to the rule characterized in Proposition E.1 are derived by Chafwehé et al. (2024) in the context of the fiscal theory of the price level. Unlike our framework, Chafwehé et al. (2024) assume constant lump-sum taxes. Although the optimal policy problems differ, their more extensive discussion provides additional intuition for the structure of optimal policy rules. To conserve space, we therefore refer the reader to that paper.

E.3 Distorted steady state.

We now derive an analogous result for the case of a distorted steady state. Before solving the full model, we first consider optimal policy in the baseline New Keynesian economy assuming that the consolidated budget is not a constraint for the planner. This allows us to briefly revisit the analysis of Benigno and Woodford (2005) and show how the target level of output and the optimal target criterion are modified in this setting. Our derivations in this subsection follow their logic, but differ in one (potentially important) respect: rather than using a quadratic approximation to household welfare and a quadratic approximation to the Phillips curve, we characterize optimal policy directly from the linear first-order conditions of the nonlinear planning problem. Likewise, when we will extend our analysis to incorporate the government budget constraint in the next subsection, our approach will differ from that of Leeper and Zhou (2021), who derive the policy objective function by combining second order approximations of the Phillips curve and the government's intertemporal budget constraint.

[28] Output may still matter through the surplus term on the left-hand side; the optimal policy rule internalizes this channel via the stochastic intercept.

Consider an optimal policy equilibrium in which the government intertemporal debt constraint is satisfied in each t. The planner is assumed to have access to lump-sum taxation so that the consolidated budget constraint does not affect optimal inflation policy; the distortionary tax rate can be positive but is assumed constant. $\widetilde{\epsilon} < 1$.

The steady-state multiplier $\overline{\psi}_{PC}$ solves

$$1 - \widetilde{\epsilon} + \overline{\psi}_{PC}\gamma\frac{1+\eta}{\omega} + \overline{\psi}_{PC}\frac{1+\eta}{\omega}\sigma\frac{\overline{Y}}{\overline{C}} = 0$$

and therefore

$$\overline{\psi}_{PC} = \frac{1-\widetilde{\epsilon}}{-\gamma\frac{1+\eta}{\omega} - \frac{1+\eta}{\omega}\sigma\frac{\overline{Y}}{\overline{C}}} = \frac{1-\widetilde{\epsilon}}{\kappa} > 0$$

where, again, κ is the slope of the Phillips curve.

The resource constraint multiplier satisfies

$$\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}} = -\widetilde{\epsilon} + \overline{\psi}_{PC}\gamma\frac{\eta}{\omega}\frac{\widetilde{\epsilon}}{1-\overline{\tau}} = -\widetilde{\epsilon} + \frac{\widetilde{\epsilon}-1}{\gamma + \sigma\frac{\overline{Y}}{\overline{C}}}\gamma < 0$$

Using our previous derivations (from the full Ramsey program but canceling terms that derive from the debt constraint) it is straightforward to show that optimal output solves the following condition:

$$-\sigma\hat{C}_t - \widetilde{\epsilon}\gamma\hat{Y}_t + \overline{\psi}_{PC}\frac{1+\gamma}{1-\overline{\tau}}\frac{\eta}{\omega}\widetilde{\epsilon}\gamma\hat{Y}_t + \overline{\psi}_{PC}\frac{1+\eta}{\omega}\sigma\hat{C}_t - \overline{Y}\overline{\psi}_{PC}\frac{1+\eta}{\omega}\sigma\overline{C}^{-1}((1+\sigma)\hat{C}_t - \hat{Y}_t) + (\widetilde{\epsilon}-1)\hat{\psi}_{PC,t} = 0$$

which, after substituting out consumption, can be written as:

$$a_1\hat{Y}_t + a_2\hat{G}_t + (\widetilde{\epsilon}-1)\hat{\psi}_{PC,t} = 0$$

$$a_1 = -\widetilde{\sigma} - \widetilde{\epsilon}\gamma - \frac{(1-\widetilde{\epsilon})}{\gamma+\widetilde{\sigma}}\left[(1+\gamma)\gamma + \widetilde{\sigma}(1-\widetilde{\sigma}+1-s_c^{-1})\right]$$

$$a_2 = \widetilde{\sigma}(1-s_c) + \frac{(1-\widetilde{\epsilon})}{\gamma+\widetilde{\sigma}}\left[(\widetilde{\sigma}(1-s_c) - \widetilde{\sigma}(s_c^{-1} - 1 + \widetilde{\sigma}(1-s_c))\right]$$

where s_c denotes the share of consumption over output in the steady state, $\widetilde{\sigma} = \sigma\frac{\overline{Y}}{\overline{C}}$ and $\widetilde{\sigma}(1-s_c) = \sigma\frac{\overline{G}}{\overline{C}}$

Combining this expression with the optimality condition for inflation ($\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}\hat{\pi}_t + \overline{\psi}_{PC}\Delta\hat{\psi}_{PC,t} = 0$) we get

$$\omega(\widetilde{\epsilon} + \frac{1-\widetilde{\epsilon}}{\gamma + \sigma\frac{\overline{Y}}{\overline{C}}}\gamma)\hat{\pi}_t + \frac{\widetilde{\sigma} + \widetilde{\epsilon}\gamma + \frac{(1-\widetilde{\epsilon})}{\gamma+\widetilde{\sigma}}\left[(1+\gamma)\gamma + \widetilde{\sigma}(1-\widetilde{\sigma}+1-s_c^{-1})\right]}{\kappa}\Delta\widetilde{Y}_t = 0 \tag{E.77}$$

Equation (E.77) is the target policy criterion for this model (the analogue of $\omega\hat{\pi}_t + \frac{\sigma\frac{\overline{Y}}{\overline{C}}+\gamma}{\kappa}\Delta\widetilde{Y}_t = 0$ in the case of the efficient steady state). As emphasized by Benigno and Woodford (2005), the presence of steady-state distortions alters both the relative weights placed on inflation stabilization and real activity stabilization in the welfare objective and the definition of the welfare-relevant output gap.

