A Dynamic Model of Banking with Uninsurable Risks

and Regulatory Constraints*

Jochen Mankart (Deutche Bundesbank)†

Alexander Michaelides (Imperial College Business School, CEPR, and NETSPAR)‡

Spyros Pagratis (Athens University of Economics and Business)§

28th August 2014

*We would like to thank Michael Haliassos, Plutarchos Sakellaris, Vania Stavrakeva, Javier Suarez, Dimitri Vayanos and seminar participants at workshops at the Central Bank of Austria, Central Bank of Cyprus, the European Central Bank, Imperial College Business School, the EABCN-INET conference in Cambridge, the MFS conference in Cyprus, the EEA conference in Toulouse, and conferences in Lyon and Surrey for helpful comments. All remaining errors are our own. This paper represents the authors’ personal opinions and does not necessarily reflect the views of the Deutsche Bundesbank.

†Deutche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main. E-mail: jochen.mankart@bundesbank.de.

‡Department of Finance, Imperial College Business School, South Kensington Campus, SW7 2AZ. Tel: +44(0)2075949177. E-mail: a.michaelides@imperial.ac.uk.

§Department of Economics, Athens University of Economics and Business, 76 Patission Street, 10434 Athens, Greece. E-mail: spagratis@aueb.gr.
Abstract

We develop stylized facts about bank behavior using U.S. commercial bank data. We then estimate the structural parameters of a quantitative banking model (new loans, liquid assets and endogenous failure) in the presence of undiversifiable background risk (problem loans, interest rate spreads and deposit shocks) and regulatory constraints. Consistent with the data, loans are highly procyclical and banks curtail new lending aggressively in response to background risk shocks, while bank failures are strongly countercyclical and increasing in leverage. Increasing capital requirements generates higher equity but also results in higher failures because equity rises proportionately less than the leverage limit.

JEL Classification: E32, E44, G21

Key Words: Bank Leverage, Uninsurable Risk, Capital Requirements, Bank Failures.
1 Introduction

Policy makers recognize the importance of developing quantitative models to assess both microprudential and macroprudential risks in the financial system. These tools aim to improve the identification and assessment of systemically important risks from high leverage\(^1\), credit growth\(^2\) or money market freezes\(^3\). Moreover, quantitative structural models can be used in real time to perform counterfactual experiments and complement the tools available to regulators before making policy decisions. Interesting counterfactuals might be a change in capital requirements by a certain pre-specified point in time. For instance, in October 2011 European political leaders agreed that Eurozone banks should increase their core-tier I capital ratios to 9% by the end of June 2012 to improve investor confidence in the solvency of the banking system. A quantitative model that can assess the range of possible outcomes in banking defaults and credit supply in the nine months following the decision would be a very useful complement to the other inputs used in analyzing the impacts of such a decision.

Given the need for such applied, quantitative models, we construct a structural model of bank lending behavior, assuming that a bank’s objective is to maximize shareholder utility.

\(^1\)Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999) are seminal examples where leverage interacts with asset prices to generate amplification and persistence over the business cycle, while Gertler and Kiyotaki (2010) and Gertler and Karadi (2010) illustrate the importance of banking decisions in understanding aggregate business cycle dynamics. Adrian and Shin (2010) provide empirical evidence further stressing the importance of leveraged bank balance sheets in the monetary transmission mechanism.

\(^2\)Bernanke and Blinder (1988) provide the macro-theoretic foundations of the bank lending channel of monetary policy transmission. Using aggregate data, Bernanke and Blinder (1992), Kashyap et al. (1993), Oliner and Rudebusch (1996) provide evidence that supports the existence of the bank-lending channel.

\(^3\)Brunnermeier (2009) discusses the freeze of money markets during the recent recession in the U.S.
Banks in our dynamic model perform a maturity transformation function as in Diamond and Dybvig (1983). In addition, banks face the possibility of financial distress that affects their decisions. Financial distress may arise from adverse shocks in profits that may lead to equity capital shortfalls, in conjunction with a debt overhang problem discussed in Duffie (2010) that prevents banks from raising external equity capital in a crisis situation. Bank financial distress may also arise due to elevated costs of wholesale funding or, in an extreme case, a money-market freeze due to market imperfections.\footnote{Banks’ limited access to equity markets could also arise due to adverse selection problems a la Myers and Majluf (1994) and the information sensitivity of equity issuance. That problem might be particularly acute in a situation where a bank faces an equity shortfall due to loan losses, in which case information sensitivities may prevent the bank from accessing external equity capital from private investors.}

Moreover, banks’ decisions are influenced by perceived profit opportunities, funding conditions and background risk perceptions. Such perceptions are driven by exogenous processes for funding costs, asset quality (such as the loan write-off levels) and shocks to certain balance sheet items, such as customer deposits and tangible equity. We emphasize that despite being exogenous, these data generating processes are calibrated using microeconomic bank balance sheet and profit and loss data and are consistent with the empirical evidence.\footnote{Chen (2010) solves a firm’s optimal capital structure problem over the business cycle. Using an exogenous stochastic discount factor, the model allows for endogenous financing and default decisions by firms and generates countercyclical default probabilities, default recovery rates and risk premia. That helps explain the large credit spreads and limited use of debt in the capital structure of investment-grade corporates. We take these risk premia as exogenous in our model and focus on matching balance sheet items of leveraged banks.} Banks therefore face different background risks in an incomplete markets setup in the spirit of Allen...
We emphasize that our approach is quantitative in nature, with the model evaluated by its ability to replicate the cross-sectional and time series evolution of bank balance sheets in the U.S.. The empirical part of the approach is inspired by Kashyap and Stein (2000) and Berger and Bouwman (2013). Kashyap and Stein (2000) use disaggregated data to show that monetary shocks affect mostly the lending behavior of smaller banks (those with lower liquid asset holdings) due to frictions in the market for uninsured funds. Berger and Bouwman (2013) emphasize that bank capital positively affects a bank’s survival probability across different (heterogeneous) asset sizes. We replicate empirically the substantial heterogeneity in bank balance sheets over time. In building our structural model, we condense this heterogeneity into a few broad categories: long term loans and short term liquid assets on the asset side; and long term deposits, short term wholesale liabilities, and equity on the liability side. We think this is a reasonable compromise between the complexities in the balance sheet data and the necessarily simpler model abstraction that retains a sufficiently rich balance sheet structure.

The quantitative model is estimated using a Method of Simulated Moments (see, for example, Hennessy and Whited (2005)) and replicates the data in a number of dimensions. In the model smaller banks rely more on deposits than larger banks because smaller banks face a larger cost of accessing wholesale markets. As a result, larger banks are also more highly levered than smaller banks. Moreover, leveraged banks are more likely to fail in a recession, both in the model and in the data. Banks invest in liquid assets along with

---

Kishan and Opiela (2000) determine that equity is another variable that affects banks' sensitivity to monetary policy shocks. By classifying banks not only by size, but also in terms of leverage ratios, they...
making loans and the model replicates the substantial component of liquid assets in the balance sheet. Liquid assets are held as a way to hedge illiquidity risk arising from long-term loan provision and also as a way to smooth background risk (deposit outflow volatility and loan write-off shocks).

In the data, a substantial cross sectional heterogeneity in the loan to asset ratio exists (this ranges between 20% and 90%). Given that we split the balance sheet of each bank across broad categories (loans and liquid assets on the asset side), this implies a substantial heterogeneity in liquid asset holdings as well. The model replicates the wide range of cross sectional heterogeneity in loans and liquid assets to total assets through the idiosyncratic risks (deposit and loan write-off shocks) that each bank faces, and the endogenous decisions in response to these risks. In the data, there also exists heterogeneity in the deposit to asset ratio, although the range there is tighter (ranging between 70% and 95%) than in the loan to asset ratio. The tighter deposit to asset ratio is also replicated through a convex funding cost to access the wholesale market. Smaller banks are estimated to face a higher cost in accessing the wholesale market than larger banks and therefore rely more heavily on deposits to finance the asset side of their balance sheet.

In a recent paper, Berger and Bouwman (2013) make the distinction between small and large banks and find that capital helps small banks increase the probability of survival, while this is true also for large banks only during banking crises. In our model we cannot distinguish between different types of macro crises as there is only one recession variable capturing “normal” business cycles. The model is consistent with the empirical evidence in show that the smallest and least capitalized banks are the most sensitive to monetary contractions. Our results are consistent with this finding.
Berger and Bouwman (2013) in the sense that failed banks, regardless of size, tend to have higher (lower) average leverage (capital) than banks that survive.

Empirically, in the time series dimension, the deposit to asset ratio, leverage and failure rates are all countercyclical, while the loan to asset ratio is procyclical. The model predicts similar cyclical properties for these variables. The deposit to asset ratio in the model is countercyclical because banks lower lending and shrink their balance sheets by reducing reliance on wholesale funding markets during recessions. The model also predicts strongly procyclical loan growth that is slightly asymmetric (positive spikes tend to happen when the economy exits the recessionary period). Procyclical loan growth generates also procyclical leverage that is stronger for banks that have easier access to the wholesale market. The model also generates strongly countercyclical failure rates, consistent with the data. Moreover, these failure rates are more likely for highly levered firms and are driven by bad loan shocks. Given the motive for smoothing dividends through the concave utility function, the model also generates a smooth dividend to profits and dividend to equity ratio, both consistent with empirical observation. Overall, we interpret these findings as consistent with quantitative features of the data.

We therefore use the model to analyze the effect of changing capital requirements, a major issue of policy concern. Maintaining a higher level of capital (lower leverage) could increase banks’ resilience to shocks and reduce the likelihood of bank failures. On the other hand, imposing tighter leverage limits can increase the likelihood of bank failures if banks

\[^7\] Higher capital might mechanically increase an individual bank’s survival probability, while higher capital can also alleviate other frictions thereby increasing the likelihood of survival (see Allen, Carletti and Marquez (2011) and Mehran and Thakor (2011)).
do not increase their equity holdings sufficiently because, for any given amount of equity, a
tighter limit is more likely to be breached than a looser limit. Thus, more stringent capital
requirements can potentially reduce banks’ financial flexibility and therefore might increase
the likelihood of failure.\footnote{For instance, Koehn and Santomero (1980) and Besanko and Kanatas (1996).} Therefore, setting capital requirements at an appropriate level is
a balancing act, especially in terms of welfare, as shown by Van den Heuvel (2008) and De
Nicolo, Gamba and Luchetta (2014), even though most studies favor the idea that higher
capital and the probability of survival tend to be positively correlated (Freixas and Rochet
(2008)).

