Accounting for changes in college enrollment and the skill premium over time*

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Abstract

The skill premium and college enrollment have increased substantially over the past few decades in the US. Accounting for variations of the skill premium and college enrollment has proven elusive. We show that a parsimonious neoclassical model featuring skill-biased technical change, endogenous education and labor supply decisions, calibrated only to initial conditions can explain simultaneously the change in the US college education rate and the skill premium between 1967 and 2000. We show analytically and quantitatively that endogenous labor supply is crucial. On the one hand, the increase in the skill premium leads to an increase in the labor supply of high-skilled workers, raising their earnings and thus strengthening education incentives. On the other hand, these two effects increase the relative supply of skilled labor. This, in turn, depresses the skill premium. Thus, both endogenous labor supply and a quantitative general equilibrium framework are necessary to analyse the evolution of the skill premium and enrollment simultaneously. Assuming constant hours biases the estimates of the effects of skill-biased technological progress on college enrollment by more than 80%.

JEL Classification: I24, J2, J31

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1 Introduction

Figure 1 shows two important developments that took place in the US in the last part of the 20th century. First, the skill premium rose from 50% in the 1960s to 80% by the end of the century. Second, college enrollment increased by a little over ten percentage points, from 52% to 63%. The existing literature accounts for some but not all of these variations. In this paper, we show that a parsimonious quantitative general equilibrium model can explain these developments. We add an intensive labor supply margin to a standard life-cycle model with an endogenous education decision. Intensive labor supply is crucial since at the individual level, working longer hours makes college more worthwhile which strengthens the incentive to go to college. At the aggregate level, however, this behavior leads to an increase in the labor supply of high-skilled workers, depressing the skill premium which lowers the incentive to go to college.

Using a standard life-cycle model with an education decision and endogenous labor supply, we show that skill-biased technological progress leads to an increase in the skill premium and the supply of skilled labor, which can take two forms: on the extensive margin, more households find it worthwhile to go to college; on the intensive margin, increasing college wage premia is an incentive for the college-educated to work more than low-skilled households. We show that ignoring the intensive margin leads to biased estimates of the extensive margin. Increasing life-time earnings by working more hours in the future, when the wage premium is high, induces some high-school graduates, who would otherwise have started to work immediately, to enroll into college.

We calibrate our quantitative model using data moments only from the initial period and not from the final period. Based on these inputs and the exogenous evolution of skill-biased technological progress, the model is able to replicate simultaneously most of the increases in the skill premium and the enrollment rate. The skill premium increases from 51.9% in 1967 to 83.2% in 2000 in the model. The corresponding increase in the data is from 51.5% to 79.5%. Enrollment in the model rises from 52.3% in 1967 to

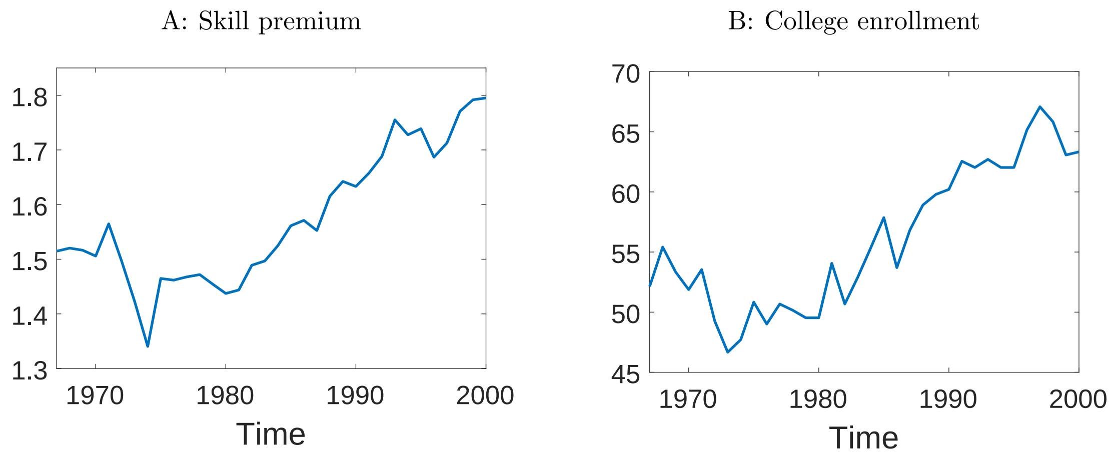


Figure 1: Skill premium and college enrollment, males, US, 1967-2000.
Panel A shows the increase in the skill premium, defined as the wage of college graduates relative to the wage of those without a college degree. Panel B shows the variation in the college enrollment rate, defined as the percentage of high-school educated persons enrolling in college. The data for panel A are from Heathcote, Storesletten, and Violante (2010) and for panel B from He (2012).

64.7% in 2000, and from 52.1% to 63.3% in the data.

Relative to the existing general equilibrium literature, we provide a more accurate account of the rise in education with a simple model addition, which allows to single out factors responsible for the rise both in the skill premium and college enrollment, our main contribution. There are two general equilibrium analyses which can account for education variations. He (2012) accounts endogenously for the rise in the US skill premium between 1950 and 2000 but underestimates the increase in college enrollment by 65%. Restuccia and Vandenbroucke (2013) investigate both US high-school enrollment and college enrollment decisions between 1940 and 2000. Their model replicates the increase in college education between 1940 and 1980 but overpredicts the increase between 1980 and 2000 by more than 45%, despite a small predicted increase in the college premium over the latter period which falls short of empirical observations.

The gap between model projections and data begs the question of missing factors. Neither He (2012) nor Restuccia and Vandenbroucke (2013) include the intensive margin of the labor supply decision. We show that adding this standard feature helps to explain the evolution of the skill premium and college enrollment. We explore the mechanism behind our main quantitative result, verifying the role of labor supply decisions with analytical results and counterfactual quantitative experiments.

Three competing effects influence education decisions.

First, there is the well-known *skill premium effect*. Skill-biased technological progress increases the wage premium for educated workers and thus the incentive to get an education. Second, there is a *relative hours effect*. People who have a medium learning ability but choose not to go to college in a setting where working hours are identical across skill groups may prefer to go to college when working hours can be freely chosen. The possibility of working more hours indeed increases lifetime earnings differentials, making education more attractive. Third, there is a *general equilibrium effect* operating through extensive and intensive margins. If the freedom to choose working hours increases education, the supply of skilled workers will be larger and, in addition, college graduates might want to increase the number of hours they work. Both effects increase the supply of skilled labor, which depresses the skill premium and reduces education incentives.

In a simplified version of the model with exogenous factor price variations, we analytically document the relative hours effect. In particular, we show that if the skill premium increases over time, high-skilled households will increase their working hours relative to low-skilled households.

A quantitative analysis of counterfactual scenarios, where the hours decision is switched off or factor prices are exogenous, allows us to gauge the importance of the three competing effects: skill premium, relative hours and general equilibrium effect. We find that all three effects matter quantitatively for education and inequality outcomes. Ignoring hours decisions would, for instance, lead to an overestimation of the increase in college enrollment by ten percentage points (73.1% instead of 63.3% in 2000) while ignoring general equilibrium responses leads to a skill premium gap of almost thirty percentage points (51.1% instead of 79.5% in 2000).

Our investigation thus shows that skill-biased technological progress and endogenous labor supply decisions along an intensive margins are the two key factors which are needed in a neoclassical framework to account both for the rise of the skill premium and the rise of college enrollment in the US between 1967 and 2000. Other factors, including regulation, preferences and social norms, may also play a role, but only if other phenomenons of the labor market require explanations.