In the distorted steady state, the output gap is given by $\widetilde{Y}_t = \hat{Y}_t + \frac{a_2}{a_1}\hat{G}_t$ where the coefficients a_1 and a_2 depend on the underlying structural distortions as defined above.

Distorted Steady state: Full model. In the presence of a binding budget constraint the optimal target criterion needs to be modified to include the Lagrange multipliers $\hat{\psi}_{gov}$. We begin by deriving the relevant formula. Subsequently, we derive an optimal rule for monetary policy. We again establish that optimal monetary policy is passive.

The steady-state expression for the Lagrange multiplier $\overline{\psi}_{PC}$ is

$$\overline{\psi}_{PC} = \frac{1-\widetilde{\epsilon}}{-\gamma\frac{1+\eta}{\omega} - \frac{1+\eta}{\omega}\sigma\frac{\overline{Y}}{\overline{C}} + \frac{\eta}{\omega}\left(\frac{\overline{\tau}}{1-\overline{\tau}}(1+\gamma)\widetilde{\epsilon} + \sigma\frac{\overline{G}}{\overline{C}}\right)} = \frac{1-\widetilde{\epsilon}}{\kappa + \frac{\eta}{\omega}s_{\hat{Y}}} > 0 \tag{E.78}$$

where $s_{\hat{Y}} > 0$ denotes the elasticity of the surplus (scaled by marginal utility) with respect to output. Moroever,

$$\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}} = -\widetilde{\epsilon} + \overline{\psi}_{PC}\frac{\eta}{\omega}\frac{\widetilde{\epsilon}}{1-\overline{\tau}}\Big[\gamma - \overline{\tau}(1+\gamma)\Big] = -\widetilde{\epsilon} + \frac{\widetilde{\epsilon}-1}{\gamma + \widetilde{\sigma} - \frac{\eta}{1+\eta}s_{\hat{Y}}}\Big[\gamma - \overline{\tau}(1+\gamma)\Big]$$

From previous derivations (repeated for convenience) we have that:

$$a_1\hat{Y}_t + a_2\hat{G}_t + (\widetilde{\epsilon}-1)\hat{\psi}_{gov,t} + \overline{\psi}_{gov}\sigma\bar{b}_L\overline{C}^{-1}\sum_{j\geq 0}\delta^j(\hat{\psi}_{gov,t-j} - \hat{\psi}_{gov,t-j-1}) = 0$$

Using the notation of the previous paragraph coefficients a_1, a_2 can be written as:

$$a_1 = -\widetilde{\sigma} - \gamma\widetilde{\epsilon} + \frac{\widetilde{\epsilon}-1}{\gamma + \widetilde{\sigma} - \frac{\eta}{1+\eta}s_{\hat{Y}}}\Big[\gamma - \overline{\tau}(1+\gamma)\Big]\Big[(1+\gamma)\gamma(1-\overline{\tau}) + \widetilde{\sigma}(1-\widetilde{\sigma}+1-s_c^{-1}) + \frac{\eta}{\eta+1}\widetilde{\sigma}(1-s_c)(\widetilde{\sigma}+s_c^{-1})\Big]$$

$$a_2 = \widetilde{\sigma}(1-s_c) + \frac{1-\widetilde{\epsilon}}{\gamma + \widetilde{\sigma} - \frac{\eta}{1+\eta}s_{\hat{Y}}}\Big[\widetilde{\sigma}(1-s_c) - \widetilde{\sigma}(s_c^{-1} - 1 + \widetilde{\sigma}(1-s_c)) + \frac{\eta}{1+\eta}\widetilde{\sigma}(1-s_c)(s_c^{-1} + \widetilde{\sigma}(1-s_c))\Big]$$

Putting all of this together we get the following solution for the output gap in this model:

$$\widetilde{Y}_t = (\hat{Y}_t + \frac{a_2}{a_1}\hat{G}_t) = -\frac{(\widetilde{\epsilon}-1)}{a_1}\hat{\psi}_{gov,t} - \frac{\overline{\psi}_{gov}}{a_1}\sigma\bar{b}_L\overline{C}^{-1}\sum_{j\geq 0}\delta^j(\hat{\psi}_{gov,t-j} - \hat{\psi}_{gov,t-j-1}) = 0$$

The target policy criterion can be found by combining the above equation with the optimality condition for inflation. The latter object is:

$$\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}\hat{\pi}_t + \overline{\psi}_{PC}\Delta\hat{\psi}_{PC,t} + \overline{\psi}_{gov}\frac{1}{1-\beta\delta}\sum_{j\geq 0}\delta^j\bar{b}_L\Delta\hat{\psi}_{gov,t-j} = 0$$