What happens when capital ratio requirements are exogenously increased? Specifically,
the leverage limit (inverse of the capital requirement) is reduced from 20 to 15 in an attempt
to evaluate the costs (lower financial intermediation) versus the benefits (lower failures) from
this regulatory change. This lowering of the leverage constraint increases bank equity since
banks are forced to accumulate more capital and lower loan issuance (consistent with the
empirical findings in Aiyar et. al (2014)). However, it also increases significantly the failure
rate, a result that goes against conventional wisdom that higher equity should make banks
safer. This is because banks endogenously move closer to the constraint and a lower leverage
limit makes it more likely for them to hit the constraint given the same level of risks they
face. Moreover, the negative loan supply effects of tighter leverage limits are much more
pronounced for smaller than for larger banks, underlying the need to better understand
cross-sectional bank heterogeneity.

We also undertake a second counterfactual to capture freezes in the wholesale funding
market. Specifically, we compare two recessions: one with a temporary (one quarter) freeze in the money market and another recession without any change in the operation of the money markets. Not surprisingly this has almost no impact on small banks since they do not borrow much in these markets. They simply lower their holdings of liquid assets during the crisis period so that their lending and survival hardly decline. Large banks, in contrast, which rely more on wholesale funding, are negatively affected. Their failure rate increases significantly. When the wholesale funding market freeze is complemented by asset fire-sales, then all banks, small and big, experience significantly larger failure rates and a credit crunch follows.

In terms of related literature, De Nicolo, Gamba and Lucchetta (2014) also model banks’ capital buffers in response to aggregate shocks and analyze the effects of capital requirements in a partial equilibrium model. We differ mainly by having a richer balance sheet structure where wholesale funding and liquid securities coexist in the bank’s balance sheet with substantial cross-sectional heterogeneity arising from background risks and bank choices. Repullo and Suarez (2013) analyze capital regulation in a general equilibrium model but we differ by emphasizing the maturity transformation role for banks and banks’ portfolio choices, albeit in a partial equilibrium setting. Corbae and D’Erasmo (2011 and 2012) also build a dynamic model of banking to investigate optimal capital requirements. Unlike our setting, they use a general equilibrium model featuring strategic interaction among a dominant big bank and a competitive fringe. We emphasize more the maturity transformation role of banks with loans having a larger duration, while banks can decide simultaneously on new loans, dividends and money market borrowing or security investments, thereby emphasizing
more the portfolio choices banks make. We should emphasize that we focus on individual banking decisions and not holding bank ones, even though this might not be a trivial assumption either theoretically or empirically. We leave to future work the more complicated model of a holding bank deciding how to move capital across different regions and regulatory environments.

One recent approach to determine optimal capital requirements is Miles, Yang and Marcheggiano (2012). They estimate the elasticity of bank cost of equity with respect to leverage and find that this is very small. As a result, given that more well-capitalized banks are safer, they provide further evidence for the message in Admati and Hellwig (2013) that banks need to hold substantially more capital than the currently prescribed regulation to avoid banking crises. Aiyar et. al. (2014) also find that increases in minimum capital requirements reduce cross-border bank credit. Our approach is different but complementary in that we can use the estimated structural quantitative model of banking to perform counterfactual experiments that can be used to inform the policy debate of the likely economic outcomes from various policy decisions, including forcing banks to hold more equity capital.

The rest of the paper is organized as follows. Section 2 discusses the data to be replicated, and section 3 the theoretical model. Section 4 shows the estimation results and section 5 compares the model with the data and discusses the model’s implications. Section 6 performs

---

9 Holod and Peek (2010) find evidence of internal capital and secondary loan markets within multi-bank holding companies that mitigate equity capital constraints and enhance the efficiency of the loan origination process. Cetorelli and Goldberg (2012) also show that internal capital markets and cross-border liquidity transfers among head offices and foreign affiliates of global banks lead to liquidity shocks at home propagating internationally.
counterfactual experiments and section 7 concludes.

2 Data

We consider a sample of individual bank data from the Reports of Condition and Income (Call Reports) for the period 1990:Q1-2010:Q4. For every quarter, we categorize banks in three size categories (small, medium and large). Small banks are those below the 95th percentile of the distribution of total assets in a given quarter, medium those between the 95th and 98th percentile and large those above the 98th percentile. We also consider the bank failures reported by the Federal Deposit Insurance Corporation (FDIC) for the same period. Bank failure occurs when either the FDIC closes down a bank or assists in the re-organization of the bank. A more detailed description of our sample is discussed in the Data Appendix.

2.1 Cross Sectional Statistics

Table 1 shows descriptive statistics for bank balance sheet compositions at year-end of the first and last year of our sample period, sorted by bank size. The significant reduction in the number of banks over time was mainly a result of regulatory changes that led to substantial consolidation in U.S. commercial banking.\footnote{According to Calomiris and Ramirez (2004), branch banking restrictions and protectionism towards unit banks (i.e. one-town, one-bank) led to a plethora of small U.S. commercial banks over the last century. But in the early 1990s protectionism was relaxed, especially following the Riegle-Neal Interstate Banking and Branching Efficiency Act (IBBEA) in 1994. That spurred a wave of mergers and acquisitions that reduced significantly the number of U.S. commercial banks. Calomiris and Ramirez (2004) provide some key facts} We abstract from endogeneizing mergers in our
Deposits (normalized by total assets) are the major item on the liability side of all commercial banks, see also Figure 1. Nevertheless, the deposit to asset ratio varies by bank size, with smaller banks relying more on deposits. Moreover, the importance of deposits has declined over time for all bank sizes until 2008. Both stylized facts can be seen in Figure 1a that graphs the mean deposit to asset ratio sorted by bank size over the period 1990-2010 (bootstrapped standard error confidence intervals are shown with dotted lines).

Larger banks tend to have more access to alternative funding sources like the Fed funds, repos and other money market instruments in the wholesale funding market. In 1990 (2010) the sum of Fed funds borrowed, subordinated debt and other non-deposit liabilities as a
Table 1: Balance sheets of U.S. commercial banks by bank size

(a) 1990

<table>
<thead>
<tr>
<th>size percentile</th>
<th>&lt;95th</th>
<th>95 - 98</th>
<th>&gt;98- 99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of banks</td>
<td>12022</td>
<td>376</td>
<td>253</td>
</tr>
<tr>
<td>Mean assets (2010 $million)</td>
<td>128</td>
<td>1701</td>
<td>14500</td>
</tr>
<tr>
<td>Median assets (2010 $million)</td>
<td>75</td>
<td>1518</td>
<td>7795</td>
</tr>
<tr>
<td>Frac. total system as.</td>
<td>26%</td>
<td>11%</td>
<td>63%</td>
</tr>
<tr>
<td>Fraction of tangible asset</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash</td>
<td>7%</td>
<td>7%</td>
<td>10%</td>
</tr>
<tr>
<td>Securities</td>
<td>29%</td>
<td>20%</td>
<td>17%</td>
</tr>
<tr>
<td>Fed funds lent &amp; rev. repo</td>
<td>7%</td>
<td>4%</td>
<td>4%</td>
</tr>
<tr>
<td>Loans to customers</td>
<td>53%</td>
<td>63%</td>
<td>59%</td>
</tr>
<tr>
<td>Real estate loans</td>
<td>27%</td>
<td>35%</td>
<td>27%</td>
</tr>
<tr>
<td>C&amp;I loans</td>
<td>10%</td>
<td>13%</td>
<td>18%</td>
</tr>
<tr>
<td>Loans to individuals</td>
<td>10%</td>
<td>14%</td>
<td>14%</td>
</tr>
<tr>
<td>Farmer loans</td>
<td>6%</td>
<td>1%</td>
<td>0%</td>
</tr>
<tr>
<td>Other tangible assets</td>
<td>4%</td>
<td>7%</td>
<td>11%</td>
</tr>
<tr>
<td>Total Deposits</td>
<td>89%</td>
<td>81%</td>
<td>73%</td>
</tr>
<tr>
<td>Transaction deposits</td>
<td>23%</td>
<td>19%</td>
<td>20%</td>
</tr>
<tr>
<td>Non-transaction deposits</td>
<td>65%</td>
<td>62%</td>
<td>53%</td>
</tr>
<tr>
<td>Fed funds borrowed &amp; repo</td>
<td>1%</td>
<td>6%</td>
<td>10%</td>
</tr>
<tr>
<td>Other liabilities</td>
<td>2%</td>
<td>6%</td>
<td>11%</td>
</tr>
<tr>
<td>Tangible equity</td>
<td>9%</td>
<td>7%</td>
<td>6%</td>
</tr>
</tbody>
</table>