In the next section, we relate our paper to the literature beyond general equilibrium analyses of education variations. Section 3 then presents our model. Section 4 provides analytical results for a simplified version of the model. Section 5 describes the calibration as well as the quantitative results and investigates the importance of endogenous labor supply. Section 6 concludes.

2 Related literature

This section provides an overview of the related literature as well as key differences with our analysis. Our paper relates to different strands in the literature on growth, education and wage inequality.

To a large extent, wage inequality is driven by skill-biased technical change, which has tilted demand and supply in the labor market in favor of high-skilled households (see, for example, Katz and Murphy, 1992). Despite the increase in education levels, demand for high-skilled labor never weakened: the race (Goldin and Katz, 2009) between technological change and education has been won by the former, leading to significant increases in inequality.[1] Our paper builds on the literature that explains the increase in the skill premium through skill-biased technological progress, specifically capital-skill complementarity, as in Krusell, Ohanian, Rios-Rull, and Violante (2000) and Lindquist (2005), and the reduction in the price of capital equipment, as in Greenwood, Hercowitz,

[1] See the reviews by Lemieux (2008) or Acemoglu and Autor (2011) for further details.

and Krusell (1997).

There are a number of general equilibrium models with endogenous education decisions.[2] Most use education information to calibrate the model. By design then, education outcomes are close to empirical observations. He (2012) and Restuccia and Vandenbroucke (2013) are two exceptions. These studies, and ours, calibrate their general equilibrium models without using education variations. Such a calibration approach leads to education outcomes which may be close to or far from the data, which allows to identify the factors accounting for education variations over time.

The model by He (2012) shows that the main determinant of the increase in the skill premium is skill-biased technological progress. Demographics, from which we abstract, play a smaller role. The model accounts endogenously for the rise in the US skill premium between 1950 and 2000 but underestimates the increase in college enrollment by 65%. Restuccia and Vandenbroucke (2013) investigate both US high-school enrollment and college enrollment decisions between 1940 and 2000. Their model replicates the increase in college education between 1940 and 1980 but overpredicts the increase between 1980 and 2000 by more than 45%, despite a small predicted increase in the college premium over the later period. In both of these models, labor supply is exogenous.

There are other analyses of endogenous education decisions, but in partial equilibrium settings. Keller (2014) investigates education decisions and on-the-job training, and matches variations in US college enrollment for cohorts born between 1920 and 1970. Hendricks and Schoellman (2014) find student abilities to influence education decisions and wage premiums for US cohorts born between 1910 and 1960. Guvenen and Rendall (2015) investigate the role of marriage and find that divorce law reforms contributed to the rise in college education by US women between 1950 and 2005. Castro and Coen-Pirani (2016) consider education levels of US cohorts born between 1932 and 1972, finding that variations in skill premium, tuition and learning ability

[2] Examples include Lee (2005); Heathcote, Storesletten, and Violante (2010); He (2012); Restuccia and Vandenbroucke (2013); Jones and Yang (2016); Kong, Ravikumar, and Vandenbroucke (2018).

all influence variations in college graduation rates, but at different periods in time.[3] Greenwood, Guner, Kocharkov, and Santos (2016) show that marriage within education groups helps to account for the difference in US education levels in 1960 and 2005. General equilibrium feedback effects from an increase in the supply of skilled labor on the skill premium cannot arise in such settings. Our simulations, however, document the quantitative importance of general equilibrium effects, with gaps on the skill premium exceeding 35%. It is unclear whether partial equilibrium results would survive in a richer, general equilibrium setting.

3 Model

We build a neoclassical, deterministic overlapping-generations model with skill-biased technical change as well as endogenous hours and education decisions in general equilibrium. Following Greenwood, Hercowitz, and Krusell (1997) and Krusell, Ohanian, Rios-Rull, and Violante (2000), we model skill-biased technical change as the result of capital-skill complementarity and variations in the price of capital equipment.[4] In order to keep the analysis transparent, we keep the model parsimonious: the only source of individual heterogeneity are differences in learning ability. Therefore, and as in Heathcote, Storesletten, and Violante (2010), education decisions depend on wage differentials and ex-ante heterogenous learning ability.[5]

3.1 Households

Population We abstract from population growth so that the size of the economy is constant. Households become economically active after high school when they start

[3]Donovan and Herrington (2019) have findings similar to Castro and Coen-Pirani (2016) over a longer time span, for US cohorts born between 1900 and 1972.

[4]The literature sometimes calls this investment-specific technological change (ISTC).

[5]The resulting model is similar to He (2012) except that our model does not feature demographics but includes endogenous labor supply.

their adult life, at which point we say that they are *born*. At that point in time a household decides whether or not to go to college. If he does not attend college, he starts to work as an unskilled worker immediately. If he enrolls in college, he will graduate four years later and will then start to work as a skilled worker. All households work until they die, J periods after birth, leaving neither debt nor bequests. As there is no bequest motive, all households are born with no assets.

Preferences Lifetime utility from consumption and leisure of a household born in time v is given by

$$\sum_{j=1}^{J} \beta^{j-1} u(c_{v,v+j-1}, l_{v,v+j-1}) \tag{1}$$

where β is the discount factor, $c_{v,t}$ denotes consumption of the household at time t and $l_{v,t}$ its labor supply. We assume that the instantaneous utility function is separable and given by

$$u(c,l) \equiv \ln(c) - \gamma \frac{l^{1+\epsilon}}{1+\epsilon}, \tag{2}$$

where ϵ is the inverse of the (Frish) elasticity of labor supply. These preferences have the well-known implication that income and substitution effects cancel out for permanent wage changes. However, we analyze the transition path of a sequence of skill-biased technology shocks which lead to a new steady-state. The relative strength of income and substitution effects changes during this transition, which can lead to changes in hours worked (see e.g. King, Plosser, and Rebelo, 1988; Blanchard and Fischer, 1989).

Education Households are born with different learning abilities. They draw an ability $i \in [0;1]$ from a uniform distribution at birth. Education comes at a utility cost $\chi(i)$, capturing psychological costs of learning in reduced form and at the cost of foregone wages during college. The education disutility cost function declines with ability, $\chi' < 0$: households born with high ability i find education easy and are more likely to choose college education. We denote college education by $s_i = c$ and high school education by

$s_i = h$.

Productivity and budget constraints Wage differences arise not only from skill differences and changes in the skill premium but also from age efficiency profiles. We assume separate exogenous time-invariant profiles $\left\{\varepsilon_j^s\right\}$ for each education level s. This implies the following budget constraint

$$a_{v,v+j} + c_{v,v+j-1} = (1 + r_{v+j-1})\, a_{v,v+j-1} + (1 - \mathcal{I}_{v,v+j-1})\, w^s_{v+j-1}\varepsilon^s_j l_{v,v+j-1}, \qquad (3)$$

where

$$\mathcal{I}_{v,v+j-1} = 1 \qquad \text{if} \quad s = c \;\wedge\; j \leq 4$$
$$\mathcal{I}_{v,v+j-1} = 0 \qquad \text{otherwise.}$$

Thus, the indicator variable captures the fact that households that go to college forgo labor income while studying. Factor prices are time-varying but, as aggregate variables, have no age subscript. For simplicity we ignore the pecuniary costs of education.[6] Despite this, households who go to college have to borrow to finance early-life consumption since they have no income.[7]

Households maximization problem Conditional on the education choice s, the maximization problem of a household born at time v, of age j and alive in period $v + j - 1$ can be expressed as

[6]We also considered cases with tuition costs. Since these models had similar implications, we do not report them here. See He (2011, 2012) for models that incorporate tuition costs.