Using the above solutions we get

$$\omega(\widetilde{\epsilon} + \frac{1-\widetilde{\epsilon}}{\gamma + \widetilde{\sigma} - \frac{\eta}{1+\eta}s_{\hat{Y}}}[\gamma - \overline{\tau}(1+\gamma)])\hat{\pi}_t + (\frac{-a_1}{\kappa + \frac{\eta}{\omega}s_{\hat{Y}}})\Delta\widetilde{Y}_t =$$

$$\underbrace{+(\frac{\overline{\psi}_{gov}}{\kappa + \frac{\eta}{\omega}s_{\hat{Y}}})\widetilde{\sigma}\bar{b}_L\sum_{j\geq 0}\delta^j(\Delta\hat{\psi}_{gov,t-j} - \Delta\hat{\psi}_{gov,t-j-1})}_{\text{Real bond price/interest rate effect}} + \underbrace{\overline{\psi}_{gov}\frac{1}{1-\beta\delta}\sum_{j\geq 0}\delta^j\bar{b}_L\Delta\hat{\psi}_{gov,t-j}}_{\text{Direct inflation effect}} = 0$$

Equation (E.3) is the analogue of (E.72) derived previously. As in the case of the efficient allocation, the optimal tradeoff between inflation and output gap is a function of the current and lagged growth of Lagrange multipliers $\hat{\psi}_{gov}$. As we explained previously, the two summations on the RHS of (E.3) reflect two features of optimal policy. The second term, 'Direct effect' is the impact of inflation on the real value of government debt. The leading term measures the effect of inflation on output and through this on the real long bond price/ the market value of debt and on the present discounted value of surpluses (discounted at the real interest rate).

Optimal interest rate rule in the distorted steady state. To derive the optimal interest rate rule, we utilize the solution for inflation and the output gap:

$$\hat{\pi}_t = \frac{\eta}{\omega}\frac{\overline{\psi}_{PC}}{\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}}\Delta\hat{\psi}_{gov,t} - \frac{\overline{\psi}_{gov}}{\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}}\frac{1}{1-\beta\delta}\sum_{j\geq 0}\delta^j\overline{b}_L\Delta\hat{\psi}_{gov,t-j} \tag{E.79}$$

and

$$\widetilde{Y}_t = -\frac{(\widetilde{\epsilon}-1)}{a_1}\hat{\psi}_{gov,t} - \frac{\overline{\psi}_{gov}}{a_1}\sigma\overline{b}_L\overline{C}^{-1}\sum_{j\geq 0}\delta^j(\Delta\hat{\psi}_{gov,t-j})$$

We then have:

$$E_t\hat{\pi}_{t+1} = -\delta\frac{\overline{\psi}_{gov}}{\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}}\frac{1}{1-\beta\delta}\sum_{j\geq 0}\delta^j\overline{b}_L\Delta\hat{\psi}_{gov,t-j} = \delta(\hat{\pi}_t - \frac{\eta}{\omega}\frac{\overline{\psi}_{PC}}{\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}}\Delta\hat{\psi}_{gov,t})$$

and

$$\widetilde{\sigma}E_t(\widetilde{Y}_{t+1} - \widetilde{Y}_t) = -(\delta-1)\frac{\overline{\psi}_{gov}}{a_1}\widetilde{\sigma}\overline{b}_L\overline{C}^{-1}\sum_{j\geq 0}\delta^j(\Delta\hat{\psi}_{gov,t-j})$$

Putting these results together we get:

$$E_t\hat{\pi}_{t+1} + \widetilde{\sigma}E_t(\widetilde{Y}_{t+1} - \widetilde{Y}_t) = \delta(\hat{\pi}_t - \frac{\eta}{\omega}\frac{\overline{\psi}_{PC}}{\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}}\Delta\hat{\psi}_{gov,t}) - (\delta-1)\frac{\overline{\psi}_{gov}}{a_1}\widetilde{\sigma}\overline{b}_L\sum_{j\geq 0}\delta^j(\Delta\hat{\psi}_{gov,t-j})$$

$$\delta(\hat{\pi}_t - \frac{\eta}{\omega}\frac{\overline{\psi}_{PC}}{\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}}\Delta\hat{\psi}_{gov,t}) + (\delta-1)(1-\delta\beta)\frac{\widetilde{\sigma}}{a_1}\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}(\hat{\pi}_t - \frac{\eta}{\omega}\frac{\overline{\psi}_{PC}}{\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}}\Delta\hat{\psi}_{gov,t}) =$$

$$\underbrace{(\delta + (\delta-1)(1-\delta\beta)\frac{\widetilde{\sigma}}{a_1}\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}})}_{=h(\delta)}\hat{\pi}_t - h(\delta)\frac{\eta}{\omega}\frac{\overline{\psi}_{PC}}{\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}}\Delta\hat{\psi}_{gov,t}$$

The following proposition summarizes:

Proposition E.2. *Assume a distorted steady state. The optimal interest rate rule is*

$$\hat{i}_t = h(\delta)\hat{\pi}_t - h(\delta)\frac{\eta}{\omega}\frac{\overline{\psi}_{PC}}{\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}}\Delta\hat{\psi}_{gov,t} + \widetilde{r}_t$$

where

$$\widetilde{r}_t = -\sigma\frac{\overline{G}}{\overline{C}}E_t(\hat{G}_{t+1} - \hat{G}_t) - \widetilde{\sigma}\frac{a_2}{a_1}E_t(\hat{G}_{t+1} - \hat{G}_t)$$

is the appropriate real rate consistent with the output target in this model. Moreover $h(\delta) \equiv \delta + (\delta - 1)(1-\delta\beta)\frac{\widetilde{\sigma}}{a_1}\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}) < 1$.