(b) 2010

<table>
<thead>
<tr>
<th>size percentile</th>
<th>&lt;95th</th>
<th>95 - 98</th>
<th>&gt;98- 99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of banks</td>
<td>6528</td>
<td>206</td>
<td>137</td>
</tr>
<tr>
<td>Mean assets (2010 $million)</td>
<td>238</td>
<td>2715</td>
<td>72000</td>
</tr>
<tr>
<td>Median assets (2010 $million)</td>
<td>141</td>
<td>2424</td>
<td>13600</td>
</tr>
<tr>
<td>Frac. total system as.</td>
<td>13%</td>
<td>5%</td>
<td>82%</td>
</tr>
<tr>
<td>Fraction of tangible asset</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash</td>
<td>9%</td>
<td>7%</td>
<td>7%</td>
</tr>
<tr>
<td>Securities</td>
<td>21%</td>
<td>21%</td>
<td>20%</td>
</tr>
<tr>
<td>Fed funds lent &amp; rev. repo</td>
<td>2%</td>
<td>1%</td>
<td>2%</td>
</tr>
<tr>
<td>Loans to customers</td>
<td>62%</td>
<td>64%</td>
<td>61%</td>
</tr>
<tr>
<td>Real estate loans</td>
<td>45%</td>
<td>49%</td>
<td>38%</td>
</tr>
<tr>
<td>C&amp;I loans</td>
<td>9%</td>
<td>10%</td>
<td>11%</td>
</tr>
<tr>
<td>Loans to individuals</td>
<td>4%</td>
<td>5%</td>
<td>11%</td>
</tr>
<tr>
<td>Farmer loans</td>
<td>4%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Other tangible assets</td>
<td>5%</td>
<td>7%</td>
<td>10%</td>
</tr>
<tr>
<td>Total Deposits</td>
<td>85%</td>
<td>79%</td>
<td>68%</td>
</tr>
<tr>
<td>Transaction deposits</td>
<td>22%</td>
<td>10%</td>
<td>7%</td>
</tr>
<tr>
<td>Non-transaction deposits</td>
<td>63%</td>
<td>70%</td>
<td>61%</td>
</tr>
<tr>
<td>Fed funds borrowed &amp; repo</td>
<td>1%</td>
<td>4%</td>
<td>6%</td>
</tr>
<tr>
<td>Other liabilities</td>
<td>4%</td>
<td>7%</td>
<td>16%</td>
</tr>
<tr>
<td>Tangible equity</td>
<td>13</td>
<td>10%</td>
<td>9%</td>
</tr>
</tbody>
</table>
fraction of total assets rises monotonically from 3% (5%) for banks in the bottom 95th percentile to 21% (22%) for banks in the largest percentile. Figure [1b] reveals that the wholesale funding markets are more important for the largest banks relative to the smallest banks throughout this period. There seems to be an increase in accessing the wholesale funding market from the largest banks during the period between 1992 and 2007 and a reduction back to around 20% of the balance sheet in 2010. We use these stark differences in access to the wholesale funding market as a defining variation between big and small banks in the structural model.

Figure [2] shows the evolution of the asset side of the bank balance sheets. The biggest components are loans which are relatively illiquid because they are contractual obligations with long term maturities. There is an upward trend in the proportion of loans in the total balance sheet across all bank sizes between 1992 and 2007, with the trend briefly interrupted during the short 2001 recession. The average loan to asset ratio is around 60% for both small and large banks.

The largest remaining part of the asset side of the balance sheet is liquid assets which comprise cash, Fed funds lent, reverse repos and securities. At the beginning of the period, which is right after the savings and loans crisis, smaller banks hold significantly more liquid assets as a proportion of total assets relative to larger banks. However, these differences across size classes become less pronounced over time as smaller banks increase their loan to asset ratio faster than larger banks do. Nevertheless, liquid assets as a proportion of total assets, remain higher on average for smaller banks throughout the sample period, consistent with Kashyap and Stein (2000).
Another variable of interest in the recent crisis is the level of leverage by bank size and over time, and this is shown in Figure 3. Leverage is defined as total tangible assets divided by tangible equity. Figure 3a reports total leverage over time for banks of different sizes. Smaller banks consistently are less levered than large banks with the exception of the recent crisis.

We are also interested in the characteristics of banks that fail or receive FDIC assistance (failed banks) at some point in time. We construct a panel of failed banks between 2008-2010 and we track some of their balance sheet characteristics over time. A stark difference between banks that fail or receive assistance, and banks that do not, is their leverage ratio.

---

11 Tangible equity equals total assets minus total liabilities minus intangible assets, such as goodwill.

12 This might reflect special government programs under TARP (Troubled Assets Relief Program) mainly affecting larger banks.
Figure 3 shows that for banks that eventually fail, leverage increases sharply before their failure. This figure is consistent with the empirical findings in Berger and Bouwman (2013) who find that higher capital can increase a bank’s survival probability.

2.2 Time Series Statistics

Banks in our model will face uninsurable idiosyncratic shocks coming from deposit growth, loan write-offs and different asset returns. At the same time banks will be exposed to aggregate uncertainty which generates cyclical fluctuations. We will use the data to constrain the data generating processes of the model’s exogenous variables. The idea will be to use these processes as inputs to the theoretical model and then examine the ability of the model to explain the endogenous variables of interest: new loans and asset growth, tangible equity, wholesale funding and failure rates. This section shows estimation results for
the relevant variables. In particular, we show that there is a large amount of heterogeneity across banks and over time.

2.2.1 Uninsurable Risk

To capture uninsurable risk from deposit growth, loan write-offs and returns on different assets, we examine the time-series statistical properties of these processes individually for each bank. We concentrate on the first and second moment and the persistence of these risks, conditional on a boom or recession state. Given our approach, that relies on computing individual statistics per bank over a twenty year period (84 quarters) and the fact that we condition on booms and recessions as well, we cannot perform a vector autoregression for all these variables at the individual bank level. A panel VAR would be an alternative approach but we prefer to emphasize the rich heterogeneity that exists at the individual bank level that can be generate substantial heterogeneity in behavior in response to regulatory changes.

We focus on four main variables: real deposit growth, loan write-offs (defined as current loan charge-offs over end of last period total loans), and the spreads of loan and liquid asset returns over transaction deposit rates. All these variables combine information that exists in either balance sheet or profit and loss accounts or both. Loan returns are defined as loan interest income (which includes income from all customer loans) divided by lagged loans, and liquid asset returns are defined as interest income on Fed funds sold and reverse repos plus gains or losses on securities over the period divided by the stock of lagged liquid assets. Deposit rates are defined as interest expense on transaction deposits divided by lagged transaction deposits. In the analysis that follows we report statistics both unconditionally
but also conditional on a boom or a recession and for both large and small banks.

Starting with real deposit growth, we run for each bank-type an AR(1) time series regression if there are more than 35 consecutive observations for a particular bank. Our statistical analysis cannot reject the null of non-stationarity on the level of deposits, consistent with the idea that in a growing economy deposits are non-stationary. We therefore produce histograms for the AR(1) coefficients for deposit growth rates. Figure 4 reports the histograms of the AR(1) coefficient for large and small banks and shows that an AR(1) coefficient for zero cannot be rejected. Nevertheless, the histograms also illustrate the large amount of heterogeneity that exists at the individual bank level with the coefficients ranging from -0.7 (-0.5) to 0.8 (0.6) for small (large) banks.

The loan write-off process is already normalized by the stock of outstanding loans and is therefore likely to be (and turns out to be) stationary. We follow the same procedure as for the deposit growth process and find that the histograms show strong positive persistence.
Figure 5: AR(1) of loan write-off process

(a) Large banks

(b) Small banks

for large banks and a milder persistence for small banks, as shown in figure 5. The figures again illustrate the large amount of heterogeneity that exists in the data.

Table 2 reports the results for the mean, standard deviation and persistence across different variables of interest, both unconditionally and conditioning on a recession and for both small and large banks. This table illustrates that there are substantial differences both across banks (small versus large) and also over the business cycle (booms versus recessions).

Starting with the idiosyncratic component of the loans’ write-off process, we observe that the persistence is higher for larger than smaller banks but does not significantly change between booms and recessions. Deposit growth tends to be higher in recessions rather than in booms,

13We count as a recession the two quarters before the start, and the six quarters after the end, of the NBER-dated recessions. There are two reasons for doing this. First, this allows us to extend the sample given the short recessions in this period. Second, loan write-off rates in the data start picking up before the official NBER recession dates and continue after well after the end of the official recession end-date.
Table 2: Time varying aggregate parameters

<table>
<thead>
<tr>
<th>Parameter (all in % except AR(1))</th>
<th>Small banks</th>
<th>Large banks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncon</td>
<td>recession</td>
</tr>
<tr>
<td>Loan write-offs: mean</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td>Loan write-offs: AR(1)</td>
<td>0.21</td>
<td>0.20</td>
</tr>
<tr>
<td>Loan write-offs: st. deviation</td>
<td>0.17</td>
<td>0.20</td>
</tr>
<tr>
<td>Deposit growth: mean</td>
<td>1.0</td>
<td>1.4</td>
</tr>
<tr>
<td>Deposit growth: AR(1)</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>Deposit growth: st. deviation</td>
<td>11.0</td>
<td>11.0</td>
</tr>
</tbody>
</table>

probably due to a flight to the safety deposits offer. The standard deviation of loan write-offs is also higher in recessions and is higher for larger banks. Consistent with Figure 4, the median persistence of real deposit growth is zero over both states and bank sizes. Moreover, even after conditioning the aggregate state of the economy, individual bank heterogeneity remains pervasive.

2.2.2 Balance Sheet Distributions

We compute the mean and the variance of key balance sheet and income statement items for each bank over time, provided that the bank has at least 20 observations. We then produce histograms for these moments that can be either unconditional or conditional on a boom or conditional on a recession.

Figure [6] shows the distribution of the deposit to asset ratio and the loan to asset ratio for small banks. Loans are the most important component on the asset side of the balance sheet. Figure [6b] shows a pretty wide dispersion of this ratio across small banks, ranging from 15% to 90%, with a mean equal to 60%. Figure [6a] shows that the dispersion of the
deposit to asset ratio, in contrast, is much smaller. Most of these banks have a deposit to asset ratio around 85%.[15] An empirically successful model should be able to replicate this heterogeneity in the data.

2.2.3 Cyclical Properties

Figure 7 shows the behavior of loan growth, the evolution of problem loans and the resulting bank failures over the sample period. Figure (7a) shows that loan growth rates are procyclical whereas problem loans are countercyclical. Problem loans are high at the beginning and at the end of the sample, coinciding with recession periods. The first period reflects the savings and loans (S&L) crisis and the second period the recent financial crises starting in 2007.