[7]Since households in the model do not face risks, they can borrow at the risk-free rate up to the natural borrowing limit, which in our model never becomes binding.

$$V^s_{v,v+j-1}\left(a_{v,v+j-1}\right) = \max_{c_{v,v+j-1},l_{v,v+j-1}} \; u\left(c_{v,v+j-1},l_{v,v+j-1}\right) \; + \; \beta V^s_{v,v+j}\left(a_{v,v+j}\right) \tag{4}$$

and is subject to the budget constraint (3). In the first period of their life (at $j=1$), households decide whether to go to college or whether to remain unskilled by comparing the corresponding value functions. The resulting choice is given by

$$s = \begin{cases} c & \text{if} \quad V^c_{t,t}\left(0\right) - \chi(i) \geq V^h_{t,t}\left(0\right) \\ h & \text{if} \quad V^c_{t,t}\left(0\right) - \chi(i) < V^h_{t,t}\left(0\right). \end{cases} \tag{5}$$

3.2 Markets

Labor market and production A representative firm uses capital K, skilled labor S and unskilled labor U to produce a single good, paying factors their marginal product. Skilled and unskilled labor are imperfect substitutes because of capital-skill complementarity. The aggregate production function is

$$Y_t = F(K_t,S_t,U_t) = \left[\mu U_t^{\theta} + (1-\mu)\left(\lambda K_t^{\rho} + (1-\lambda)\,S_t^{\rho}\right)^{\theta/\rho}\right]^{1/\theta}, \tag{6}$$

with $\mu,\lambda \in (0,1)$ and $\rho < \theta < 1$. The condition $\rho < \theta$ implies capital-skill complementarity, the elasticity of substitution between skilled labor and capital $1/\left(1-\rho\right)$ being smaller than the elasticity of substitution between unskilled labor and the capital-skilled labor aggregate $1/\left(1-\theta\right)$.

Production capital is derived from households' assets

$$A_t = \sum_{s\in\{c,h\}} \left(A^s_t\right) \equiv \sum_{s\in\{c,h\}} \left(\sum_{v=0}^{t} N^s_{v,t} a^s_{v,t}\right) \tag{7}$$

where $N^s_{v,t}$ is the number of households who choose education s, born at time ν and alive at time t and $a^s_{\nu,t}$ are their assets. Firm capital depreciates at rate δ. Savings (investment) $X_t = A_{t+1} - A_t$ can be transformed into production capital K_t thanks to a technology which is exogenously improving over time and represented by a price q_t. The law of motion for capital is

$$K_{t+1} = (1-\delta) K_t + X_t q_t. \tag{8}$$

The exogenous time-invariant labor input efficiency process $\left\{\varepsilon^s_j\right\}$ depends on the skill level and on age. The aggregate efficient labor supply of the unskilled and skilled workers are given, respectively, by

$$U_t = \sum_{v=0}^{t} N^h_{v,t} \varepsilon^h_{t-\nu} l^h_{v,t} \qquad S_t = \sum_{v=0}^{t} N^c_{v,t} \varepsilon^c_{t-\nu} l^c_{v,t}. \tag{9}$$

Goods market Defining aggregate consumption C_t in a way similar to aggregate assets A_t, the goods market must clear:

$$Y_t = C_t + X_t. \tag{10}$$

We provide first order conditions with other equilibrium properties and a formal definition of the competitive equilibrium. We then continue with an analysis of the role of hours in a simplified version of the model, which exhibits an intertemporal relative labor supply effect, at the heart of our study.

3.3 Equilibrium properties

Change of variable We perform a permanent change of variable, borrowed from Greenwood, Hercowitz, and Krusell (1997), which simplifies the analysis: $\tilde{K}_{t+1} \equiv K_{t+1}/q_t$. Then the production function (6) and the capital law of motion (8) are equivalent to

$$Y_t = \left[\mu U_t^\theta + (1-\mu)\left(\lambda\left(B_t\tilde{K}_t\right)^\rho + (1-\lambda)\, S_t^\rho\right)^{\theta/\rho}\right]^{1/\theta},$$

$$\tilde{K}_{t+1} = \left(1-\tilde{\delta}\right)\tilde{K}_t + X_t,$$

with additional notation $B_t = q_{t-1}$ and $\tilde{\delta} = 1-(1-\delta)q_{t-1}/q_t$. After the change of variable, the capital law of motion has the familiar neoclassical growth expression. Assets market clearing $\tilde{K}_t = A_t$ then implies goods market clearing (10), a property which simplifies the numerical implementation of the model.

Optimality conditions First order conditions, applying to all life-cycle periods of the household, are given by:

$$\frac{c_{v,t+1}^s}{c_{v,t}^s} = (1+r_{t+1})\beta \qquad\qquad l_{v,t}^s = (1-\mathcal{I}_{\nu,t})\left(\frac{1}{\gamma}\frac{w_t^s\varepsilon_{t-\nu}^s}{c_{v,t}^s}\right)^{\frac{1}{\epsilon}}.$$

The first condition is the Euler equation. The only role of the term $(1-\mathcal{I}_{\nu,t})$ in the second condition is convenience in notation: when households obtain education ($\mathcal{I}_{\nu,t} = 1$), they do not work.

Factor prices As factors are paid their marginal products, the net interest rate and wage rates are equal to:

$$r_t = \lambda(1-\mu)\, B_t^\rho H_t\left(\lambda\left(B_t\tilde{K}_t\right)^\rho + (1-\lambda)\, S_t^\rho\right)^{\theta/\rho-1}\tilde{K}_t^{\rho-1} - \tilde{\delta} \tag{11}$$

$$w_t^c = (1-\mu)(1-\lambda)\, H_t\left(\lambda\left(B_t\tilde{K}_t\right)^\rho + (1-\lambda)\, S_t^\rho\right)^{\theta/\rho-1} S_t^{\rho-1} \tag{12}$$

$$w_t^h = \mu H_t U_t^{\theta-1} \tag{13}$$

where $H_t = \left[\mu U_t^\theta + (1-\mu)\left(\lambda\left(B_t\tilde{K}_t\right)^\rho + (1-\lambda)\, S_t^\rho\right)^{\theta/\rho}\right]^{1/\theta-1}$.

Skill premium Dividing (12) by (13), the skill premium follows:

$$\frac{w_t^c}{w_t^h} = \frac{(1-\mu)(1-\lambda)}{\mu}\left(\lambda\left(\frac{B_t\tilde{K}_t}{S_t}\right)^{\rho} + (1-\lambda)\right)^{\theta/\rho-1}\left(\frac{S_t}{U_t}\right)^{\theta-1}.$$

The expression illustrates the impact of skill-biased technical change, or skilled labor demand, and skilled labor supply on the skill premium. Since $\mu, \lambda \in (0,1)$ and $\rho < \theta < 1$, a larger supply of skilled labor S_t depresses the skill premium, ceteris paribus. Increases in the demand for skilled labor follow from increases in $B_t = q_{t-1}$, which represents an improvement in investment technology and leads to skill-biased technical change. When this happens the skill premium increases, as expected.

Competitive equilibrium A competitive equilibrium over the time periods $\mathcal{T} = \{t_0, t_0+1, ..., t_0+T\}$ is a sequence

$$\left\{\ s_{\nu,t}, c_{v,t}, l_{v,t}, w_t^c, w_t^h, r_t, N_{v,t}^s\ \mid v, t \in \mathcal{T}\ \right\}$$

such that education decisions $s_{\nu,t}$ solve (5), consumption and leisure decisions $(c_{v,t}, l_{v,t})$ solve the Bellman equations (4) subject to budget constraints (3), factor prices (w_t^c, w_t^h, r_t) are paid their marginal products as summarized in conditions (11,12,13), factor markets clears as expressed in (7) and (9) and the goods market clears as expressed in (10).

4 Wage inequality and endogenous labor supply: analytical results

We use a simplified version of the model to analyze the intertemporal impact of wage inequality on labor supply decisions along the intensive margin.