Proposition E.2 confirms that, in the distorted steady state, optimal monetary policy follows a rule, in which the inflation coefficient is strictly less than one. As in the case of the efficient steady state, optimal monetary policy is passive.

E.4 Optimal Taxation.

We now characterize the optimal tax schedule in the model. The formulas derived here are complementary to those presented in Section B.1, where we study optimal taxation in the nonlinear model. In contrast to that analysis, the results in this section abstract from financial frictions and do not impose fully rigid prices.

From the Phillips curve, the tax rate in period t satisfies:

$$-\frac{1+\eta}{\omega}\frac{\overline{\tau}}{1-\overline{\tau}}\hat{\tau}_t = \hat{\pi}_t - \kappa\hat{Y}_t - \beta E_t\hat{\pi}_{t+1} - \sigma\frac{1+\eta}{\omega}\frac{\overline{G}}{\overline{C}}\hat{G}_t =$$

Using the previous derivations for the processes of inflation and the output gap, it is straightforward to derive a closed-form solution expressing the tax rate in t as a function of the Lagrange multipliers. We get:

$$\overbrace{-\frac{1+\eta}{\omega}\frac{\overline{\tau}}{1-\overline{\tau}}}^{>0}\hat{\tau}_t = \overbrace{\frac{\eta}{\omega}\frac{\overline{\psi}_{PC}}{\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}}\Delta\hat{\psi}_{gov,t}}^{\text{Stochastic Intercept}} + \overbrace{(-\frac{\overline{\psi}_{gov}}{\omega\frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}} + \kappa\frac{\overline{\psi}_{gov}}{a_1}\sigma\overline{C}^{-1})\overline{b}_L\sum_{j\geq 0}\delta^j\Delta\hat{\psi}_{gov,t-j}}^{\text{Intertemporal Distortions}}$$
$$+ \underbrace{\kappa\frac{(\widetilde{\epsilon}-1)}{a_1}}_{\text{Random Walk}}\hat{\psi}_{gov,t} - \underbrace{(\sigma\frac{1+\eta}{\omega}\frac{\overline{G}}{\overline{C}} - \kappa\frac{a_2}{a_1})}_{\text{Markup Management}}\hat{G}_t \tag{E.80}$$

Equation (E.80) highlights four components of the optimal tax schedule.

First, consider the (perhaps less interesting) term labeled *Stochastic Intercept*. This term operates through the Phillips curve. Because the path of taxes affects inflation, an increase in the tax rate at date t raises current inflation (abstracting from the output effects, which the planner has already set optimally). The Stochastic Intercept ensures consistency between the planner's solution and the policy promises made in $t-1$ which determined the conditional expectation $E_{t-1}\hat{\pi}_t$. This margin depends solely on the realization of the shock at date t since, as we highlighted previously, $\Delta\hat{\psi}_{gov,t} = \nu\epsilon_{G,t}$.

Next, consider the term labeled *Markup Management* (also appeared in subsection B.1 for the model with financial frictions). At the efficient equilibrium we have:

$$\underbrace{\left(\sigma\frac{1+\eta}{\omega}\frac{\overline{G}}{\overline{C}} - \kappa\frac{a_2}{a_1}\right)}_{=0},$$

and so this component disappears. Under our preference specification (constant Frisch elasticity of labor supply), optimal taxes do *not* vary with government spending when the economy is free of distortions from monopolistic competition. However, in a distorted steady state, spending directly influences the optimal tax schedule. Intuitively, monopolistic competition introduces a wedge between the marginal rate of substitution and the marginal product of labor. The government therefore uses

taxes to partially manage this markup-induced wedge over the business cycle (e.g. Schmitt-Grohé and Uribe, 2004, among others.)

A similar reasoning applies to the term labeled *Random Walk*. A standard result in the Ramsey-optimal taxation literature is that taxes follow a random walk (Aiyagari et al., 2002), reflecting the planner's desire to smooth the welfare losses associated with tax distortions over time. When $\widetilde{\epsilon} = 1$, however,

$$\kappa \frac{(\widetilde{\epsilon} - 1)}{a_1} = 0,$$

and this property no longer holds in the neighborhood of the efficient steady state. In such a context, allowing taxes to drift permanently away from the efficient steady state level increases the distortions generated by monopolistic competition.

Finally, the term labeled 'Intertemporal Distortions' corresponds to the familiar D_3 term of subsection B.1. Under the efficient allocation we have $\overline{b}_L < 0$ and (for a plausible parameterization of the model)

$$(-\frac{1}{\omega \frac{\overline{\psi}_{RC}}{\overline{C}^{-\sigma}}} + \frac{\kappa}{a_1}\sigma \overline{C}^{-1}) \approx \frac{1}{\omega}(1 + (1+\eta)\widetilde{\sigma}) < 0$$

Consequently, a positive spending shock at time t leads to an initial increase in taxes, followed by a gradual decline.