[15] The histograms for the other two ratios (wholesale funding to asset and securities to asset ratio) can be found in section 5.2, where we compare them to their model equivalents.
Figure 7: Cyclical properties of loan growth, problem loans and bank failures

(a) Loan growth and problem loans

(b) Aggregate problem loans and bank failures

Figure (7b) shows the business cycle behavior of aggregate problem loans and bank failures. Not surprisingly, these two series are highly correlated and strongly countercyclical. Banks do fail over the business cycle in a countercyclical way and the possibility of banks failing will be an important ingredient in our model. The unconditional failure rate is 0.05% (0.07%) for small (big) banks, which rises in recessions to 0.17% (0.18%) and falls in booms to 0.01% (0.01%).

2.3 Summary

In the cross section, larger banks tend to rely less on deposits and more on wholesale funding and they tend to be more levered. Moreover, banks that fail tend to have more levered balance sheets before eventual failure. Moreover, a large heterogeneity exists in the cross section on items that are either on the bank’s balance sheet (deposit and loan to asset
ratios, for instance) but also in their profit and loss statements (loan write-offs and profits).

In the time series, real loan growth is procyclical as it falls in recessions, whereas problem loans and failures are countercyclical as they tend to increase during recessions. We next build a structural model to replicate these stylized facts.

3 The Model

3.1 The model environment

We consider a discrete-time infinite horizon model. We assume that banks are run by managers whose incentives are fully aligned with those of bank shareholders. Therefore, banks maximize the present discounted value of utility of their existing shareholders and have limited liability. We consider interest income from relatively illiquid loans and liquid assets as the key driver of decisions by commercial banks. Banks in our model have the following stylized balance sheet: their liabilities consist of deposits, wholesale funding (equivalent in the data to the sum of Federal Funds borrowed, subordinated debt and other non-deposit liabilities) and equity. Their assets consist of loans and liquid assets (securities). A stylized balance sheet is shown in Table 3, which also reports the real rate of return on each asset and liability.
3.1.1 The Asset Side of the Balance Sheet

Consistent with the maturity transformation role of banks, we assume that loans ($L_t$) are long term and these loans are funded through deposits, wholesale funding and equity capital. Both deposits and wholesale funding are assumed to be of shorter maturity than customer loans. Such a maturity mismatch gives rise to funding liquidity risk. To capture this risk we assume that a fraction of outstanding loans ($\theta$) gets repaid every period. This generates an exogenous deleveraging process, which we calibrate to our data. At the same time, in every period the bank issues (endogenously) new long term loans ($N_t$) to customers.

The income from customer lending is the interest income from long term loans. The interest rate earned on outstanding customer loans equals $(r_{Lt} - w_t)$ where $r_{Lt}$ is the real return on loans, and $w_t$ measures the loans that banks have to write-off every period. Issuing new loans requires banks to assess and screen their clients. This screening cost is assumed to be convex in new loans either because bank resources get stretched over more projects or because the quality of additional projects is declining. The specific functional form is discussed in Section 4.2.

Loan write-offs follow a process with both aggregate and idiosyncratic components. We model this by assuming (consistent with the data) that the idiosyncratic first and second moments depend on the aggregate state (the state of the economy). Empirically, there is more uncertainty during recessions than booms in the loan write-off process. Therefore, loan

---

16 We omit using an i-subscript for banks but all bank-specific variables must be understood to have an i-subscript.
write-offs have a higher mean and a higher variance during recessions than during booms. We calibrate these moments to what we calculate from our data set.

Instead of investing in long term loans, banks can also invest in short term liquid assets ($S_t$ denoting securities). The return on these liquid assets $r_{St}$ is stochastic with both aggregate and idiosyncratic components that are specified in the calibration section.

### 3.1.2 The Liability Side of the Balance Sheet

The main liability of most commercial banks are customer deposits $D_t$. We assume that the deposit growth rate, similar to loan write-offs, follows a process where the mean and variance of the idiosyncratic shocks depend on the aggregate state. Conditional on the aggregate state, the growth rate of deposits is i.i.d. over time, and can be well approximated by a log-normal distribution.

$$\log (G_{D_t}) \sim N(\mu_{D_j}, \sigma^2_{D_j})$$

where $j$ refers to a boom or a recession. This is consistent with the idea that there might be higher uncertainty in recessions than in booms. We use the empirical counterparts to determine specific values for the means and variances.

A second source of external funds for banks is the wholesale funding market where banks can borrow short term (wholesale funding, $F_t$). However, as discussed in the data section, there is an important difference between small and large banks in their reliance on short-term borrowing from the wholesale market. For most small banks, wholesale funding is a small fraction of their overall liabilities even in recent years, as shown in Table 1a and Figure 1b.
To capture this difference in the model we specify a size-dependent net cost function (over
the interest rate cost) of accessing the wholesale market. We assume a convex function to
reflect that higher short term borrowing implies that more risk is borne by lenders, thereby
justifying a higher external finance premium to access this market. The specific functional
form is discussed in Section 4.2.

3.1.3 Equity

Equity is defined as assets minus liabilities. Equity is the sum of past earnings (positive
or negative), reduced by the amount of dividends the bank has paid to shareholders. At any
period \( t \), the bank has the option to pay out dividends \( (X_t > 0) \). If, in addition, we denote
by \( \Pi_{t+1} \) the bank profits at time \( t + 1 \), then the amount of equity at the beginning of next
period is given by

\[
E_{t+1} = E_t + \Pi_{t+1} - X_t \tag{1}
\]

3.1.4 Capital Ratio Requirement

Banks are subject to regulatory constraints regarding their capital adequacy ratios,
namely a minimum ratio between measures of bank capital and measures of bank assets.
We consider an exogenously specified leverage ceiling that regulators set and banks must
respect. Leverage is defined as the ratio of total assets (total loans plus liquid assets) to
equity\(^{17}\). *Ceteris paribus*, the higher the profitability of the bank in a given period, the

---

\(^{17}\)Our model has only equity whereas in the data there is the distinction between tangible and non-tangible
equity. All our empirical results use only tangible equity since this measure is closer to what regulators
consider as loss-absorbing capital.
higher its retained income and therefore equity, and the less likely it is to breach its regul-
atory leverage limit in the future. This gives the bank the incentive to extend more lending
to customers to boost its return on equity or to pay out dividends to its owners, and these
are the two key endogenous decisions studied by the model.

The leverage constraint (the inverse of the capital ratio requirement) is captured by
parameter $\lambda$ which gives the maximum ratio of assets to equity that the bank must respect:

$$
\frac{\omega_L(L_t + N_t) + \omega_S S_t}{E_t - X_t} \leq \lambda
$$

The numerator in (2) calculates risk-weighted assets under different assumptions about
risk weights. The denominator denotes equity after dividends. We experiment with different
assumptions about risk weights both to understand the implications of the model but also
as part of counterfactuals.\(^{18}\)

### 3.1.5 Objective function

Banks discount the future with a constant discount factor $\beta$. They maximize the present
discounted value of a concave function of dividends:

$$
V = E_0 \sum_{t=0}^{\infty} \beta^t \frac{X_t^{1-\gamma}}{1-\gamma}
$$

where $E_0$ denotes the conditional expectation given information at time 0. Bankers are risk
averse: $\gamma > 0$ is the coefficient of relative risk aversion. The concavity from risk aversion
captures the idea that banks (like other firms) might want to smooth dividends over time, as

---

\(^{18}\)One particular case of interest involves using a weight on liquid assets equal to 0%, that is, studying the
case where $S_t$ disappears from (2).
suggested by empirical evidence in Acharya, Le and Shin (2013). In the data set we analyze, it is indeed the case that dividends to equity (assets) are smoother than profits to equity (assets) and this becomes one of the endogenous moments that the model can match.\footnote{Dividends need to always be positive in this world due to the concavity of the utility function but can be set very close to zero, if needed.}

3.1.6 Profits

The profits of bank $i$ attributable to shareholders are\footnote{We now introduce the $i$ subscript to make the distinction between aggregate and idiosyncratic variables.}

$$
\Pi_{i,t+1} = \left( r_{L,t+1} - w_{i,t+1} \right) L_{it} + r_{L,t+1} N_{it} + r_{S,t+1} S_{it} - r_{D,t+1} D_{i,t} - g_N (N_{it}) - g_F (F_{it}) - c_{D_{i,t}}
$$

where the first two term is the interest income on performing loans when we assume that new loans do not experience any losses; the third term reflects income from holding liquid assets, the forth term is the cost from servicing deposits, the fifth term is the cost of issuing new loans and the sixth term is the cost of accessing the wholesale funding market. The seventh term is the control variable denoting new loans being issued and the final term is the non-interest expense associated with operating the bank and is modelled as proportional to deposits. This term includes various costs to operate a bank (operating expenses) and also the FDIC insurance surcharge to fund deposit insurance.

We could introduce corporate taxation in the model, but prefer to leave it out for three reasons. First, we would like to determine whether a model without a tax shield can replicate the observed high leverage in the banking sector. Second, the tax-shield benefit from interest expenses can be quite low in this sector given the low observed deposit rates in the last ten
years. Last, to the extent that the value of the tax shield is fairly constant over this period, adding a tax shield would most likely affect only the estimate of our operating expense cost. Introducing tax changes over the period of interest would further complicate the analysis and we leave this for future research.

3.1.7 Entry and exit

Exit is endogenous in this model. We assume that following bank failure, bankers pursue another career (outside banking) that we do not endogenize. The outside option yields a constant amount of consumption $C^D$ and a level of utility equal to $V^D$. Since the banker takes this continuation value into account when making decisions, exit is endogenous. In the simulation, whenever a bank exits, we exogenously add another bank that takes over the deposits of the failed bank but which starts at a good idiosyncratic state, i.e. low loan losses.