We consider a partial equilibrium version of the model with a constant interest rate and an exogenous skill premium variation given by

$$w_t^c = (1 + \alpha_t)\, w_t^h$$

for a given path of the skill premium $\alpha_t \geq 0$. We also assume that education does not take time and an identical age efficiency profile for unskilled and skilled workers ($\varepsilon_{t-\nu}^h = \varepsilon_{t-\nu}^c$ for all $t \geq \nu$).

First we express optimality conditions for the simplified model and derive its implications. Consider two households born at the same time but making different education choices. The first order conditions for labor can be rewritten as $c_v^s(t) = \frac{1}{\gamma}\frac{1}{(l_t^s)^\epsilon} w_t^s \varepsilon_{t-\nu}^s$. Comparing the consumption decisions of the two households,

$$\frac{c_v^c(t)}{c_v^h(t)} = \left(\frac{1}{\gamma}\frac{w_t^c \varepsilon_{t-\nu}^c}{(l_v^c(t))^\epsilon}\right) \Big/ \left(\frac{1}{\gamma}\frac{w_t^h \varepsilon_{t-\nu}^h}{(l_v^h(t))^\epsilon}\right) = \left(\frac{l_v^h(t)}{l_v^c(t)}\right)^\epsilon \frac{w_t^c}{w_t^h} = \left(\frac{l_v^h(t)}{l_v^c(t)}\right)^\epsilon (1 + \alpha_t)\,.$$

From the Euler equation it follows that consumption grows at the same speed for the two households. Hence the quantity expressed above is constant. Lemma 1 consolidates optimality conditions and sums up the additional derivations:

Lemma 1 ***(Optimality conditions):*** *For households born at time v, optimal labor supply and consumption decisions are characterized by the following:*

$$l_v^s(t) = \left(\frac{w_t^s \varepsilon_{t-\nu}^s}{\gamma c_v^s(t)}\right)^{\frac{1}{\epsilon}} \qquad \frac{c_v^s(t+1)}{c_v^s(t)} = (1+r_{t+1})\,\beta \qquad \left(\frac{l_v^h(t)}{l_v^c(t)}\right)^\epsilon (1+\alpha_t) = \frac{c_v^c(t)}{c_v^h(t)} = \textit{constant.}$$

Assume that the skill premium α increases over time. The last relationship says that labor supply l^h of the low-educated will increase more slowly than labor supply l^c of the high-educated, a *relative intertemporal labor supply effect.* Intuitively, it makes more sense for high-educated households to work relatively more in the future rather than in the present, when they earn an even higher wage.

We provide formal derivations of this intuitive discussion in Lemma 2. The lemma first shows that labor supply is the same for households when the skill premium is constant over their lifetime. It also shows that high-educated households work less at the start of their life, and more at the end, if the skill premium never decreases and increases at least once over their lifetime.

Lemma 2 ***(Relative hours):*** *Compare an educated household with an uneducated household born at the same time v. Then...*
(a) if the skill premium is constant over their lifetime ($\dot{\alpha} = 0$), households work the same (and consume according to their productivity):

$$l_v^c(t) = l_v^h(t) \qquad c_v^c(t) = (1 + \alpha_v)\, c_t^h(t) \qquad t \in [v, v + J]$$

(b) if the skill premium never decreases and increases once or more over their lifetime ($\dot{\alpha} \geq 0$ with $\dot{\alpha} > 0$ at least once), educated households initially work less than uneducated households, but more before they die: there exists t_1 and t_2 such that

$$l_v^c(t) < l_v^h(t) \quad \forall v \leq t < t_1 \qquad l_v^c(t) > l_v^h(t) \quad \forall t_1 < t_2 \leq t \leq v + J.$$

Further, $l_v^c(t)/l_v^h(t)$ is constant over intervals where the skill premium is constant.

Proof : See Appendix A.1.

The converse to case (b) of the lemma also holds: if the skill premium never increases and decreases at least once in the lifetime of the two households, educated households initially work more than uneducated households, and then less. The intuition for case (a) of the lemma is the following: in the absence of transitory shocks to wages, substitution and income effects cancel out so labor supply is identical, and lifetime income and consumption differences between educated and uneducated households equal the productivity difference (skill premium).

The intuition for case (b) is based on the relative intertemporal labor supply effect and case (a): when the premium is constant, households work an equal number of hours; when the premium increases, educated households work more when the premium is higher, which is towards the end of their life.

As shown in Figure 1, the skill premium has been increasing at the end of the 20th century. There is also evidence that low-skilled employees worked more hours than high-skilled employees early in the century in the US, a trend reversed by the end of the century (Costa, 2000). Case (b) of lemma is consistent with this stylized evidence.

So far, we have assumed that the skill premium changes are exogenous. The changes in hours induced by changes in the skill premium will, however, dampen the initial change in the skill premium. The magnitude of this general equilibrium effect depends on the (changing) size of each education group and the age structure of the economy. Therefore, we use a quantitative version of our model to analyze the magnitude of these general equilibrium effects in the next section.

5 Quantitative results

In the previous section, we have shown analytically how an exogenous increase in the skill premium can lead to an increase in labor supply of high-skilled households in partial equilibrium. However, any increase in the labor supply of high-skilled will in turn decrease the skill premium. Therefore, in this section, we ask whether a quantitative general equilibrium model with endogenous labor supply can generate the increase in the skill premium and the increase in college enrollment rates simultaneously. Our calibration strategy is to use only data for the beginning of our sample period (1967). The only exception is the exogenous evolution of the price of capital between 1967 and 2000. Thus, the resulting changes in college enrollment and the skill premium are equilibrium outcomes and used to evaluate the model's performance.

To solve the model numerically, initial and final steady states are needed since the economy is not in a steady state in 1967. In particular, the stock of skilled workers is not the same as the flow of new skilled workers entering the labor market since the college graduation has increased steadily between 1950 and 1967. Similarly, the economy is not in its new steady state in 2000. Therefore, we choose a counterfactual initial steady state in 1900 and final steady state in 2070 and a smooth transition between these two steady states. With these choices, the model representations in 1967 and in 2000 are not steady states but points on the transition path. Parameters are chosen so that the economic equilibrium in the model is close to the data in 1967.[8] For our key exogenous driving force, the price of capital equipment, we have only data between 1967 and 2000. We assume that this price continues to fall at the average rate (based on our data) for another 20 years and then remains constant until 2070.[9]

We use standard balanced-growth preferences, implying offsetting income and substitution effects in steady states. However, our analysis takes place along the transition path, where ISTC and life-cycle effects lead to transitory changes in wages. Along the transition path thus, income and substitution effects may not cancel out, which can lead to changes in labor supply (see e.g. Blanchard and Fischer, 1989).

We first discuss our calibration in Section 5.1. Then, we show our baseline results in Section 5.2 before discussing the role of endogenous labor supply in Section 5.3.

5.1 Calibration

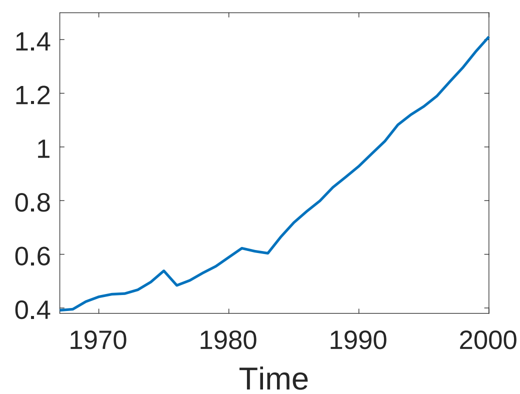
Our calibration strategy has two components. One group of parameters, shown in Panel A of Table 1, is based on previous research. Another group of parameters, shown in Panel B of Table 1, is calibrated to match crucial stylized facts.[10]

[8]This approach is standard in the literature on structural change but has an impact on counterfactual analyses, as will be discussed later.