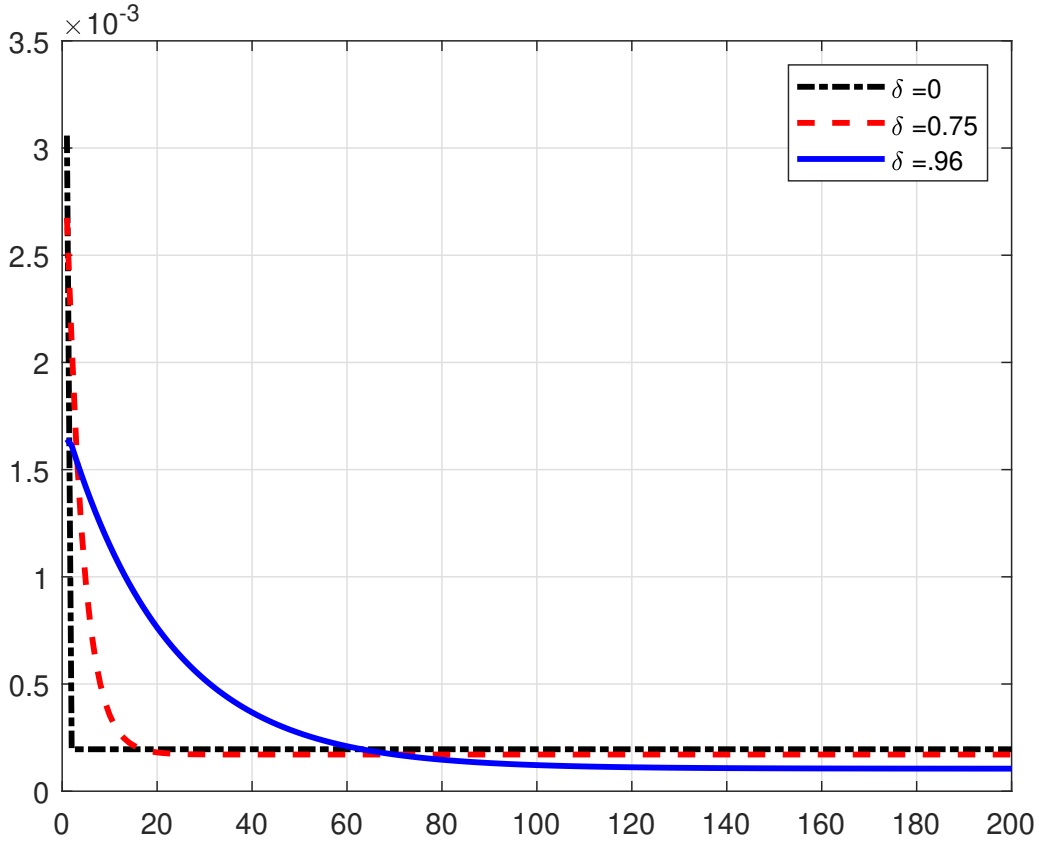
As was explained in previously, through this channel, the planner introduces intertemporal distortions in the tax schedule that reprice long-term government bonds and adjust the market value of public debt so as to absorb the fiscal shock.

By contrast, when public debt is positive—as in a distorted steady state—the planner initially cuts taxes and subsequently raises them over time. This policy lowers the market value of outstanding government debt at the time the shock occurs (Faraglia et al., 2016).

F Optimal Inflation Targets: Numerical Output

This subsection examines the implications of the model for the behavior of optimal inflation targets. We begin by presenting numerical solutions that illustrate the dynamic response of inflation to fiscal shocks (Section F.1). We then analyze how debt maturity shapes both the transition dynamics and the evolution of the inflation target, highlighting the role of financial frictions and liquidity rents (Section F.2). Finally, we compare the statistical properties of inflation and taxation across the model with financial frictions and the canonical Ramsey benchmark (Section F.3). Together, these results provide a quantitative assessment of the mechanisms discussed in the main text and in the analytical sections of the appendix.

Figure 4: Inflation Targets: Quasilinear Model



Notes: The figure shows the impulse response function of inflation to a large spending shock in the quasilinear utility model. The blue line sets $\delta = .96$ our baseline value, the red line sets the maturity of non-safe and illiquid debt to 1 year and the black dotted line to one model period.

F.1 Numerical Solution

We investigate the response of inflation to a positive spending shock in the model with quasi-linear preferences. Figure 4 plots the response (blue line).[29]

[29] We consider a spending shock that is sufficiently large to induce an increase in inflation on impact that is comparable to the analogous increase we observe in the nonlinear model under our baseline calibration. See below.

For the quasi-linear model illustrated in this paragraph, we adjust the parameters relative to the baseline calibration in order to obtain a positive inflation target (2 percent per annum) and a ratio of short to long term bonds close to the data and term premium close to the baseline. Achieving these targets required a lower value of ω and a higher

The figure shows the two key properties of optimal inflation. First, there is a transition period over which inflation decays at rate $\frac{\delta}{\bar{\pi}}$ (with dynamics governed by the term $\sum_{j\geq 0}(\frac{\delta}{\bar{\pi}})^j \Delta\hat{\psi}_{gov,t-j}$). Second, inflation exhibits a time-varying target, in that it does not revert to its steady-state level even after 200 model periods.

F.2 Debt Maturity and Inflation Dynamics

The black dotted and red dashed lines isolate the effect of varying debt maturity on these properties. A lower value of the maturity parameter δ amplifies the impact response of inflation and shortens the transition period. Moreover, it strengthens the response of the inflation target.

The first set of observations is consistent with the effects of shortening debt maturity in the canonical model. The intuition follows directly from the government's intertemporal budget constraint. If all government debt (including illiquid debt) is short term, then only contemporaneous inflation can absorb fiscal shocks. By contrast, when the government issues long-term debt, a rise in government spending can be offset by both current and future inflation, which jointly reduce the real value of outstanding debt. In standard models, longer debt maturity therefore implies smaller movements of inflation on impact and greater inflation persistence, as the planner optimally smooths the inflationary burden over time to absorb fiscal shocks efficiently.