3.2 Timing

Figure 8 shows the timing of the model for a bank that continues in period $t$ with a stock of loans $L_t$, deposits $D_t$, and equity $E_t$. Since the various interest rates $r_t$ and the idiosyncratic loan write-off process $w_{it}$ are persistent, these are state variables in the bank’s problem as well. At the end of period $t$, decisions about new loans ($N_t$), dividends ($X_t$), liquid assets ($S_t$) and wholesale funding ($F_t$) are made. At this stage the leverage constraint must be respected. At the beginning of the next period the exogenous shocks (returns, 21 We have to assume that a failed banker can consume after exiting, otherwise no banker would ever choose to fail given the concave utility function.
deposit shocks and problem loans) are realized: the bank learns the various rates of return \( r_{t+1} \); deposit withdrawals and how many loans are repaid and how many loans have to be written off \( w_{t+1} \).

The bank decides whether to continue or fail at that stage. If the bank fails, it exits the market forever. If it continues, it repays wholesale funds and receives the payment on the liquid assets. These cash-flows, the flow profits and the new dividend payment \( X_{t+1} \) determine the equity \( E_{t+1} \) at the end of period \( t + 1 \). Deposits depend only on the initial value \( D_t \) and the exogenous shock realization in the current period and are therefore equal to \( D_{t+1} \) after the shock realizes. The stock of loans \( L_{t+1} \) is the sum of the old loan stock and the new loans made in period \( t \), adjusted for the exogenous repayment fraction \( \vartheta \) and the fraction of loans the bank has to write-off \( w_{t+1} \).
3.3 Value functions

A banker who has failed in the past cannot become a banker again. This banker enjoys an exogenous constant level of consumption $C_D$ yielding utility $V^{D^2}$.

A banker who has not failed in the past solves the following continuation problem that takes into account the fact that failure is possible in the future

$$V^C (L_t, D_t, E_t; w_t, r_t) = \max_{X_t, S_t, F_t, N_t} \left\{ \frac{(X_t)^{1-\gamma}}{1-\gamma} + \mathbb{E}_t [\beta V (L_{t+1}, D_{t+1}, E_{t+1}; w_{t+1}, r_{t+1})] \right\}$$

where the last term is defined as the upper envelope

$$V (L_t, D_t, E_t; w_t, r_t) = \max [V^D, V^C (L_t, D_t, E_t; w_t, r_t)]$$

subject to the equity evolution equation (1), the leverage constraint (2), the profit evolution (4) and the evolution of the loan stock

$$L_{t+1} = (1 - \vartheta - w_{t+1}) L_t + N_t.$$  

The first decision of the bank is to decide whether to continue operating. If the bank continues its operations, it chooses the optimal level of pay-out to shareholders $X_t$, how many new loans $N_t$ to issue, how many liquid assets $S_t$ to buy and how much funding $F_t$ to borrow on the wholesale market. If it ceases operations, it is liquidated.

---

22Specifically, the value $V^D$ is given by the formula $V^D = \frac{1}{1-\beta} \frac{(C_D)^{1-\gamma}}{1-\gamma}$.
4 Estimation

In this section, we first discuss the normalization that is necessary to make the model stationary. Second, we specify the two cost functions. Third, we present the exogenous parameters which are either common across both banks or are based on estimates for small and big banks, respectively. Lastly, we show the results from the Method of Simulated Moments estimation of the remaining four parameters that involves one estimation for small, and one for big banks.

4.1 Normalization

The estimated process of deposits contains a unit root. To render the model stationary, we normalize all variables by deposits, e.g. equity \( E_t \) is transformed into \( e_t \equiv \frac{E_t}{D_t} \). For this transformation to work, all components of the profit function have to be homogenous of degree one in deposits. Details of these transformations are in the solution appendix in Section [10.1]

4.2 Cost functions

The functional forms for the cost functions are chosen to satisfy different objectives. First, to limit the volatility of new loans and wholesale funding, we choose the cost of screening new loans and the cost of accessing the wholesale funding market to have a convex component. Second, to be able to normalize the model by deposits, these functions have to be homogenous of degree one in deposits. Both assumptions are common in the investment literature, see, for example, Abel and Eberly (1994).
We assume a convex screening cost in the ratio of new loans to deposits. To capture that the screening cost rises with the scale of the bank, we multiply it by deposits. Thus, the resulting cost function is

\[ g_N(N_t, D_t) = \phi_N n_t^2 D_t \]

where \( n_t \equiv \frac{N_t}{D_t} \) is the normalized variable, generating a Hayashi-type convex cost function.

A similar reasoning leads to the following cost of accessing the wholesale funding market

\[ g_F(F_t, D_t) = r_{Ft} F_t + \phi_F f_t^2 D_t \]

where the first term is the interest rate cost and the second term reflects a convex external finance premium. The external finance premium is increasing in the bank’s reliance on the wholesale funding market.

### 4.3 Calibrated parameters

We eventually estimate four parameters. Given the complexity associated with solving and estimating the model, we also have to choose certain other parameters exogenously. We next discuss these choices.

Table [I] reports the calibrated parameters that are the same for both small and large banks. The model period is one quarter. Therefore, we set the discount factor \( \beta \) to 0.98 and the risk aversion to 2. The weighted leverage limit is set to 12.5 reflecting an 8% risk-weighted capital adequacy rule consistent with Basel II. In the baseline we assign a risk weight equal to 65% on loans \( (L_t) \) and a risk weight equal to 10% on liquid assets \( (S_t) \), both consistent with Basel II capital adequacy rules (see appendix for further details).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>discount factor $(\beta)$</td>
<td>0.98</td>
</tr>
<tr>
<td>risk aversion $(\gamma)$</td>
<td>2</td>
</tr>
<tr>
<td>leverage limit $(\lambda)$</td>
<td>12.5</td>
</tr>
<tr>
<td>risk weight loans $(\omega_L)$</td>
<td>0.65</td>
</tr>
<tr>
<td>risk weight securities $(\omega_S)$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 4: Fixed parameters

The model features aggregate and idiosyncratic uncertainty. In general, we estimate the stochastic processes generating these variables from the data discussed in section 2. In order to keep the state space tractable, we assume that the aggregate component of all variables follows the same two-state persistent process. We label the bad aggregate state a recession and the good one a boom. We choose the transition probabilities to obtain recessions that last for 2 years on average and booms that last for 5 years on average. The deposit interest rate depends on the aggregate state only, whereas conditional on the aggregate state and bank size, idiosyncratic uncertainty has different properties (the variance of the shocks is different for example).

Idiosyncratic uncertainty (or background risk) is captured by four different variables: loan write-offs, the deposit growth rate, the loan spread, and the spread on liquid assets. As discussed in section 2.3, see Table 2, idiosyncratic bad loans behave very differently in booms and recessions. Due to this asymmetry, we model the bad loan process as state dependent. During a recession, the bad loan process is significantly worse for banks. The mean is around 50% higher, while the standard deviation and the persistence also increase significantly. We use the means, standard deviation and persistence from Table 2 as inputs in the structural model. Note that these are conditional on a boom or a recession and are also conditional on
bank size (small versus large).

4.4 Estimated parameters

There are four parameters left to be estimated: the flow cost of operating the bank $c$, the new loans screening cost parameter $\phi_N$, the external finance premium for accessing wholesale funding $\phi_F$, and the value of consumption after failure $c^D$. We estimate the model separately for small and big banks by the Method of Simulated Moments using eleven moment conditions. We use the standard deviation of the chosen moments in the cross-section to weight the moment conditions and minimize their squared differences from their simulated counterparts.

Table 5 shows the estimated moments for big and small banks in columns 2 and 4, respectively. Their corresponding data counterparts are in columns 3 and 5. Overall, the model matches the moments reasonably well but the OID (Overidentifying restrictions test) rejects the model, implying that further work is needed to match the data. In terms of specific results, the mean failure rate is matched. Similarly, the means of the loan to asset and the deposit to asset ratio are fairly well matched too. The model overpredicts equity holdings, i.e. the bankers in our model have a stronger precautionary motive than what observed in the data. This could come from preference parameters, they might be too patient or too risk averse. The model overpredicts the dividend to profit ratio significantly. This might be due to the omission of all taxes. The second moments of the balance sheet variables are slightly overpredicted but the model does generate a smooth dividend process, as in the data.

Table 6 shows the estimated parameters. The estimated parameters for big and small
Table 5: Estimated and data moments

<table>
<thead>
<tr>
<th>Moments</th>
<th>Big banks</th>
<th></th>
<th>Small banks</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>model</td>
<td>data</td>
<td>model</td>
<td>data</td>
</tr>
<tr>
<td>mean default rate (in %)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>mean loans/assets</td>
<td>0.580</td>
<td>0.599</td>
<td>0.602</td>
<td>0.604</td>
</tr>
<tr>
<td>mean deposits/assets</td>
<td>0.628</td>
<td>0.667</td>
<td>0.824</td>
<td>0.850</td>
</tr>
<tr>
<td>mean equity/assets</td>
<td>0.110</td>
<td>0.077</td>
<td>0.131</td>
<td>0.102</td>
</tr>
<tr>
<td>mean profit/equity</td>
<td>0.055</td>
<td>0.046</td>
<td>0.043</td>
<td>0.031</td>
</tr>
<tr>
<td>mean dividends/profits</td>
<td>1.067</td>
<td>0.678</td>
<td>1.167</td>
<td>0.527</td>
</tr>
<tr>
<td>std. loans/assets</td>
<td>0.126</td>
<td>0.068</td>
<td>0.088</td>
<td>0.080</td>
</tr>
<tr>
<td>std. deposits/assets</td>
<td>0.082</td>
<td>0.065</td>
<td>0.083</td>
<td>0.038</td>
</tr>
<tr>
<td>std. equity/assets</td>
<td>0.039</td>
<td>0.013</td>
<td>0.040</td>
<td>0.020</td>
</tr>
<tr>
<td>std. profit/equity</td>
<td>0.022</td>
<td>0.059</td>
<td>0.017</td>
<td>0.031</td>
</tr>
<tr>
<td>std dividends/profits</td>
<td>0.830</td>
<td>0.990</td>
<td>1.21</td>
<td>0.806</td>
</tr>
</tbody>
</table>

Table 6: Estimated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Big banks</th>
<th></th>
<th>Small banks</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>operating cost (c)</td>
<td>0.006</td>
<td></td>
<td>0.0065</td>
<td></td>
</tr>
<tr>
<td>screening cost new loans (\phi_N)</td>
<td>0.740</td>
<td></td>
<td>0.771</td>
<td></td>
</tr>
<tr>
<td>risk premium wholesale funding (\phi_F)</td>
<td>0.008</td>
<td></td>
<td>0.063</td>
<td></td>
</tr>
<tr>
<td>consumption after bank failure (\phi_D)</td>
<td>0.002</td>
<td></td>
<td>0.0018</td>
<td></td>
</tr>
</tbody>
</table>

banks are rather similar with the exception of the weight on the convex cost of accessing wholesale funding markets. As shown in section 2.1 the crucial difference between small and big banks is the differential access to the wholesale funding market. Thus, the result that the estimated \(\phi_F\) is eight times lower for big than small banks is reasonable. This leads to a significantly lower share of deposits in total assets for the big banks, as can be seen in the third row in Table 5. Moreover, better access to this alternative funding source also allows big banks to operate with lower equity, despite the fact that we assume the same preferences across banks.
5 Results

We first present individual policy functions to enhance our intuition about the economics behind the model and then proceed with analyzing the implications of the model through simulations.