[9]To avoid jumps in household decisions around 1967, we also assume that the price of equipment starts to grow 25 years before 1967, at half the yearly average growth rate between 1967 and 2000.

[10]Further details on the calibration and the solution algorithm can be found in Appendix A.2.

Figure 2: Technological progress



Since households enter in the model only when joining the labor market or starting education (age 18) and exit after they stop working (age 65), we follow He (2012) and set lifetime duration J to 48. Age-efficiency profiles for educated $\left\{\varepsilon_j^c\right\}$ and uneducated workers $\left\{\varepsilon_j^h\right\}$ are derived from average wages in the second half of the century, as reported in Figure 8 in He (2012).[11] One period corresponds to one year, so we set the time discount factor β to 0.96.[12] There is a debate in the literature on the value of labor supply elasticities: we follow Shimer (2009) and choose a value of 1. We thus set ϵ such that the elasticity $1/\epsilon = 1$.[13]

Figure 2 shows the (normalized) inverse of the price of capital equipment which is our measure of ISTC.[14] It is denoted $\{q_t\}$ in the model. The capital depreciation rate δ is taken from Imrohoroglu, Imrohoroglu, and Joines (1999). The parameters θ and ρ which defines the elasticity of substitution between capital, skilled and unskilled labor are taken from Krusell, Ohanian, Rios-Rull, and Violante (2000).

Following Heathcote, Storesletten, and Violante (2010), we assume that the utility

[11]Jeong, Kim, and Manovskii (2015) find variations in US returns to experience between 1968 and 2007, which, in our setting, calls for time-varying age-efficiency profiles. We decide, however, to use constant age-efficiency profiles to allow for comparisons with He (2012) and other general equilibrium analyses of education decisions, which all use constant profiles.

[12]The resulting capital-output ratio K/Y is 3.18 in the initial steady-state.

[13]In Appendix A.5, we perform a robustness test and recalibrate the model with a lower labor supply elasticity ($1/\epsilon = 0.5$). The results hardly change relative to the benchmark case.

[14]It is derived from the estimates of the quality-adjusted prices for total investment from Cummins and Violante (2002).

Table 1: Exogenous model parameters

	Parameter	Value	Source
	A: Exogenous parameters		
J	Lifetime duration	48	Real average working life
$\left\{\varepsilon_j^s\right\}$	Age efficiency profiles	...	He (2012) based on CPS
β	Time discounting factor	0.96	Aiyagari (1994)
$1/\epsilon$	Labor supply elasticity	1	Shimer (2009)
$\{q_t\}$	Price of capital equipment 1967-2000	...	Cummins and Violante (2002)
δ	Capital depreciation rate	0.069	Imrohoroglu et al. (1999)
θ	Elasticity param. unskilled labor/capital	0.401	Krusell et al. (2000)
ρ	Elasticity param. skilled labor/capital	-0.495	Krusell et al. (2000)
	B: Calibrated parameters		
η	Mean in dist. education disutility	0.196	
κ	Variance in dist. education disutility	2.25	
γ	Scale labor supply disutility	7.44	Calibrated jointly
μ	Share for unskilled labor in production	0.409	
λ	Share for capital in production	0.882	

cost function for education has a log-normal form with mean η and variance κ. Therefore, we have five parameters in Panel B of Table 1 that need to be determined jointly. These are the scale factor for labor supply disutility γ, the share factors for unskilled labor and the capital-skilled labor composite in the production function μ and λ in addition to η and κ.

As mentioned before, we calibrate these five parameters to match five moments in 1967. Table 2 shows the target variables, their data counterparts and the model outcomes. Overall, the model fits the data well. The initial wage-hours correlation is negative, implying that the high-skilled worked fewer hours than the low-skilled. The college enrollment rate is 52% in the model and in the data.[15] The skill premium in

[15]Following Heathcote, Perri, and Violante (2010) and He (2012), we classify workers with a college degree as skilled and workers with a high school degree and no college as unskilled. This leaves those who attended college but did not obtain a degree. We classify half of the workers with some college but no college degree as skilled and the other half as unskilled. Our resulting college enrollment data is identical to Figure 4 in He (2012). Athreya and Eberly (2015) show that enrollment and attainment, i.e. college completion, follow a similar trend over the period analyzed.

Table 2: Model moments

Variable	Model	Data
log wage - log hours correlation in 1967	-0.148	-0.140
Average number of hours worked in 1967	0.370	0.370
Investment (% GDP) in 1967	0.222	0.221
College enrollment rate in 1967	0.523	0.521
Skill premium in 1967	0.519	0.515

The log wage - log hours correlation data comes from Heathcote, Storesletten, and Violante (2010). The investment share data are from the World Bank. Due to the volatility of investment we use the 1966-1968 three-year average. The other numbers are from He (2012).

the model is 52% in 1967, consistent with the data. The quality of the calibration will be assessed in the next section when we look at two moments that were not targeted.

5.2 Simulation results

Given variations in the price of capital equipment, our model makes predictions on labor supply, education decisions and equilibrium wages. Figure 3 shows the skill premium and college enrollment predicted by the model (dashed line) and compares them to the data (solid line). The calibration strategy implies that the values for the skill premium and enrollment in 2000, as well as the time-path between 1967 and 2000 are model outcomes.

Panel A in Figure 3 shows that the skill premium predicted by the model for 2000 is very close to its empirical counterpart. It is 83.2% in the model and 79.5% in the data (4.6% gap). The model-implied time-path of the skill premium is also close to the empirical trend, except for the initial 5 years drop which the model fails to reproduce. Behind this trend for the skill premium is the continuous fall in the relative price of capital equipment which benefits high-skilled workers. Since the relative price does not show a drop in the first few years (see Figure 2), the model fails to reproduce the initial

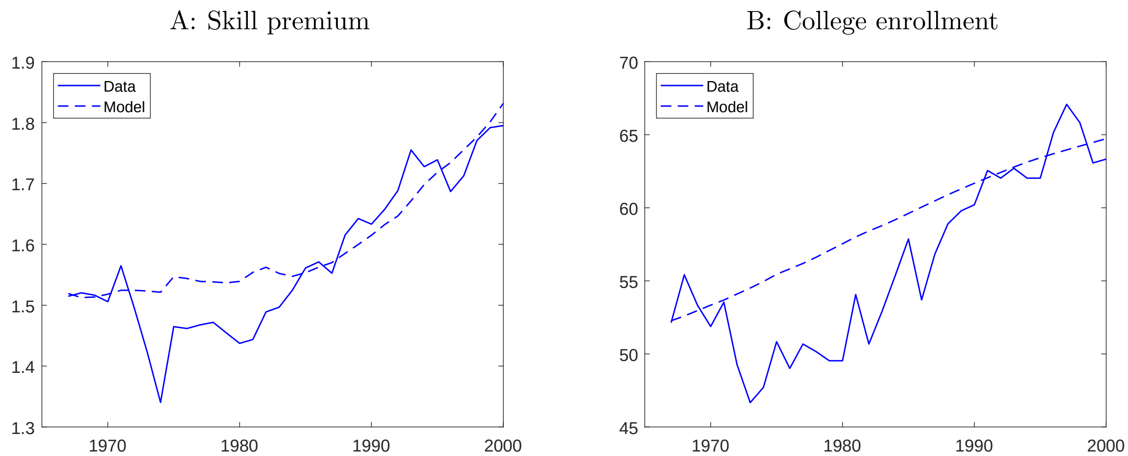


Figure 3: Data and model predictions for education and the labor market.
Panel A shows the increase in the skill premium and Panel B the evolution of the college enrollment rate, defined as in Figure 1.

drop observed in the data.[16]

Panel B shows not only that the model generates an increase in college enrollment endogenously but also that the magnitude is very close to the data. The enrollment rate in 2000 in the model is 64.7% while it is 63.3% in the data. Thus, again the model is close to the data: while enrollment increases 11.2 percentage points between 1967 and 2000 in the data, the model predicts an 12.4 points increase (11% gap). Similar to the skill premium, the time-path in the model is smoother and, in particular, misses the initial drop. The reason for this is again that the evolution of the relative price of capital equipment is fairly smooth, generating smooth adjustments in the premium and enrollment.