The introduction of financial frictions activates a second channel: a time-varying endogenous inflation target. Inflation acquires a random-walk component driven by movements in the target. Interestingly, the lower the average debt maturity, the stronger is the response of the inflation target to the shock.

As explained in the main text, in the presence of financial frictions, the left-hand side (LHS) of the government's intertemporal budget constraint depends on inflation. Higher permanent inflation relaxes the constraint by increasing the value of the rents extracted from the provision of safe and liquid assets. According to our numerical results, the planner relies more heavily on this channel when the average maturity of government debt is shorter, thereby compensating for the weaker shock-absorption role of inflation on the right-hand side (RHS) of the constraint.

Curved utility. The top panel of Figure (5) reproduces the transition path shown in Figure (4) under the baseline calibration, extending the impulse response to 200 model periods. Consistent with the results shown in the main text, the optimal inflation response to a fiscal shock continues to exhibit a random-walk component.

The middle and bottom panels of Figure (5) display the responses of short- and long-term government bonds, respectively. Consistent with the results shown in text,[30] an increase in government spending induces the planner to reduce the supply of short-term bonds. The resulting decline in short-term debt, together with the higher fiscal deficit, is financed through greater issuance of long-term debt.

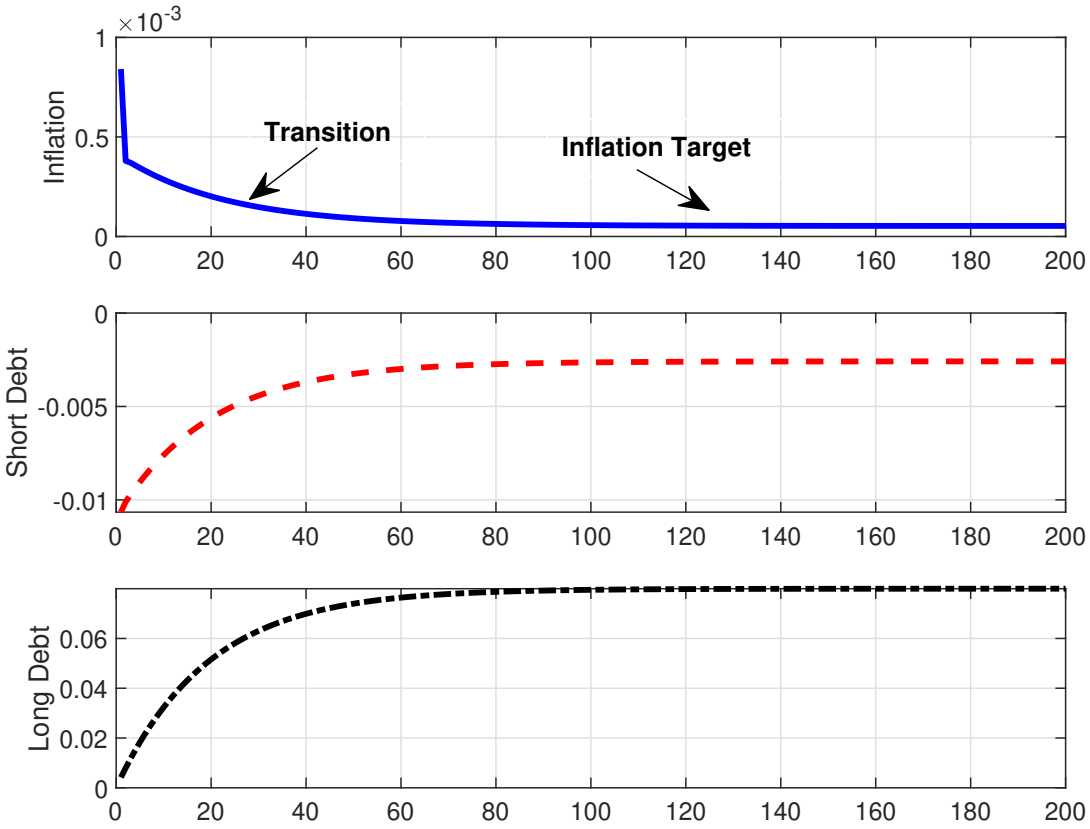
Varying the quantity of total/long-term debt. Figure (6) illustrates how changes in the maturity structure—implemented by varying the steady-state level of long-term debt—affect inflation dynamics. The solid blue line corresponds to a scenario with a large supply of long-term bonds. The

value of γ than in the baseline calibration. Interestingly, setting $\gamma = 1$ implies that long-term debt is negative in the steady state for a reasonable term premium.

Since our goal with the quasi-linear model is to illustrate properties for which we have already provided analytical results we did not recalibrate the model fully to match the evidence in Greenwood, Hanson, and Stein (2015). In contrast to the analytical results in the previous subsection, we assume that f is log-normal, as in the baseline model.

[30] In text, these responses were shown using the nonlinear model. The analytical result in Proposition 5 was derived under quasi-linear utility.