5.1 Policy functions

To understand the workings of the model, we first present the policy functions. Having normalized the model by deposits, we are left with two continuous state variables: normalized loans and equity. Due to the persistence in the aggregate state and idiosyncratic loan losses, there are two additional discrete state variables, an aggregate state that can be a boom or a recession and idiosyncratic problem loans that can be either high or low. All the policy functions shown are for a big bank and are for the same aggregate and idiosyncratic state. All policy functions share the feature that the leverage constraint becomes binding if loans exceed equity by the allowed multiple. For instance, all banks with equity equal to 0.06 and loans exceeding 1.2 will be closed down immediately. In the graphs, this region is at the very right end of the loan state.

Figure 9a shows the dividend policy function which has three noteworthy implications. The first one is standard. For a low amount of loans and sufficiently high equity (so that a bank is not closed down), dividends are monotonically increasing in equity. This happens because equity is the measure of the banker’s wealth and a richer banker can consume more. Second, there is a small hump in the direction of loans, keeping equity fixed. For low levels

\[23\text{To be precise, the policy functions are for a big bank in a recession but with low idiosyncratic bad loans. The policy functions in other states and for small banks look similar.}\]
of loans, the bank wants to expand its loan exposure. This, however, incurs screening costs for issuing new loans which the bank has to pay from available equity. When the bank is already close to its desired level of loan holdings, it does not have to pay this cost and can therefore enjoy higher dividends. The third region is for low levels of equity and high levels of loans. In this region, banks, at first, pay out low amounts of dividends since they get close to the leverage constraint. When equity is so low that banks could pay out only a very small dividend to stay in business and therefore not violate the leverage constraint, they pay out all remaining equity as a dividend and close the bank.

Figure 9b shows the wholesale borrowing of these banks. Due to the convex cost in most parts of the state space banks borrow the optimal amount. The area where borrowing is higher is the one where banks do not have sufficient deposits and equity to fund their outstanding loan book. Thus, to fund their loans, they need to borrow on the wholesale market, and they borrow more the less initial equity they have.
Figure 10: Policy functions with low idiosyncratic loan losses during a boom

(a) New loans

(b) Securities

Figure 10a shows the issuance of new loans. At low initial loan levels and with sufficient equity, banks can reach their desired level of loan holdings. However, due to the convex loan issuance cost, banks do not go to this desired level of loan holdings in one step. Thus, as equity increases, new loans increase monotonically to a desired level given a low level of initial loans. At higher levels of equity, banks prefer to invest in liquid assets, see Figure 10b. Buying liquid assets does not incur any adjustment cost, therefore the investment in liquid assets is monotonic in initial equity. Liquid securities decline as a function of initial loans due to a hysteresis effect arising from the illiquidity of old loans.

To take stock, given a particular state, banks have a desired balance sheet structure. Due to the direct adjustment costs for issuing new loans and for borrowing in the wholesale market and the dividend smoothing motive, it takes time until they reach that state. When the states change, the policy functions change in the expected way. If the idiosyncratic persistent bad loans increase, banks issue less new loans and instead invest more in liquid
assets. If the aggregate state changes to a boom, they expand their overall balance sheet by borrowing more in the wholesale market. The resulting cyclical pattern is shown in Section 5.3.

5.2 Cross section

In this section, we provide further empirical evidence for heterogeneity and compare the model outcomes to their data counterparts. The results here are the outcome of simulating the model for the small banks. Figures 11 to 14 all show histograms of the respective variable; the data (simulations) are depicted on the left (right) hand side of each figure.

Figure 11 shows the distribution of the unconditional average deposit to asset ratios in the data and in the model simulation, respectively. For most banks, this ratio is between 70 and 90 percent. The model replicates this distribution relatively well, even though the model distribution is somewhat more symmetric than the data.

---

24 Details for the simulation procedure can be found in the computational appendix in Section 10.1.
25 The results for big banks are similar and skipped for brevity.
Figure 12 shows the distribution of wholesale borrowing. While the distribution in the data is smoothly right-skewed, the model is more symmetric. Moreover, the model misses the large mass of the distribution at $F = 0$.\footnote{We experiment with a fixed cost in the model. This generates a mass at zero. However, the shape of the distribution still differs significantly from the data. We leave this issue for future research.} These model results can be understood from the policy function for wholesale borrowing, Figure 9b. Banks have a preferred value for this borrowing which does not vary much with the state. This explains the concentrated distribution in the model.

Figure 13 shows the distribution of loan to asset ratios. This ratio is a lot more dispersed than, for example, the deposit to assets ratio. This wide dispersion, even when we focus on small banks, shows that heterogeneity is an important feature of the data. The model captures this wide dispersion reasonably well, even though its mass is closer to the mean than the data.

The distributions for liquid assets (not shown) are similarly well captured. Lastly, Figure
Figure 13 shows the distributions of leverage. Leverage in the data is distributed symmetrically around its mean. The model dispersion is somewhat wider and in particular is skewed to the left and not as symmetrical as in the data. The mean in the model is lower since equity holdings (which are the inverse of leverage) are higher, as was shown in the estimation section.

5.3 Time series behavior

Figures (15) to (18) provide a more detailed view on the time series behavior of the model. The corresponding figures in the data are Figures (1) to (7). Figure (15a) shows the deposit to asset ratio in the model over time for big and small banks and Figure (15b) shows the wholesale funds to total assets ratio. Consistent with the data, small banks rely significantly more on deposits to fund their operations, while large banks rely on wholesale funds to a significant extent. However, even for large banks deposits are the main funding

\[\text{[27] The shaded areas denote model recessions.}\]
Figure 14: Distribution of leverage ratios

(a) Data

(b) Model

Figure 15: Evolution of deposit to asset ratio and wholesale funding to asset ratio in the model

(a) Deposits

(b) Wholesale funding
source. During recessions, when the perceived return on assets falls, the deposit to asset ratio rises for both big and small banks because banks reduce their borrowing in the wholesale funding markets. Because equity is a small component of the balance sheet, reduced borrowing in wholesale funding markets translates into a relative increase in the share of deposits in the total balance sheet during recessions. Thus, even though total assets shrink in a recession, the relative importance of deposits in the balance sheet increases. This cyclical pattern seems consistent with the evidence in Figure 1, even though the empirical counterparts are based on a limited number of recessions and should therefore be treated with caution.

Figure (16) shows the asset side of the balance sheet. As a proportion of the total balance sheet, small banks invest more (less) in loans (securities) than large banks because smaller banks face less uncertainty for loan write-offs and lower deposit growth uncertainty than larger banks.

Moreover, the fraction of loans in total assets is strongly procyclical for both types of
banks and arises because loan write-offs are very countercyclical. However, the evolution of loans and liquid assets at the onset and during the recession is not the same across the two bank sizes. For large banks, at the onset of the recession the share of loans jumps up and then declines during the recession, a behavior that is different from smaller banks. The explanation is that large banks also invest less in liquid assets during recessions. Thus, they want to reduce their holdings of both liquid assets and loans. Banks can reduce their liquid assets immediately at the onset of a recession. Loans are illiquid, however, since only a small fraction gets repaid every period. Thus it takes some time until banks deleverage by reducing their loans, which means that the loan to asset ratio takes a few quarters to fall below the level it had just before the recession. This effect is more pronounced for big than for small banks because big banks fund their activities to a larger degree with wholesale funds which can be quickly cut back. For small banks it takes only a few quarters until the loan to asset ratio is back to its original level, while for big banks it takes significantly longer. For example, in the long recession in periods 165-194, it takes 10 quarters until the loan to asset ratio falls below its initial value for the big banks.

Figure 17a shows the leverage ratio of the banks in the model. Consistent with the data, see Figure 3a, big banks are more highly levered than small banks. Since small banks have less access to the wholesale funding market, they rely more on deposit and equity funding. This increase in equity funding translates into a lower leverage ratio. Leverage is countercyclical for big and small banks. They use the profits to build up equity during good times through retaining some of their earning. This is similar to the role of precautionary savings in the consumer literature. During recessions, equity declines and leverage increases.
because profits are lower. Since banks want to smooth dividends to some degree, they do not lower dividend payments as much as profits fall. Again, there is a non-monotonicity at the onset of a recession for big banks. Since they are able to cut their liquid assets and liquid liabilities quickly, the share of equity in total assets actually rises, and so leverage falls, at the onset of a recession. However, the reduced profits deplete equity rather fast so that it takes less than a year until the leverage level rises above its initial level.

Figure 17b is the model counterpart to Figure 3b. It shows the leverage for banks that ultimately fail and those that survive in a model recession. Leverage rises significantly for those banks that ultimately fail. The different evolution is similar for big and small banks. Thus, an increase in leverage is an indicator for successive vulnerability. This is consistent with the evidence in Berger and Bouwman (2013).