5.3 The role of endogenous labor supply

In this subsection we look at the role of the choice of hours, endogenous labor supply at the intensive margin, for the enrollment decision and the evolution of the skill premium. There are three competing effects: a *skilled premium effect*, a *relative hours* effect and

[16] He (2012) shows that demographics can explain this drop to a significant degree.

a *general equilibrium* effect. The first two operate at the individual level and the last one is a general equilibrium feedback effect.

First, an increase in the skill premium increases the return on costly education investments and thus strengthens education incentives. Second, suppose labor supply, i.e. hours, and the skill premium were exogenously fixed. If the marginal person whose disutility of education is slightly too large to go to college were able to work longer hours, he would be able to generate a larger income and therefore might decide to go to college. Thus, for a given skill premium, the possibility to work more hours increases the incentive to obtain an education. Third, the skill premium will, however, not remain constant in general equilibrium. The supply of skilled labor will increase at the intensive and extensive margin. Those with a low disutility of education who would have enrolled in college anyway now choose to work more hours when the skill premium increases, which, in general equilibrium, depresses the skill premium. The interplay of these three effects determines the outcome for the skill premium and enrollment jointly.

We evaluate the contribution of the three effects with the following experiments, the results of which are shown in Figure 4. The experiments are carried out in three steps. The first step (Panels A(i)-(ii)) will illustrate the *skill premium effect*. The first and second steps (Panels B(i)-(ii)) will jointly illustrate the importance of endogenous labor supply and the *relative hours effect*. The second and third steps (Panels C(i)-(ii)) will jointly illustrate the influence of the third channel, *general equilibrium effects*.

In the first step, we use a model version with constant hours, which we re-calibrate. As noted at the start of Section 5, the initial steady state differs from 1967. A recalibration of the model is thus required for our counterfactual experiments, as a preparation step.[17] The results are shown in Panels A(i)-(ii).

[17] If 1967 was our initial steady state, we would take the model as it is calibrated, change a parameter and then analyze the effects of this change. However, 1900 is our initial steady state. If we ignored this fact and went ahead without recalibration, our model would be already far from the data in 1967. We therefore have to recalibrate the entire model. Details on the recalibration can be found in Appendix A.3. In Appendix A.4, we show the results when we do not recalibrate the model. The results are qualitatively the same but the initial skill premium in 1967 is too high.

We again target the initial values of the skill premium and of college enrollment. The evolution of the skill premium is close to the data. However, now the enrollment rate overshoots its observed value. Panel A(ii) shows that it increases 21.1 percentage points to 73.1%, overshooting the empirical 11.2 percentage points increase by 88%. By comparison, in the baseline case the model enrollment is only 11% above its data counterpart. Thus, despite following the same calibration strategy as for the baseline case in Figure (3), a model with fixed labor supply fares worse. These quantitative differences between the two versions of the model, with and without endogenous labor supply, show the importance of that margin for a quantitative analysis of the skill premium and enrollment. Panels A(i) and A(ii) illustrate the *skill premium effect*, the increase in education incentives as the wage differential between high-skilled and low-skilled workers becomes larger over time.

The *relative hours effect* is shown in Panels B(i)-(ii), where we perform the following partial equilibrium experiment, as a second step: We keep all parameters and the evolution of factor prices as in the model with constant hours but allow households to decide on their hours and education. The skill premium in B(i) is thus identical to the one in A(i). Panel B(ii) shows that in this scenario, the enrollment rate is even higher than it was under constant hours in Panel A(ii). This is true for all periods. For example, the enrollment rate with endogenous hours is more than ten percentage points higher in 1967. In 2000, the gap is still more than three percentage points.

The experiment shows that the relative intertemporal labor supply effect, exhibited analytically in Section 4, matters quantitatively: ceteris paribus (under partial equilibrium with an exogenous skill premium), education incentives are increased if educated households can boost the return on education by working additional hours. Moreover, there is also a significant response at the intensive margin. Those households who obtain an education work on average 17% more than those who remain unskilled (unreported in Figure 4).

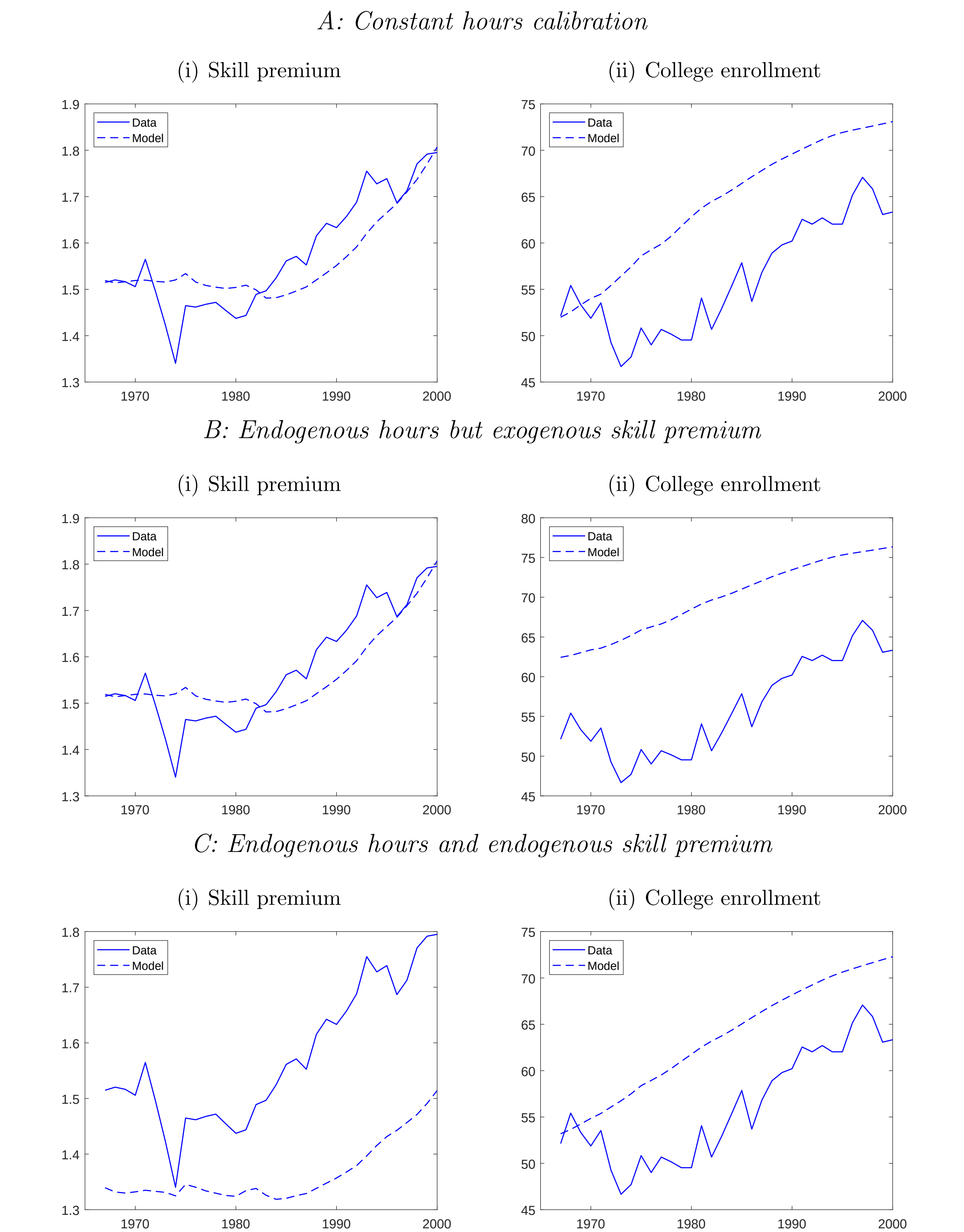


Figure 4: Inspecting the mechanism.
Row A shows the results for the constant hours calibration. Row B shows the partial equilibrium outcome where factor prices remain at the values from Row A but agents can choose hours. Row C shows the general equilibrium outcome when hours and factor prices can adjust.