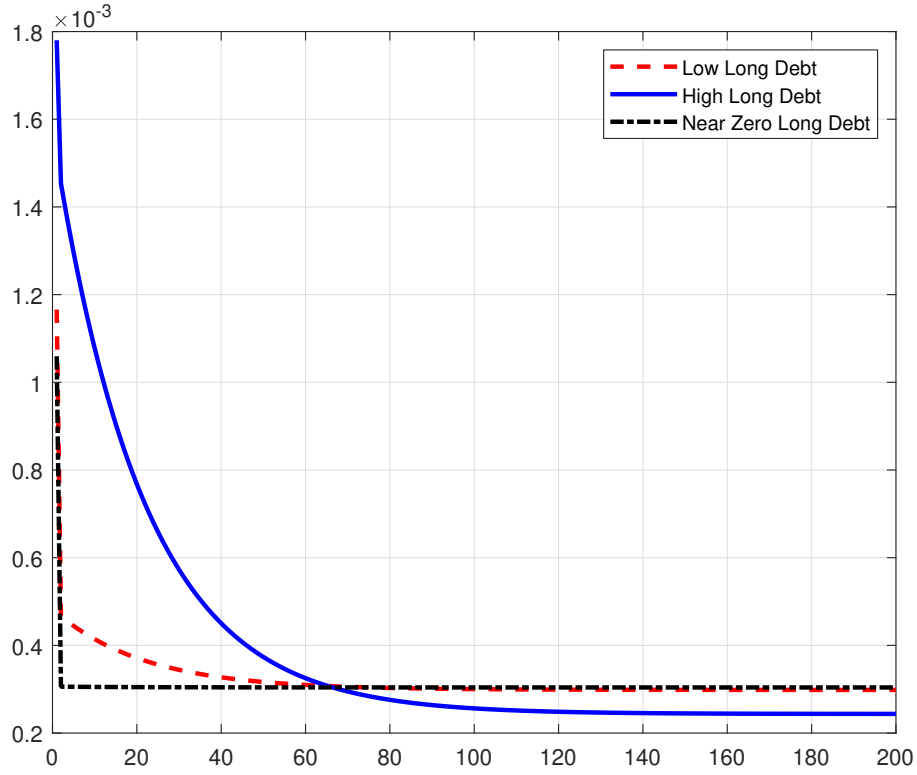
Figure 5: Inflation Target, Short and Long bond IRFS: Baseline



Notes: The figure shows the impulse responses of inflation, short-term and long-term bonds to a spending shock, in the baseline model.

black dotted line represents a case in which $\bar{b}_L$ is close to zero, while the red line lies between these two extremes, with $\bar{b}_L$ taking an intermediate value (approximately half of the baseline calibration).[31]

Figure 6: Inflation Targets and the Supply of Long Bonds: Quasilinear Model



Notes: The figure shows the impulse response function of inflation to a large spending shock in the quasi-linear utility model. Each of the three lines corresponds to a different steady state value of $\bar{b}_L$. The legend explains the mapping between colours and long debt levels.

Qualitatively, the responses are similar to those shown in Figure (4). Hence, a shorter maturity of debt—achieved either through a lower value of δ or a lower quantity of long-term bonds $\bar{b}_L$—has a similar effect on the path of inflation. There is, however, one striking difference in the impact response of inflation.

When the government's portfolio is tilted toward shorter-term bonds but this debt is predominantly liquid and safe (i.e., following a reduction in $\bar{b}_L$), the period-0 response of inflation is weaker than in the case in which a shorter maturity arises from a reduction in δ. Similarly, reducing total debt attenuates the inflationary response when most of the outstanding debt provides liquidity and safety services. As noted above, in canonical models, the opposite predictions hold: shortening the maturity or reducing total government debt leads to larger fluctuations in inflation.

The key difference lies in the response of short-term debt in the presence of financial frictions. As discussed above, short-term bonds act as a shock absorber when frictions are present, allowing the government to mitigate the inflationary effects of fiscal shocks by adjusting the supply of liquid and

[31]To calibrate the model, we solve for the corresponding Ramsey steady states conditional on the quantity of long-term bonds. Consistent with our analytical findings, a reduction in long debt leads to a higher steady-state value of $\widetilde{\theta}$. At the same time, the total quantity of government debt in the economy declines. Consequently, Figure (6) can also be interpreted as illustrating the effects of changes in overall government debt on the equilibrium properties of inflation. With binding frictions, reducing long-term bonds is the only way to lower total government debt in the steady state, since $\bar{b}_S$ is endogenously determined.

safe assets. When shortening debt maturity shifts the portfolio toward liquid and safe assets, the planner can rely more heavily on adjustments in short-term bonds to absorb shocks. This reduces the required adjustment of inflation on impact, though as is evident in the figure, this strengthens the time-varying target channel.

F.3 Statistical Properties of Inflation and Taxes

We now compare the properties of equilibrium inflation and taxes in the baseline model with financial frictions and in the canonical Ramsey model. The content of this paragraph was referenced in Section 3 of the main text.

Panel A of Table F1 reports the standard deviation of inflation (in percentage points), the standard deviation of taxes, and the first-order autocorrelation of inflation under alternative levels of long-term debt in the models with and without financial frictions.[32] The "Benchmark Long Debt" scenario corresponds to the calibration used in the steady-state analysis and in the first column of Table 1 in text. The "Near-Zero Long Debt" specification sets $\bar{b}_L = 0.001$.[33]

Even though inflation is more volatile and more persistent under financial frictions, the quantitative differences relative to the canonical model are modest. In both models, taxes display substantially higher volatility than inflation. However, tax volatility is lower in the presence of financial frictions—a finding consistent with the impulse responses shown in Section 3 of the main text.