Figure 18 is the model counterpart to Figure 7 in the data section. Figure 18a shows that aggregate loan growth in the model is strongly procyclical. Loan growth hardly differs
between big and small banks. It is positive during booms and declines during recessions. While loan growth is fairly stable during recessions, meaning that banks delever continuously, it is somewhat different in booms. As explained above, at the beginning of a boom, banks have invested a relatively small share of their assets in loans, therefore as soon as aggregate conditions improve, they start lending more to replenish their loan book. This makes loans very procyclical. Note that they do this despite the convex cost function for new loans. Thus, at the beginning of the boom, they are willing to incur these high costs because the benefits of increasing their loan exposure are high enough.

Figure 18b shows that failures are higher for small than for big banks. The reason is that big banks have cheaper access to wholesale funding markets which enables them to deal better with shocks. Figure 18b also shows that failures are strongly countercyclical. There are almost no failures in good times. The intensity of failures increases strongly with the length of a recession. In short recessions there are only few failures, while in long recessions
the failure rate rises above one percent. The reason is that the equity stock of banks gets more and more depleted during recessions. This is confirmed in Figure 17a where leverage is at its highest level at the end of the longest recession.

6 Counterfactual policy experiments

In this section we use our structural model of the banking system to perform counterfactual experiments that can provide guidance to policy makers about the possible effects on behavior from changing policy-controlled parameters or the economic environment.

6.1 Financial crisis

The recent financial crisis has two important elements: first, a wholesale funding market freeze and second, forced sales of liquid assets generating losses for banks. We use our model to assess the implications of such events in our model economy. We model the freeze in the wholesale funding market as an unexpected loss of non-deposit funding for all banks. Thus, for one (crisis) period, banks are unable to obtain any wholesale funds $F_{i,t}$. Forced sales are modeled as an unexpected drop in the value of liquid assets. We assume however that banks know that normal conditions in the wholesale funding and liquid asset markets will be restored in the following period.\textsuperscript{28} This is roughly in line with the events after the Lehman bankruptcy where it took a while for policy makers to respond adequately to the freeze in the wholesale funding market.

\textsuperscript{28}Thus, the freeze lasts only for one quarter.
6.1.1 Wholesale funding market freeze

In our model, we assume that at the end of the recession in period 45, banks can not borrow any wholesale funds $F_{i,t}$. This makes all banks that rely on this market, mainly the larger banks, vulnerable to a liquidity crisis.

We implement this by using the regular continuation value functions all the time, also in the crisis period when unexpectedly no bank has access to wholesale funds. We compute new policy functions in this particular period for new loans issued, liquid assets bought and dividends issued. Borrowing in the wholesale market is, by definition, zero in the crisis period. We then simulate the model up to the crisis period as before using the regular policy functions. In the crisis period we use the new policy functions. After the crisis period we use again the normal ones.

Figure (19) shows the results for large banks and is compared to a standard simulation without a freeze in the wholesale funding market. Figure (19a) shows the drop of borrowing in
the crisis period. Large banks, however, hold significant amount of liquid assets. Thus, upon losing access to short term funds, they lower their investments in short term liquid assets, see Figure 19b. Thus, liquid assets are an important insurance device against problems in funding markets for large banks. However, those large banks that rely extensively on wholesale markets do not survive such a crisis. The failure rate increases from 0.65% to 1.1% for large banks. Small banks (not shown) are a lot less affected because they hardly use the wholesale funding market. Their failure rate does not increase in such a crisis.

6.1.2 Forced sales

We now analyze losses due to forced sales in the crisis period. Fire sale losses are modeled as an unexpected reduction in the return on liquid assets $r_{S,t}$ in the crisis period. As in the previous experiment, normal times resume after the crisis period.

Table 7 shows the failure rate of small and large banks for the same period across four different simulations. Column 2 shows the results in a normal recession without further stress in financial markets. Column 3 shows the effect of a freeze in wholesale funding markets, as discussed previously. The failure rate of large banks increases significantly while small banks are not affected. Column 4 shows the failure rate when there are unexpected losses on the return on liquid assets of 1 percent, i.e. $r_{S,t} = -0.0129$. The failure rate in this case increases six fold. There is no difference between big and small banks. Both types hold on average around 40% of their total balance sheet as liquid assets. Thus, an unexpected loss on these holdings reduces equity significantly which then triggers failures.

If both types of crises occur in the same period, as has happened during the Lehman

\footnote{A quarterly loss of 1% is in line with returns during the Lehman crisis.}
<table>
<thead>
<tr>
<th></th>
<th>Normal recession</th>
<th>Freeze</th>
<th>Forced sale losses</th>
<th>Freeze &amp; Forced sale losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small banks</td>
<td>0.58%</td>
<td>0.58%</td>
<td>3.2%</td>
<td>3.3%</td>
</tr>
<tr>
<td>Large banks</td>
<td>0.65%</td>
<td>1.1%</td>
<td>3.5%</td>
<td>3.6%</td>
</tr>
</tbody>
</table>

Table 7: Failure rates in different crisis periods

Figure 20: Cyclical properties of loan growth and bank failures with different leverage limits

(a) Aggregate loan growth

(b) Bank failures

... crisis, the forced sale losses dominate the overall effect completely, see Column 5. Thus, in our model solvency problems are more important than liquidity problems.

6.2 Tightening leverage constraint

One important policy change currently being implemented is a move towards tighter leverage limits. In this section we show the consequences of such a policy by comparing results across steady states. The partial equilibrium nature of our model does not allow us to make a welfare comparison between the status quo and a tighter leverage limit. Nevertheless, the model does provide instructing results from limiting cases. We solve the model for different values of the leverage limit, leaving all other parameters at their benchmark values. The
simulation uses exactly the same shock sequence. Figure (20) shows loan growth rates and failure rates for the benchmark economy with a leverage limit of 20 and a counterfactual economy where the leverage limit is tightened to 17 for small banks. As can be seen in Figure (20a), the average growth rate of loans is not affected much. However, and in line with, for example, Repullo and Suarez (2013) loan growth becomes more procyclical. In particular, the growth rate during recessions is negatively affected. Figure (20b) shows the impact on the bank failure rate. While this rate is hardly affected during booms, its peaks during recessions increase more than three times. Thus, despite reducing their loan supply somewhat, banks still fail at a much higher rate under a reduced leverage limit.

Figure (21) shows the average equity holding for different leverage limits for big and small banks. It is clear that a tighter leverage constraint leads banks to hold more equity. This effect is very similar for large and small banks. In that sense, a tighter leverage limit leads to safer bank balance sheets. However, a tighter leverage also means that, ceteris paribus, a bank with a given balance sheet becomes more likely to breach this lower limit. It is this
latter effects that dominates. Therefore a tighter leverage limit leads to an increase in the failure rates, as shown in Figure (22a). Moreover, the increase for small banks is somewhat larger than for big banks.

This cross-sectional difference is even more pronounced for loan supply, as can be seen in Figure (22b) which shows the percentage change in average loan supply. This effect is mostly negative, even though, there are some non-monotonicities. But again, the effect is much more pronounced for small banks. Lowering the leverage constraint to \( \lambda = 15 \) leads to a decline in loan supply of about 1.8 percent in the new steady state for small banks but only of about 1.2 percent for big banks. The reason is that big banks through their better access to the wholesale funding market can easier cope with shocks. Thus, they are less likely to break the leverage constraint and this effect is more pronounced at lower levels.

Note however, one important motivation for tighter leverage limits are the fiscal costs of bank bail-outs. Even though we do not model bail-outs directly, we can use our model to assess its implications. Bail-out costs depend on two quantities: first the failure rate and
second the expected loss conditional on failure. We have already seen that a tighter leverage limit increases the frequency of bank failures. However, the second component works in the opposite direction. A tighter leverage limit leads to higher equity holdings, see Figure 21. Thus, it is likely that banks will have more equity capital left in the case of a failure which lowers the cost of third parties.

7 Conclusion

We use individual U.S. commercial bank balance sheet information to develop stylized facts about bank behavior in both the cross section and over time. We then estimate the structural parameters of a quantitative banking model choices (new loans, liquid investments and endogenous failure) that are made in the presence of undiversifiable background risk (problem loans, interest rate spreads and deposit shocks) and regulatory constraints. The model replicates many features of the data and can therefore provide a useful approximation of reality to perform counterfactual experiments. In particular, and in contrast to previous papers, we show that a tighter leverage limit can counter-intuitively lead to more bank failures since banks might not increase their equity holdings by a sufficient amount in response to tighter capital requirements. Future work can extend the model in general equilibrium to determine whether the results continue to hold. In addition, it would be interesting to introduce a central bank as a lender of last resort and to investigate the implications of different recapitalization mechanisms for various stakeholders and public policy.
8 References


Corbae, Dean and Pablo D’Erasmo, 2011, “A Quantitative Model of Banking Industry Dynamics,” Mimeo


Duffie, Darrell, 2010, How Big Banks Fail and What to Do about It, Princeton University Press.

Freixas, Xavier and Jean-Charles Rochet, 2008, Microeconomics of Banking, MIT Press.


9 Data Appendix

9.1 Call reports

The analysis draws on a sample of individual bank data from the Reports of Condition and Income (Call Reports) for the period 1989:Q1-2010:Q4. For every quarter, we categorize banks in three size categories: banks of size 1 are those below the 95th percentile of the distribution of total assets in the given quarter, of size 2 those between the 95th and 98th percentile and size 3 those above the 98th percentile.

Our initial dataset is a panel of 890,252 quarterly observations corresponding to 17,226 different identification numbers of U.S. commercial banks. We drop 38,563 observations that
have zero FDIC identification number and 4,313 observations due to missing values. We exclude the effect of exceptional growth in bank size (e.g. due to mergers and acquisitions or winding down of bank activities) by winsorizing at the 1st and 99th percentile of the sample distribution of growth rates in customer loans and tangible assets at every quarter. By this criterion we drop 25,292 outlier observations and also 22,647 observations due to missing values in growth rates. The final sample is a panel of 799,437 quarterly observations from 16,564 uniquely identified U.S. commercial banks.