The quantitative importance of the *general equilibrium effect* is shown in the third step, where we continue the previous, partial equilibrium experiment but now impose market clearing. In partial equilibrium, the increase in the skill premium leads to an increase in skilled labor supply, both at the intensive and at the extensive margin. One can thus clearly expect that the skill premium will fall in the general equilibrium setting. Since a smaller increase in the skill premium implies lower incentives to go to college, the supply of skilled agents will fall, as will their hours worked, which will dampen the general equilibrium effect. The results are shown in Panels C(i)-(ii).[18]

Panel C(i) shows that the general equilibrium effect on the skill premium is large. The premium falls in all periods by between 18 and 28 percentage points. For example, it is 51% instead of 80% in 2000, a relative difference larger than 35%. This drop in the skill premium takes place despite the fact that the smaller skill premium leads to a smaller rise in the enrollment rate, Panel C(ii). For example, instead of rising to 76%, it now rises only to 72.5% in 2000.

Overall, these results demonstrate the importance of analyzing the skill premium, the extensive margin (enrollment) and the intensive margin (hours) of skilled labor supply jointly. The joint analysis is important to understand the quantitative effects of skill-biased technological progress. As Panel A(ii) shows, neglecting the hours decisions leads to a significant bias (88%) in the increase in college enrollment, while Panel C(i) shows that ignoring general equilibrium responses leads to a similarly significant bias (35%) in the skill premium. By assuming constant hours, one ignores the incentive for the high-skilled to increase their labor supply which, through a general equilibrium effect, also counteracts the effects of skill-biased progress on the skill premium. This implies that, for a given change in technology, the effect on wage inequality is overstated.

[18] Models in Figure 3 and Panel C of Figure 4 are identical but differ in their calibration. The calibration approach in Panel C of Figure 4 indeed serves decomposition purposes only.

6 Concluding remarks

Education is an important investment decision for individuals: it has a larger impact on lifetime earnings inequality than shocks during the working life (Huggett, Ventura, and Yaron, 2011) and provides better self-insurance than savings or working hours (Heathcote, Storesletten, and Violante, 2010).

Recent decades have seen a large increase in the skill premium and a steady increase in college enrollment. Understanding changes in education decisions and the forces behind these changes is crucial for a number of policy questions. For example, a larger pool of high-skilled workers implies a larger income tax base; the quality of the labor force affects firms' investment decisions; and the political demand for redistribution depends on the distribution of pre-tax income.

We show that a standard neoclassical model with endogenous education and labor supply decisions with capital-skill complementarity and changes in the prices of capital equipment can simultaneously generate the observed increases in the skill premium and in college enrollment.

Since households are forward-looking, education and labor supply decisions are taken jointly. Education incentives are supported not only by an increase in the wage premium, but also by the possibility for educated workers to reap additional benefits by working longer hours. However, the longer working hours of high-skilled households will endogenously depress the skill premium. We show that neglecting hours decisions leads to a 88% bias in the increase in college enrollment, while ignoring general equilibrium responses leads to a 35% bias in the skill premium level.

General equilibrium effects on education-specific labor supply and the skill premium can have important policy implications. Limiting the labor supply, for example through union negotiations or overtime regulation, can, through the general equilibrium effect, lead to a larger wage increase for the skilled workers. A similar increase of wage inequality can take place with means-tested childcare policy, as the supply of labor is made easier for low-skilled workers, a possibly unintended consequence of the policy. A quantitative analysis of the impact of policy on education, labor supply and wage inequality is left for future research.

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A Appendix

Appendix A.1 shows the proofs for Section 4, while Appendix A.2 describes the numerical solution technique, Appendix A.3 provides details on the model calibration with constant hours, Appendix A.4 investigates a constant hours scenario without recalibration and Appendix A.5 is a sensitivity analysis with an alternative value for the labor supply elasticity.

A.1 Proofs

Proof (Lemma 2): Consider two households born at the same time v, one educated and one non-educated. For ease of reading, we drop the index v. Because households are born with no assets, the lifetime budget constraint is $\sum_{t=1}^{J} c_t^s / (1+r)^{t-1} = \sum_{t=1}^{J} w_t^s \varepsilon_t^s l_t^s / (1+r)^{t-1}$. From lemma 1 with a constant interest rate, $c_t^s = ((1+r)\beta)^{t-1} c_1^s$, so the lifetime budget constraint is

$$c_1^s = \frac{1}{\bar{\beta}} \sum_{t=1}^{J} \frac{w_t^s \varepsilon_t^s l_t^s}{(1+r)^{t-1}} \qquad \bar{\beta} \equiv \sum_{t=1}^{J} \beta^{t-1} = \frac{1-\beta^J}{1-\beta}.$$

Define $\kappa = c_1^c / c_1^h$. By the lemma 1, $(l_v^h(t)/l_v^c(t))^{\epsilon}(1+\alpha_t) = c_v^c(t)/c_v^h(t)$ is constant, so

$$l_t^c = \left(\frac{1+\alpha_t}{\kappa}\right)^{1/\epsilon} l_t^h. \tag{14}$$

Plugging the lifetime budget constraint, one has

$$\kappa = \frac{\sum_{t=1}^{J} \frac{(1+\alpha_t)^{1+1/\epsilon} w_t^h \varepsilon_t \left(\frac{1}{\kappa}\right)^{1/\epsilon} l_t^h}{(1+r)^{t-1}}}{\sum_{t=1}^{J} \frac{w_t^h \varepsilon_t l_t^h}{(1+r)^{t-1}}} \qquad \Leftrightarrow \qquad \kappa^{1+1/\epsilon} = \frac{\sum_{t=1}^{J} \frac{(1+\alpha_t)^{1+1/\epsilon} w_t^h \varepsilon_t l_t^h}{(1+r)^{t-1}}}{\sum_{t=1}^{J} \frac{w_t^h \varepsilon_t l_t^h}{(1+r)^{t-1}}}. \tag{15}$$

Assume first that the skill premium is constant over the lifetime of both households ($\alpha_t = \alpha$). From (15), one has $\kappa = 1 + \alpha$, so, with (14):

$$l_t^c = l_t^h \qquad\qquad c_t^c = (1+\alpha)\, c_t^h.$$

Assume otherwise that the skill premium strictly increases at some point of the lifetime of both households and never decreases. Then $\alpha_{v+J} \geq \alpha_t \geq \alpha_v$ for all $t \in [v, v+J]$ and $\alpha_t > \alpha_v$ for $t \geq u$ for some index u. From (15) one has $(1+\alpha_v) < \kappa < (1+\alpha_{v+J})$. From (14) one has

$$l_v^c = \left(\frac{1+\alpha_v}{\kappa}\right)^{1/\epsilon} l_v^h < l_v^h \qquad\qquad l_{v+J}^c = \left(\frac{1+\alpha_{v+J}}{\kappa}\right)^{1/\epsilon} l_{v+J}^h > l_{v+J}^h.$$

Lemma 1 says that $(l_v^h(t)/l_v^c(t))^\epsilon (1+\alpha_t)$ is constant. Thus, when the skill premium α_t is constant (or increases) over periods of the life of the households, the ratio l_t^c/l_t^h is constant (or increases).