Panel B of the table considers an alternative calibration designed to amplify the differences between the two models. In particular, we assume a higher degree of price flexibility relative to the baseline, while maintaining a strictly positive resource cost of price adjustment.[34] Under this specification, inflation volatility is an order of magnitude higher in the presence of financial frictions when long-term debt is near zero. The gap in inflation persistence between the two models is also substantial.

Table F1: Inflation and Tax Moments

		Std. Dev. Inflation (%)	Std. Dev. Taxes	Autocorr. Inflation
Panel A: Baseline Price Stickiness				
Benchmark Long Debt	Frictions	0.20	0.25	0.86
	Canonical	0.17	0.27	0.74
Near-Zero Long Debt	Frictions	0.09	0.29	0.62
	Canonical	0.06	0.30	0.03
Panel B: More Flexible Prices				
Benchmark Long Debt	Frictions	2.54	0.15	0.99
	Canonical	1.25	0.13	0.94
Near-Zero Long Debt	Frictions	3.37	0.22	0.99
	Canonical	0.16	0.30	0.08

Figure (7) complements the table by reporting the unconditional standard deviations of optimal inflation in the two models (left panels): the model with financial frictions (solid blue lines) and the canonical Ramsey model (dashed red lines), plotted as functions of the price-rigidity parameter ω.

[32] We compute these moments from a long simulated sample (20000 observations).

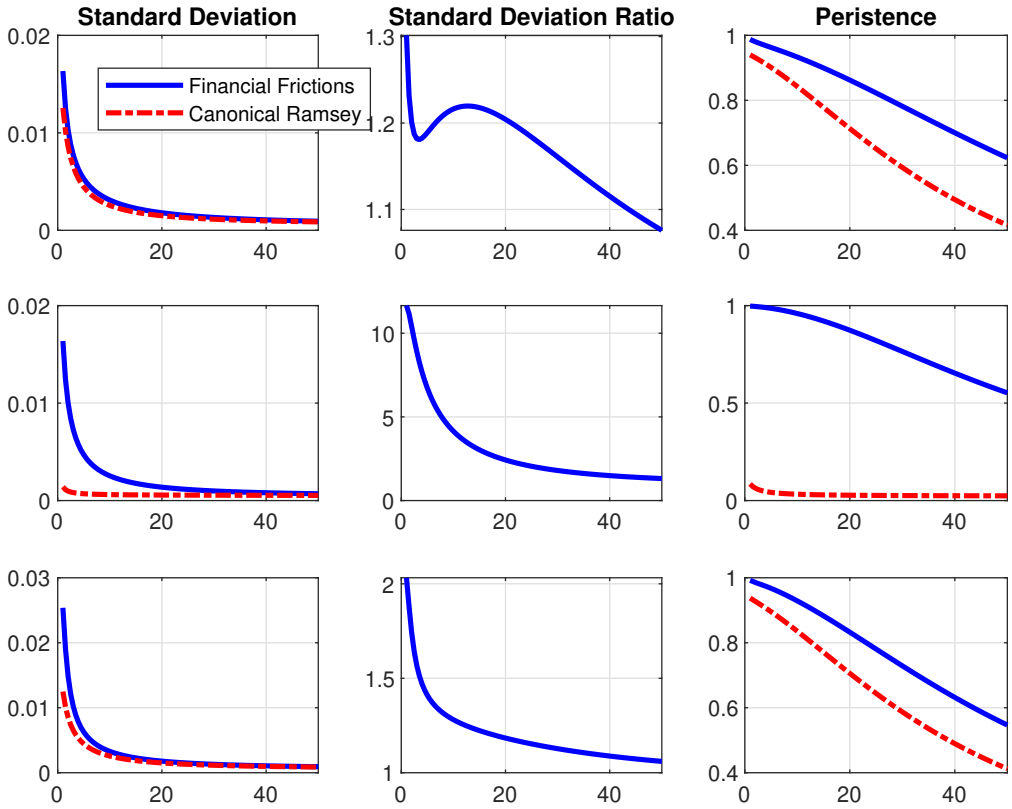
[33] This case is analogous to the Zero Long Debt specification in Table 1 of the main text. However, because we solve the model using a log-linear approximation, we calibrate it with a small but strictly positive value of b_L.

[34] To provide a sense of magnitudes, the parameter ω is set to approximately 5 percent of its baseline value. Fully flexible prices would correspond to $\omega = 0$, in which case inflation becomes indeterminate under optimal policy.

The middle panels show the ratios of standard deviations across the two models. The top panels correspond to the case of positive long debt (baseline). The middle panels set $\overline{b}_L = .001$.

Finally, in the bottom panels of Figure (7) we calibrate the model assuming that a higher standard deviation of $\widetilde{\theta}$ (equal to 0.2 and representing a tenfold increase relative to the baseline). Quantitatively, this change generates only small differences relative to the baseline results.

Figure 7: Inflation Process



Notes: The figure reports statistical properties of the optimal inflation in the model with financial frictions (solid blue lines) and the canonical Ramsey model (dashed red lines). The left panels plot the unconditional standard deviations of the logarithm of inflation (computed using a simulation with 20,000 observations). The middle panels show the ratios of the standard deviations. The right panel report the persistence of inflation, computed as the first-order autocorrelation coefficients. The x-axis measures the degree of price stickiness ω. The top panels are the baseline calibration of the model. The middle panels set long term debt to a very small number $\overline{b}_L = 0.01$. The bottom panels assume a higher variance of the idiosyncratic preference shock.

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