The loan write-off ratio (the analog of $w$ in the model) is calculated by dividing quarterly loan charge-offs by lagged gross loans, where the latter is defined as total loans plus quarterly charge-offs. Real deposit growth is calculated by taking the log difference in broad deposits, where the latter is defined as the sum of transaction and non-transaction deposits. In order to avoid the impact of outliers when estimating the exogenous processes for loan write-offs and real deposit growth, we winsorize their sample distributions at the 1st and 99th percentile every quarter, by bank size. The autoregressive processes for loan write-offs and deposit growth are estimated at the individual bank level, considering only banks with at least 35 observations in booms and 35 observations in recessions, i.e. at least 70 observations in total. For deposit growth in particular, the autoregressive process is estimated taking into account seasonal effects at a bank level by adding quarterly dummies. The model parameters that we consider for the autoregressive processes are the averages of the estimated ones across banks by size, after winsorizing them at the 1st and 99th percentile of their estimated sample distribution.

We also calculate first and second moments of balance sheet and profit and loss variables

---

30 Tangible assets equal total assets minus intangible assets, such as goodwill.
that we target for estimating the model using a Method of Simulated Moments. With respect to balance sheet variables, we consider the ratios of broad deposits, wholesale funding, liquid assets and tangible equity over tangible total assets, as well as the ratios of tangible total assets and total loans over tangible equity, if the latter is strictly positive.\footnote{Tangible equity equals tangible assets minus total liabilities.} Regarding profit and loss variables, we consider the ratios of quarterly profits (before tax, extraordinary items and other adjustments) and dividends over tangible equity. In order to derive targeted moments for these ratios, we first winsorize the ratios at the 1st and 99th percentile of their sample distribution every quarter, by bank size. Moments of ratios are calculated at an individual bank level by considering only banks with at least 20 observations in booms and equally in recessions, i.e. at least 40 observations in total. For the Method of Simulated Moments estimation we use average moments across banks, as well as the standard deviations around these averages for weighting purposes.

The same approach as for targeted moments of ratios is used for estimating average real return on loans, liquid-asset returns and deposits rates from individual bank data. For loan returns we use the ratio of quarterly interest income on loans over lagged loans. For liquid-asset returns we use the ratio of quarterly interest income on Fed funds sold and reverse repo plus gains or losses on securities over lagged liquid assets. For deposit rates we use the ratio of quarterly interest expense on deposits over lagged deposits.

To calculate the fraction of loans that are repaid every quarter (the analog of $\theta$ in the model), we use one fourth of the ratio of loans that mature in less than 1 year divided by total loans outstanding. The resulting average estimate is 7\% which assumes both a uniform repayment rate over time and that banks cannot take action to scale down their existing
loans before maturity, e.g. by outright loan sales, or securitization. It also assumes no difference in the ability of small and large banks to delever, e.g. due to different access to loan sales and securitization technology.

We have also identify 670 bank failures, which are basically all bank failures reported by the FDIC for the period 1991Q1-2010Q4. For the second half of this period (i.e. 2000Q4-2010Q4), names and FDIC identification numbers of failed banks were obtained from an FDIC list. For the first half of the period (i.e. 1990Q1-2000Q3), names of failed banks were obtained manually from FDIC reports. For those banks, we are able to uniquely identify their FDIC identification numbers from Call Reports by matching bank-name, city and state information. From the 670 bank failures reported by the FDIC, it was not possible to identify the FDIC identification numbers in 59 cases. As a result, the number of bank failures considered was reduced to 611. Among those, 19 failed banks had the same FDIC identification number with other banks in Call Reports and were dropped from the sample, reducing the number of bank failures considered to 568.

9.2 Capital adequacy rules and leverage ratio

Basel II defines different risk weights for different asset classes. The risk weight on government bonds is 0% and on safe financial assets 20%. Banks in our sample hold approximately 50% of their liquid assets in each of these categories. Therefore, we use a risk weight \( \omega^* \) equal to 10% . The risk weights on residential mortgages are 35%, on commercial real estate 100%, and on personal loans, including small business loans, 75%. Given the average loan portfolio

---

in our sample, we use a risk weight weight $\omega^s$ equal to 65%.

### 9.3 Further empirical results

In this section, we report the empirical results we have left out in the main text, which we did to preserve space.

Figure 23 is the equivalent to Figure 6 and shows the deposit to asset ratio and loan to asset ratio of big banks.

The deposit to asset ratio is much more dispersed for big banks, reflecting that some of them rely significantly on wholesale funds. The loan to asset ratio, however, has a similar distribution to the one for small banks. The mean is almost identical and most banks have a ratio between 40% and 80%. The distribution is not as smooth, at least partly, because there are around 35 times less observations for big than for small banks.
10 Solution Appendix

This section first shows the normalization of the model and then the computational approach to solve it numerically.

10.1 Normalization

The deposit process contains a unit root but is i.i.d. in growth rates. Therefore we normalize the entire model by deposit growth rates. For this approach to work, all equations have to be homogenous of degree 1. Denote normalized variables as lower case variables, for example \( f_t = \frac{F_t}{D_t} \) and the growth rate of deposits with \( \Gamma_{t+1} = \frac{D_{t+1}}{D_t} \).

The leverage limit (2) becomes
\[
\frac{l_t + n_t + s_t}{e_t - x_t} \leq \lambda. \tag{8}
\]

The equity evolution (1) becomes
\[
e_{t+1} \equiv \frac{E_{t+1}}{D_{t+1}} = \frac{E_t - X_t + \Pi_{t+1}}{D_{t+1}} = (e_t - x_t) \frac{1}{\Gamma_{t+1}} + \pi_{t+1}. \tag{9}
\]

Profits (4) become
\[
\pi_{i,t+1} = (r_{L,t+1} - w_{i,t+1})l_{it} + n_{it} + r_{S,t+1}s_{it} - r_{D,t+1}D_{it} - g_N(n_{it}) - g_F(f_{it}) - c. \tag{10}
\]

10.2 Computational appendix

After the normalization there are 2 continuous state variables: normalized equity \( e_t \) and normalized loans \( l_t \). The aggregate state is approximated by a two state Markov chain, where the good state is interpreted as a boom, and the bad state as a recession. The transition
probabilities are chosen to generate boom and recessions that last, on average, 5 and 2 years, respectively. The state dependent stochastic process for bad loans follows an AR(1) process which is discretized using the procedures described by Adda and Cooper (2003). The numerical solution algorithm is as follows.

1. Values for all exogenous parameters are assigned.

2. Two grids are made for the two continuous state variables equity $e$ and loans $l$.

3. A sequence for all shocks for the simulation is drawn.

4. Values for the four estimated parameters are assigned.

   The remaining computational steps have two components: solution of the value functions and simulation.

**Solution of value function problem**

5. The value for consumption after failure $ar{c}$ implies a continuation value after failure $V^d$

6. A guess is made for the (normalized) value function $v(l, e, w, r, g)$

7. The optimization problem is solved for all discrete states: boom and recession, and nodes for bad loans and for all values on the grids for $e$ and $l$. At each such node, the bank chooses dividends $x$, new loans $n$, liquid assets (securities) $s$ and wholesale borrowing $f$ simultaneously to maximize the normalized value function. The details for this step are as follows:

   (a) at each node $(e, l)$ three nested grids are made for $(x, n, f)$, $s$ follows from the balance sheet constraint that $s = 1 + f + e - x - l - n$. 

64
(b) if the candidate for \((x, n, f)\) violates the leverage limit, the bank is closed down and the failure utility level \(V^d\) is assigned.

(c) if the candidate for \((x, n, f)\) is feasible and obeys the leverage limit, a loop is made over all possible future states and profits in each state are calculated. The shocks and the choices imply a certain level of profits in each state which leads to a different level of equity and loan \((l', e')\) in the future period. The continuation value is computed in each of these states. This is either \(V (l', e', w', r', g')\) or if failure is preferred \(V^d\).

(d) Since future values of \((l', e')\) will not, in general, lie on the grid, a two-dimensional linear interpolation routine is chosen to obtain the values \(V (l', e', w', r', g')\) at this node.\(^{34}\)

8. The solution to the optimization problem at each node provides an update value function \(\tilde{v} (l, e, w, r, g)\).

(a) if \(\tilde{v} (l, e, w, r, g)\) is close to \(v (l, e, w, r, g)\) at every single node, i.e. if the maximum absolute difference is below the tolerance level, the value function has converged;

(b) otherwise the value function has not converged and \(v (l, e, w, r, g)\) at the beginning of step 7 is replaced with \(\tilde{v} (l, e, w, r, g)\) and step 7 repeated.

9. After convergence the decision rules for dividends \(x\), new loans \(n\), and wholesale borrowing \(f\) are saved for the simulation

Simulation

\(^{34}\)Linear interpolation is chosen because, being a local method, it is more stable than, for example, cubic splines.
10. The previously drawn shock sequences and the saved decision rules are used to simulate

\[ N = 100,000 \] banks for \( T = 2,000 \) periods.

11. Each bank starts with some specific initial value for \((e_t, l_t)\) aggregate state and idiosyncratic bad loan state. The decision rule is then used to compute new loans \(n_t\), dividends \(x_t\), wholesale borrowing \(f_t\). The shocks \(t + 1\) are realized which in turn yield profits \(\pi_{t+1}\). This yields new equity \(e_{t+1} = e_t + \pi_{t+1} - x_t\). Similarly, the state of loans next period is \(l_{t+1} = (1 - \vartheta - w_{t+1})l_t + n_t\).

12. A bank that fails during the simulation is replaced by a new one which starts with mean equity and mean loans.

13. After the simulation is concluded the first 1,500 periods are excluded and all statistics reported are calculated based on the last 500 periods.

14. The criterion function of the estimation is calculated.

   (a) The squared differences between model and data moments are calculated.

   (b) These are weighted by the efficient weighting matrix which uses the standard deviations of the empirical moments.

15. If the criterion function is too high, a new set of values is tried in step 4. For this optimization, we use a standard derivative-free simplex method.

---

\[ ^{35} \text{However, due to the very low number of defaults, this choice has no influence on aggregate statistics.} \]