QED

A.2 Computational details

We provide a generic algorithm for computing a steady state, in a normalized system where the change of variable from K to $\tilde{K}$ has been performed (see Section 3.3). The extension to a transition path version is standard.

Algorithm: Steady-state computation with normalized system

1. Make guesses on capital stock $\tilde{K}$ and labor supply stocks S and U

2. Derive factor prices r, w^c and w^h with (11,12,13)

3. Compute value functions and household decision policy functions by solving backward the maximization system (4) subject to (3)

4. Using the fact that newborns start their life with zero assets, compute the path for simulation assets $a_{v,t}$, consumption $c_{v,t}$, education $s_{v,v}$ and labor supply $l_{v,t}$

5. Compute the resulting aggregate assets A with (7) and, from asset market clearing, derive simulated capital stock $\tilde{K}' = A$

6. Compute the resulting supplies of skilled labor S' and unskilled labor U' from (9)

7. Compare guesses $\tilde{K}$, S, U with simulated outcomes $\tilde{K}'$, S', U'; stop if they are close; otherwise update guesses and return to step 2.

A.3 Calibration with constant hours

We calibrate the model with exogenous labor supply to the same moments as the baseline model except for hours worked and the wage-hours correlation, which are not defined in a model with exogenous and constant hours. The exogenous parameters remain the same as for the baseline model (see Table 1) and are not reported. Thus, we set the η, κ, and μ to match the college enrollment rate, the skill premium, and the investment share in 1967.[19] Table 3 shows the resulting parameters.

Table 3: Calibrated model parameters with constant hours

	Parameter	Value
η	Mean in dist. education disutility	-0.818
κ	Variance in dist. education disutility	2.25
μ	Share for unskilled labor in production	0.447
λ	Share for capital in production	0.910

[19]Since we have only three targets, we leave the variance of the education disutility distribution κ at its baseline value.

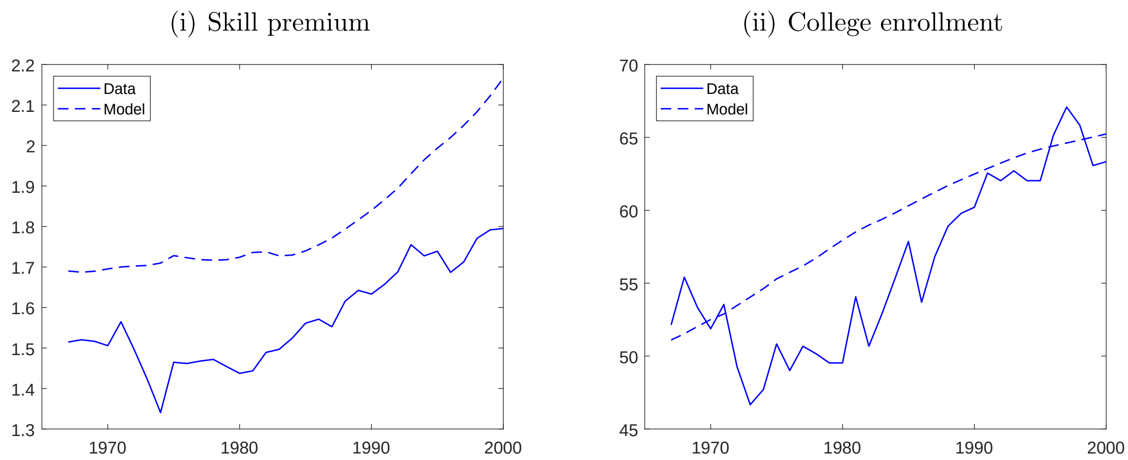

Figure 5: Data and model predictions for education and the labor market: No recalibration.

This Figure shows the results when we leave all parameters at their baseline values. Panel A shows the increase in the skill premium and Panel B the evolution of the college enrollment rate, defined as in Figure 1.

A.4 Constant hours analysis without recalibration

In the Section 5.3, we have analyzed the effects of keeping hours constant. Due to the fact that the situation in 1967 is not our initial steady state, this counterfactual experiment required a recalibration of the five endogenous parameters. This appendix shows the results when we do not recalibrate these parameters. Figure 5 shows the result when we leave all parameters unchanged. Because our initial steady state is 1900 and not 1967, keeping hours constant with the initial calibration leads to a different equilibrium in 1967, which motivates the recalibration approach used in the main text. In particular, the skill premium is counterfactually large.

A.5 Lower labor supply elasticity

This section shows that our results do not depend on the choice of the labor supply elasticity. In the baseline, we chose a value of $1/\epsilon = 1$, an intermediate value between micro- and macro-based estimates based on the literature discussion by Shimer (2009). Here we use a value $1/\epsilon = 0.5$, closer to micro-based estimates. We calibrate the model

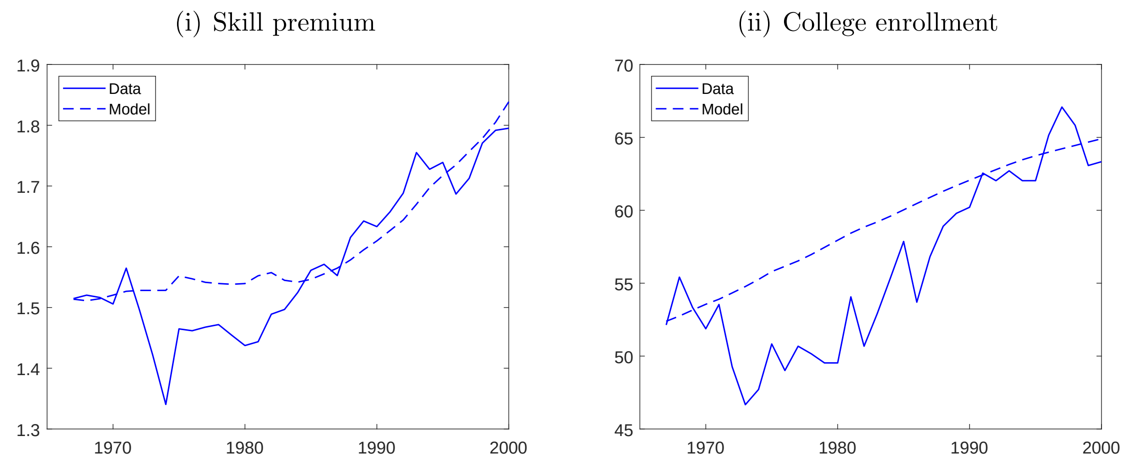

Figure 6: Data and model predictions for education and the labor market for lower labor supply elasticity.
Panel A shows the increase in the skill premium and Panel B the evolution of the college enrollment rate, defined as in Figure 1.

to the same moments as the baseline model. The exogenous parameters, except the labor supply elasticity, remain the same as for the baseline model (see Table 1) and are not reported. The endogenous parameters are recalibrated to match exactly the same moments as the baseline model. Table 4 shows the resulting parameters.

Table 4: Model parameters with lower labor supply elasticity

	Parameter	Value
η	Mean in dist. education disutility	-0.003
κ	Variance in dist. education disutility	2.50
γ	Scale labor supply disutility	21.70
μ	Share for unskilled labor in production	0.423
λ	Share for capital in production	0.878

As can be seen in Figure 6, the results hardly change relative to the baseline case (provided in Figure 3).